ANALYSIS OF A THEORETICAL-EXPERIMENTAL MODEL OF A ROTOR-BEARING-FOUNDATION SYSTEM

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Abstract. *This work presents a theoretical-experimental model of a rotor-bearing-foundation system. The subsystem rotor-bearing is modelled through the finite element method, using Timoshenko beam theory, and the bearings stiffness and damping coefficients are calculated using a finite difference method to solve the Reynolds hydrodynamic equation. The foundation structure of the rotating machinery is determined through the modal analysis of the frequency response function of a real structure. Four methods were used to calculate the modal parameters: Peak Picking, Circle Fit, Circle Fit with an optimization method and a Levenberg-Marquardt algorithm. The frequency response function calculated through these methods, were compared to the response of the experimental test, demostrating the accuracy and precision of each one of the methods used to process the experimental response of the foundation structure.The Method of the Mixed Co-ordinates joins the subsystems, rotor-bearing and foundation. This method works with physical and principal co-ordinates, reducing the processing time to calculate the response of the complete system. From this response it is possible to observe the interaction between all the components of the rotating machinery.*

Keywords*: Rotordynamics, Foundation Structure, Modal Analysis*

1. INTRODUCTION

Rotating machines have an important role in the majority of electrical power generation systems. The turbine, their main component, can be found in hydro, wind, thermo e nuclear power plants. Because of that, it is essential to study the working behaviour of these particular group of machines, in order to get a cleaner and safer generation of energy.

Turbomachinery using gas or steam as fuel, generally employed in thermoelectrical and nuclear plants, can be described as large and horizontal turbines, that transform the fuel energy in rotation, by driving a generator that converts it into electrical energy. These fuels can be found as derivatives of petroil, coal or biogas (from the decomposition of organic matter). The horizontal turbines have components interacting with the rotor (the main part of a rotating machinery), and they influence its dynamical behaviour. To study these interactions is necessary to know the characteristics of this kind of machinery.

The bearings are supported by a structure, also known as foundation or pedestal. This structure, which is generally made of steel, is connected to large blocks or columns of concrete embedded in soil. These blocks or columns act as mass dampers, absorbing the rotor vibration transmitted to foundation. This vibration excites the movement of the foundation, