# PARAMETERS ESTIMATION OF SANDWICH BEAM MODEL WITH RIGID POLYURETHANE FOAM CORE

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**Abstract.** In this work the physical parameters of sandwich beams made with the association of hot-rolled steel, Polyurethane rigid foam and High Impact Polystyrene, used for the assembly of household refrigerators and food freezers are estimated using measured and numeric frequency response functions (FRFs). The mathematical models are obtained using the Finite Element Method (FEM) and the Timoshenko beam theory. The physical parameters are estimated using the amplitude correlation coefficient and Genetic Algorithm (GA). The experimental data are obtained using the impact hammer and four accelerometers displaced along the sample (cantilevered beam). The parameters estimated are the Young modulus and the loss factor of the Polyurethane rigid foam and the High Impact Polystyrene.

Keywords: sandwich beam, genetic algorithm, amplitude correlation coefficient, parameter updating.

# **1. INTRODUCTION**

The proved efficiency of sandwich beams and its current usage in a growing rate demands a higher level of acknowledgment of the mechanical properties, even when the structure is submitted to dynamic loads. For household refrigerators and food freezers, one of the main complaints to the customer care centers is related to noise generation, that is related most of the times with vibration of the cabinet that produces sound irradiation from internal components like shelves and containers, leaking to the outside of the unit.

The efficient numerical models are necessary to simulate (estimate) the dynamical behavior of such systems. When the complex sandwich beams are used the physical parameters are difficult to be estimated.

Sandwich structures are extensively used in engineering because of their high specific stiffness and strength. The modeling of sandwich structures has been studied extensively, but less attention has been paid to their material identification (Shi et al., 2006). The work proposes an inverse method for the material identification of sandwich beams by measured flexural resonance frequencies.

Caracciolo et al. (2004) presented an experimental technique for completely characterizing a viscoelastic material, by determining the Poisson ratio and the complex dynamic Young's modulus of a small beam-like specimen subject to seismic excitation, together with the theoretical background. The same experimental device is used basically for both kinds of tests: the specimen is instrumented, placed into a temperature controlled chamber and excited by means of an electrodynamic shaker. The longitudinal and the transversal deformations are measured by strain gauges to get the Poisson ratio, whereas the vertical displacement of the specimen and the acceleration of the support are measured to get Young's modulus of the tested material. The experimental curves of the Poisson ratio and of Young's modulus, obtained at different temperatures, are then gathered into a unique master curve by using the reduced variables method. The two master curves, respectively, represent the Poisson ratio and Young's modulus for the tested material in a very broad frequency range.

Park (2005) used experimental methods to measure frequency-dependent dynamic properties of complex structures. Flexural wave propagations are analyzed using the Timoshenko beam, the classical beam, and the shear beam theories. Wave speeds, bending and shear stiffnesses of the structures are measured through the transfer function method requiring small number of vibration measurements. Sensitivity analysis to investigate the effects of experimental variables on the measured properties and to study optimal sensor locations of the vibration measurements is performed. Using the developed methods, the complex bending and shear stiffnesses of sandwich beams of different core materials and a polymer beam are measured. Continuous variations of the measured bending and shear stiffnesses and their loss factors with frequency were obtained. To further illustrate the measurements of frequency-dependent variation of dynamic properties of complex structures, the damping of structural vibration using porous and granular materials is investigated.

Kim and Kreider (2006) studied the parameter identification in nonlinear elastic and viscoelastic plates by solving an inverse problem numerically. The material properties of the plate, which appear in the constitutive relations, are recovered by optimizing an objective function constructed from reference strain data. The resulting inverse algorithm consists of an optimization algorithm coupled with a corresponding direct algorithm that computes the strain fields given a set of material properties. Numerical results are presented for a variety of constitutive models; they indicate that the methodology works well even with noisy data.

Pintelon et al. (2004) analyzed the stress-strain relationship of linear viscoelastic materials characterized by a complex-valued, frequency-dependent elastic modulus (Young's modulus). Using system identification techniques it is shown the elastic modulus can be measured accurately in a broad frequency band from forced flexural (transverse) and longitudinal vibration experiments on a beam under free-free boundary conditions. The approach is illustrated on brass, copper, plexiglass and PVC beams.

Yang et al. (2005) analyzed the vibration and dynamic stability of a traveling sandwich beam using the finite element method. The damping layer is assumed to be linear viscoelastic and almost incompressible. The extensional and shear moduli of the viscoelastic material are characterized by complex quantities. Complex-eigenvalue problems are solved by the state-space method, and the natural frequencies and modal loss factors of the composite beam are extracted. The effects of stiffness and thickness ratio of the viscoelastic and constrained layers on natural frequencies and modal loss factors are reported. Tension fluctuations are the dominant source of excitation in a traveling sandwich material, and the regions of dynamic instability are determined by modified Bolotin's method. Numerical results show that the constrained damping layer stabilizes the traveling sandwich beam.

Singh et al. (2003) formulated a system identification procedure for estimation of parameters associated with a dynamic model of a single-degree-of-freedom foam-mass system. Ohkami and Swoboda (1999) presented two parameter identification procedures for linear viscoelastic materials. Chang (2006) uses the genetic algorithm for parameters estimation of nonlinear systems.

Backström and Nilsson (2007) indicate the need for simple methods describing the dynamics of these complex structures. By implementing frequency-dependent parameters, the vibration of sandwich composite beams can be approximated using simple fourth-order beam theory. A higher-order sandwich beam model is utilized in order to obtain estimates of the frequency-dependent bending stiffness and shear modulus of the equivalent Bernoulli–Euler and Timoshenko models. The resulting predicted eigenfrequencies and transfer accellerance functions are compared to the data obtained from the higher-order model and from measurements. It can be noticed that for lower order wavenumber the ordinary Timoshenko theory and the higher order theory show satisfactory agreement.

In this work the physical parameters of sandwich beams made with the association of hot-rolled steel, Polyurethane rigid foam and High Impact Polystyrene, used for the assembly of household refrigerators and food freezers are estimated using measured and numeric frequency response functions (FRFs). The mathematical models are obtained using the Finite Element Method (FEM) and the Timoshenko beam theory. The physical parameters are estimated using the amplitude correlation coefficient (Grafe, 1998) and Genetic Algorithm (GA) (Chang, 2006). The experimental data are obtained using the impact hammer and four accelerometers displaced along the sample (cantilevered beam). The parameters estimated are the Young modulus and the loss factor of the Polyurethane rigid foam and the High Impact Polystyrene. To estimate the initial values of the parameters, separated tests were conduced using cantilevered beams of Polyurethane rigid foam and High Impact Polystyrene.

#### 2. MATHEMATICAL MODEL

A lot of research has been done on Finite Element Models of cantilever beams based on Euler Bernoulli beam theory. In Euler Bernoulli beam theory the assumption made is, plane cross section before bending remains plane and normal to the neutral axis after bending. This assumption is valid if length to thickness ratio is large and for small deflection of beam. However if length to thickness ratio is small, plane deflection before bending will not remain normal to the neutral axis after bending. In practical situations a large number of modes of vibrations contribute to the structure's performance. Euler Bernoulli beam theory gives inaccurate results for higher modes of vibration. Timoshenko beam theory corrects the simplifying assumptions made in Euler Bernoulli beam theory. In this theory cross sections remain plane and rotate about the same neutral axis as the Euler Bernoulli model, but do not remain normal to the deformed longitudinal axis. The deviation from normality is produced by a transverse shear that is assumed to be constant over the cross section. Thus, the Timoshenko beam model is superior to the Euler Bernoulli model.

The equation of motion for the vibration of a beam according to the Timoshenko beam theory (Zenkert, 1995) is:

$$D\frac{\partial^4 w}{\partial x^4} + \rho^* \frac{\partial^2 w}{\partial t^2} - \frac{\rho^*}{S} \left( D\frac{\partial^4 w}{\partial x^2 \partial t^2} - \Re \frac{\partial^4 w}{\partial t^4} \right) - \Re \frac{\partial^4 w}{\partial x^2 \partial t^2} = f(x)e^{i\omega t}$$
(1)

where: w(x,t) is the transverse displacement of the beam from an equilibrium state; D is bending stiffness;  $\rho *$  is the mass per unity of surface; S is the shear stiffness;  $\Re$  is the rotational inertia; x is the coordinate along the beam axis; t is the time; f(x) is the amplitude of the external force applied along the beam span;  $\omega$  is the excitation frequency and  $i = \sqrt{-1}$ .

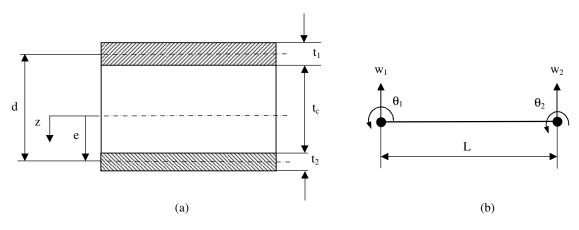


Figure 1-a) Sandwich beam geometric parameters, b) finite element d.o.f.

The dimensions and parameters of sandwich beam showed in Fig. 1a are:  $E_1$  = Young Modulus;  $\rho_1$  = density and  $t_1$  = thickness (steel);  $E_2$  = Young Modulus;  $\rho_2$  = density and  $t_2$  = thickness (High Impact Polystyrene);  $E_c$  = Young Modulus,  $G_c$  = shear Modulus;  $\rho_c$  = density and  $t_c$  = thickness (Polyurethane rigid foam); e = position of the neutral line; d = distance between centerline of the steel and High Impact Polystyrene beam; z and z<sup>\*</sup> positions of the reference axis. According Fig. 1a and detailed theory of sandwich beam (Zenkert, 1995):

$$d = t_c + \frac{t_1}{2} + \frac{t_2}{2}$$
(2)

$$e = \left(E_{1}t_{1}\left(\frac{t_{1}}{2} + t_{c} + \frac{t_{2}}{2}\right) + E_{c}t_{c}\left(\frac{t_{c}}{2} + \frac{t_{2}}{2}\right)\right) / [E_{1}t_{1} + E_{c}t_{c} + E_{2}t_{2}]$$
(3)

$$D = \frac{E_1 t_1^3}{12} + \frac{E_2 t_2^3}{12} + \frac{E_c t_c^3}{12} + E_1 t_1 (d - e)^2 + E_2 t_2 e^2 + E_c t_c \left(\frac{t_c + t_2}{2} - e\right)^2$$
(4)

$$\rho^* = \rho_1 t_1 + \rho_2 t_2 + \rho_c t_c$$
(5)

$$\Re = \frac{\rho_1 t_1^3}{12} + \frac{\rho_2 t_2^3}{12} + \frac{\rho_c t_c^3}{12} + \rho_1 t_1 (d - e)^2 + \rho_2 t_2 e^2 + \rho_c t_c \left(\frac{t_c + t_2}{2} - e\right)^2$$
(6)

$$S = \frac{G_c d^2}{t_c}$$
(7)

Making w = w(x,t) an harmonic function, it is possible to admit that:

$$w(x,t) = W(x)e^{i\omega t}$$
(8)

Replacing Equation (8) into (1) we obtain:

$$D\frac{\partial^4 W}{\partial x^4} - \rho^* \omega^2 W(x) - \frac{\rho^*}{S} \left( -D\omega^2 \frac{\partial^2 W}{\partial x^2} - \Re \omega^4 W(x) \right) + \Re \omega^2 \frac{\partial^2 W}{\partial x^2} = f(x)$$
(9)

The exact solution W(x) needs to satisfy (9) at every point x and in general is unknown. We seek instead an approximate solution  $\widetilde{W}(x)$ . This approximate solution is interpolated over a finite element, Fig.1b, with 2 nodes according to the formula:

$$\widetilde{\mathbf{W}}(\mathbf{x}) = [\phi] \{\mathbf{q}\} \tag{10}$$

where  $[\phi(x)]$  is the shape function matrix (1x4) and the four  $\phi_j(x)$  are the well known Hermitian interpolation functions (Cook et al., 1989) for C<sup>1</sup> continuity. The vector  $\{q\}$  is the generalized displacement vector,  $\{q\}=\{w_1, \theta_1, w_2, \theta_2\}^t$  where  $w_i$  denote the nodal displacement and  $\theta_i$  is its first derivatives at element node i.

Replacing the approximate solution into (9) it introduces an residual error,  $\epsilon(x,\omega)$ , which is minimized using the Galerkin weighted residual method. In mathematical terms, the residual error is made orthogonal to the weight functions:

$$\int_{0}^{L} [\phi]^{t} \left( D \frac{\partial^{4} \tilde{W}}{\partial x^{4}} - \rho^{*} \omega^{2} \tilde{W}(x) - \frac{\rho^{*}}{S} \left( -D \omega^{2} \frac{\partial^{2} \tilde{W}}{\partial x^{2}} - \Re \omega^{4} \tilde{W}(x) \right) + \Re \omega^{2} \frac{\partial^{2} \tilde{W}}{\partial x^{2}} - f(x) \right) dx = 0$$
(11)

where L is the element length.

After integrations can be obtained the standard finite element equation:

$$\left[\mathbf{K}_{e}(\boldsymbol{\omega})\right]\left\{\mathbf{q}\right\} = \left\{\mathbf{F}\right\} \tag{12}$$

where:

$$\left[\mathbf{K}_{e}(\boldsymbol{\omega})\right] = \mathbf{D}\left[\mathbf{K}\right] - \boldsymbol{\omega}^{2} \left[\boldsymbol{\rho}^{*}\left[\mathbf{M}\right] + \left(\frac{\boldsymbol{\rho}^{*}\mathbf{D}}{\mathbf{S}} + \boldsymbol{\Re}\right)\left[\mathbf{K}_{\sigma}\right]\right] + \boldsymbol{\omega}^{4} \frac{\boldsymbol{\rho}^{*}\boldsymbol{\Re}}{\mathbf{S}}\left[\mathbf{M}\right]$$
(13)

The matrices and [K], [M] and  $[K_{\sigma}]$  are (Cook et al, 1989):

$$[\mathbf{K}] = \frac{1}{\mathbf{L}^3} \begin{bmatrix} 12 & 6\mathbf{L} & -12 & 6\mathbf{L} \\ 6\mathbf{L} & 4\mathbf{L}^2 & -6\mathbf{L} & 2\mathbf{L}^2 \\ -12 & -6\mathbf{L} & 12 & -6\mathbf{L} \\ 6\mathbf{L} & 2\mathbf{L}^2 & -6\mathbf{L} & 4\mathbf{L}^2 \end{bmatrix}$$
(14)

$$\begin{bmatrix} K_{\sigma} \end{bmatrix} = \frac{1}{30L} \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix}$$

$$\begin{bmatrix} M \end{bmatrix} = \frac{1}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L \\ 54 & 13L & 156 & -22L \\ -13L & -3L & -22L & 4L^2 \end{bmatrix}$$
(15)

and

$$\left\{F\right\} = \int_{0}^{L} f(x)[\phi]^{t} dx$$
(17)

## 2.1. Numerical estimation methods

To approximate the experimental and numeric FRF data, the predictor-corrector updating technique (Grafe,1998) based in two correlation coefficients (shape and amplitude) and their sensitivities can be used. In this work, only the amplitude correlation coefficient is used. This coefficient is defined as:

$$\chi_{a}(\omega_{k}) = \frac{2\left[\left\{H_{X}(\omega_{k})\right\}^{T}\left\{H_{A}(\omega_{k})\right\}\right]}{\left[\left\{H_{X}(\omega_{k})\right\}^{T}\left\{H_{X}(\omega_{k})\right\}\right]\left[\left\{H_{A}(\omega_{k})\right\}^{T}\left\{H_{A}(\omega_{k})\right\}\right]}$$
(18)

where  $H_X(\omega_k)$  and  $H_A(\omega_k)$  are the measured and predicted response vectors at matching excitation/response locations.

The corresponding sensitivity is:

$$\frac{\partial \chi_{a}(\omega_{k})}{\partial \phi} = 2 \frac{\partial \left[ \{H_{x}\}^{T} \{H_{A}\} \right]}{\partial \phi} \frac{\left( \{H_{x}\}^{T} \{H_{x}\} + \{H_{A}\}^{T} \{H_{A}\} \right)}{\left( \{H_{x}\}^{T} \{H_{x}\} + \{H_{A}\}^{T} \{H_{A}\} \right)^{2}} - 2 \frac{\partial \left( \{H_{A}\}^{T} \{H_{A}\} \right)}{\partial \phi} \frac{\left| \{H_{x}\}^{T} \{H_{A}\} \right|}{\left( \{H_{x}\}^{T} \{H_{A}\} \right)^{2}}$$
(19)

It is therefore proposed to make use of  $\chi_a(\omega_k)$  and its sensitivity in a combined manner to improve the overall level of correlation. Based on a truncated Taylor series expansion, one can write therefore one equation for frequency point  $\omega_k$ :

$$\left\{1 - \chi_{a}(\omega_{k})\right\} = \left[\frac{\partial \chi_{a}(\omega_{k})}{\partial \varphi_{1}} \quad \frac{\partial \chi_{a}(\omega_{k})}{\partial \varphi_{2}} \quad \dots \quad \frac{\partial \chi_{a}(\omega_{k})}{\partial \varphi_{N_{\phi}}}\right]_{k = N_{\phi}} \left\{\Delta \varphi\right\}$$
(20)

where  $N_{\phi}$  is the number of updating parameters and equation (20) is recognized to be in the standard form of sensitivity-based model updating formulations:

$$\{\varepsilon\} = [S]\{\Delta\phi\}$$
(21)

An extended weighted least-square approach is proposed which minimizes:

$$\mathbf{J}(\{\boldsymbol{\varphi}\}) = \{\boldsymbol{\varepsilon}\}^{\mathrm{T}} [\mathbf{W}_{\mathrm{f}}] \{\boldsymbol{\varepsilon}\} + \{\Delta \boldsymbol{\varphi}\}^{\mathrm{T}} [\mathbf{W}_{\boldsymbol{\varphi}}] \{\Delta \boldsymbol{\varphi}\}$$
(22)

where  $[W_f]$  and  $[W_{\phi}]$  are diagonal weighting matrices for the frequency points and updating parameters respectively.

Another update method uses Genetic Algorithm (GA). This method is vastly used and it is based in evolutionary biological process (Chang, 2006) and the GA's parameters used in this application are: mutation rate=0,02; population size=50 and number of generations = 5000. The objective function is defined by:

$$f = \sum_{i=1}^{np} \left| FRF_{exp.} - FRF_{FEM} \right|$$
(23)

where  $FRF_{exp}$  is the FRF experimental obtained with laser transducer or accelerometer;  $FRF_{FEM}$  is the numeric FRF estimated using GA or amplitude correlation coefficient; np is the number of points (normally np = 1600). The data were collected using frequency range varying from 0 to 400 Hz and the frequency increment  $\Delta \omega = 0.25$  Hz.

FRF numeric is obtained using frequency sweeping in the range of interest with the same increment of the measurements. The final finite elements system of equations is solved for each frequency and the unitary external force applied in the node of the excitation. Adopting this procedure, the FRF's numeric values correspond to the acceleration of the nodes of fixation of accelerometers because the amplitude of force is unitary.

## **3. RESULTS**

To validate the mathematical model was considered the three layered cantilever sandwich beam model and the results are compared with Helgesson (2003), Mead and Markus (1969), Ahmed (1972), Sakiyama et al. (1996). The beam specific properties such as width, height, modulus of elasticity, etc. are chosen to be the same as the former investigators in order to be able to compare the results.

The specific dimensions and material properties of the sandwich beam are:

Length of beam L= 0.7112 m ; Modulus of elasticity E =  $6.89 \times 10^{10}$  N/m<sup>2</sup> ; Core shear modulus G = $82.68 \times 10^{6}$  N/m<sup>2</sup>; Thickness of faces h<sub>1</sub> = h<sub>3</sub> =0.45720 mm ; Thickness of core h<sub>2</sub> = 12.7 mm ; Core density  $\rho_2$  = 32.8 kg/m<sup>3</sup> ; Face density  $\rho_1 = \rho_3 = 2680$  kg/m<sup>3</sup> ; Width of beam = 1 m.

The out come of the program and analysis are natural modes and frequencies. The numerical results of the natural mode frequencies or eigen-frequencies are presented and compared with earlier investigators in Table 1.

| Mode | Present | Helgesson | Mead    | Ahmed  | Sakiyama |
|------|---------|-----------|---------|--------|----------|
| 1    | 33.75   | 33.75     | 34.24   | 32.79  | 33.15    |
| 2    | 198.81  | 198.77    | 201.85  | 193.50 | 195.96   |
| 3    | 511.45  | 511.27    | 520.85  | 499    | 503.43   |
| 4    | 905.34  | 904.87    | 925.40  | 886    | 893.28   |
| 5    | 1346.58 | 1345.60   | 1381.30 | 1320   | 1328.50  |
| 6    | 1811.94 | 1810.30   | 1867    | 1779   | 1790.70  |
| 7    | 2288.17 | 2285.80   | 2374    | 2249   | 2260.20  |
| 8    | 2767.82 | 2764.70   | 2905    | 2723   | 2738.90  |
| 9    | 3247.08 | 3243.30   | -       | -      | 3212.80  |

Table 1. Natural frequency (Hz) and comparison with earlier authors.

The resulting natural frequencies in Tab. 1 compare quite well with the results obtained by earlier investigators, principally with Helgesson. Worth noting is that the obtained frequencies are always below Mead and always above Ahmed and Sakiyama. The fact that frequencies of Mead are always above the results from this work is due to the fact that Mead simplifies the harmonic forces by using complex damped modes. Ahmed has not considered axial forces and the theory presented is therefore softer then the present hence is the frequencies below the present. Sakiyama has derive the governing differential equations by using the Green function and complex shear module  $G = G_0 (1 + i\eta)$  (Helgesson, 2003) where  $\eta$  is the loss factor. Figure 2 shows the convergence rate for the first vibration mode. The fast convergence is obtained using the mathematical model with 300 finite elements. Higher modes in Table 1 presented the same convergence behavior.

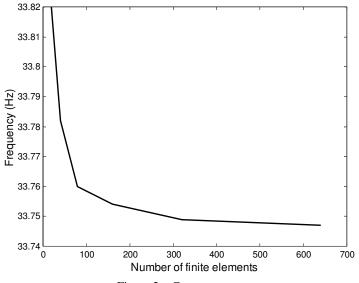
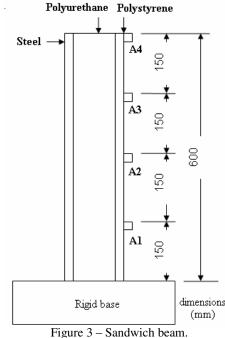


Figure 2 - Convergence rate

The experimental sample of sandwich beam made with the association of hot-rolled steel, Polyurethane rigid foam and High Impact Polystyrene is shown in Fig. 3. The thickness of the steel is 0.6 mm; the Polyurethane is 38.25 mm and the Polystyrene is 1.25 mm and the beam width is 39.18 mm.



The experimental data are obtained using the impact hammer and four accelerometers displaced along the sample (A1, A2, A3 and A4). The Rational Fraction Polynomial (RFP) (Maia and Silva, 1997) method was used to estimate the damping ratio ( $\xi$ ) and the natural frequencies ( $\omega$ ) of the three mode shapes. Table 2 shows the values of these parameters to the four accelerometers.

|       | Accelerometers |        |        |        |        |        |        |        |  |
|-------|----------------|--------|--------|--------|--------|--------|--------|--------|--|
| Mode  | A1             |        | A2     |        | A3     |        | A4     |        |  |
| shape | ω              | ξ      | ω      | ګ      | ω      | ξ      | ω      | ګ      |  |
|       | [Hz]           |        | [Hz]   |        | [Hz]   |        | [Hz]   |        |  |
| 1     | 25.42          | 0.050  | 25.39  | 0.050  | 25.33  | 0.052  | 25.32  | 0.053  |  |
| 2     | 109.41         | 0.0171 | 109.45 | 0.0165 | -      | -      | 109.39 | 0.0167 |  |
| 3     | 223.97         | 0.0154 | 224.00 | 0.0152 | 224.05 | 0.0159 | 224.18 | 0.0157 |  |

Table 2 – Experimental damping ratio and natural frequencies.

The position of the accelerometer A3 is near to the nodal point of the second mode shape. This justifies the results suppressed in Table 2.

According to the results shown in Table 2, it tried estimate some physical parameters of the system: Young modulus and the loss factor of the Polyurethane rigid foam and the High Impact Polystyrene.

The loss factor  $\eta$  was estimated considering the complex Young modulus  $E^* = E(1 + j\eta)$ .

To obtain initial values of these parameters separated studies were conduced to the Polyurethane rigid foam and High Impact Polystyrene. The first approximation to the loss factor was  $\eta = \xi \cong 0.05$ .

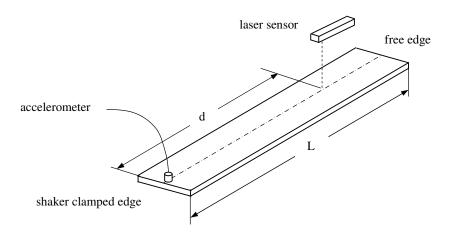


Figure 4 - Experimental specimen of cantilever beam with position sensor

Figure 4 shows the experimental specimen for the isolated High Impact Polystyrene and Polyurethane rigid foam cantilever beam. The High Impact Polystyrene beam dimensions and material property are: length L = 0.145m; width = 0.02 m; thickness = 0.0018 m; density  $\rho = 1040 \text{ kg/m}^3$ . The mini shaker was used to the beam random excitation with the frequency range varying from 0 to 400 Hz. One accelerometer (PCB model 353B18) and one laser velocity transducer (B&K model 3544) were used to collect the vibration data. The accelerometer was placed in the base excitation point (shaker) and the laser sensor to the position d=0.135 m. Figure 5 shows the experimental and estimated curves of the acceleration ratio (non-dimensional parameter) Paa (acceleration of laser sensor / acceleration of the accelerometer). The parameters were estimated using the Genetic Algorithm and the objective function as being the difference between the values of the experimental and numeric Paa. The parameter updated is the Young modulus of the High Impact Polystyrene and the loss factor. The optimal values found for these parameters are E = 1.425 GPa e  $\eta = 0.045$ .

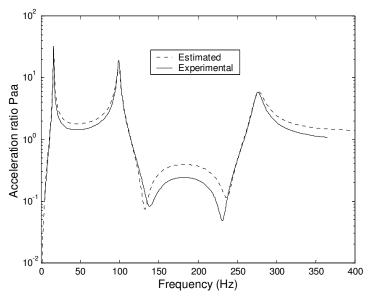


Figure 5 - Acceleration ratio curves of High Impact Polystyrene cantilever beam.

The Polyurethane rigid foam cantilever beam dimensions and material property are: length L = 0.225m; width = 0.03 m; thickness = 0.03 m; density  $\rho = 29 \text{ kg/m}^3$ . The laser sensor position is d=0.1125 m. Figure 6 shows the experimental e estimated curves of the acceleration ratio (non-dimensional parameter) Paa (acceleration laser sensor / acceleration of the accelerometer). The parameter updated is the Young modulus of the Polyurethane rigid foam and the loss factor. The optimal values found for these parameters are E = 8.394 MPa e  $\eta = 0.045$ .

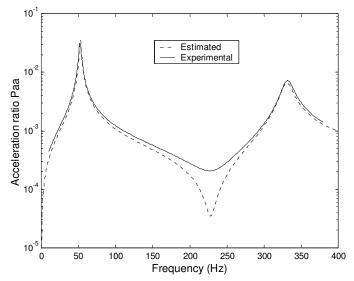


Figure 6 - Acceleration ratio curves of Polyurethane rigid foam cantilever beam

Figures 5 and 6 show good agreement between the estimated and experimental curves, principally near to the resonances.

The physical parameters of the cantilever beam shown in Fig. 3 were estimated using the amplitude correlation coefficient (ACC) and Genetic Algorithm (GA). The frequency range varying from 0 to 250 Hz was choosing due to the good relation signal/noise. Figure 7 shows the experimental and numeric (estimated) FRF curves for the sandwich beam (Fig. 3). The curves obtained using ACC and GA are practically superposed.

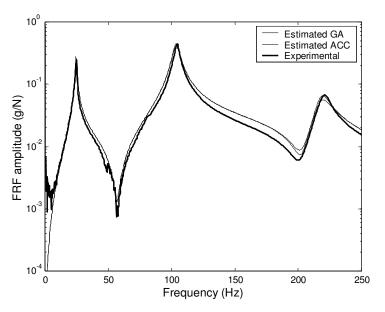


Figure 7 – Experimental and estimated FRF curves.

The optimal parameter values found with the two numeric methods are shown in Table 3.

Table 3 – Optimal parameters

|     | High Impact | Polystyrene | Polyurethane rigid foam |        |  |
|-----|-------------|-------------|-------------------------|--------|--|
|     | E (GPa)     | η           | E (MPa)                 | η      |  |
| GA  | 1.5830      | 0.0446      | 9.5159                  | 0.0645 |  |
| ACC | 1.5800      | 0.0471      | 9.5284                  | 0.0635 |  |

#### 4. CONCLUSIONS

Two methods, Genetic Algorithm and amplitude correlation coefficient, were used to update the values of physical parameters of mathematical models of sandwich beam made with the association of hot-rolled steel, Polyurethane rigid foam and High Impact Polystyrene, used for the assembly of household refrigerators and food freezers.

The physical parameters estimated were: the Young modulus and the loss factor of the Polyurethane rigid foam and the High Impact Polystyrene.

Both methods, Genetic Algorithm and Amplitude Correlation Coefficient, presented good results when it is compared the estimated and experimental FRF curves.

Genetic Algorithm method does not use derivatives, thus is a good estimated method even for resonance region.

Amplitude Correlation Coefficient method use derivatives, even so it was possible to obtain good estimation of the parameters.

The conventional Timoshenko beam theory was applied to model the sandwich beam and the numeric response presented good results when compared with results of literature and experimental data (only three vibration modes). At the moment other theories are being studied and implemented for evaluation of the effect of compression of the rigid foam core which is much softer than the other two layers.

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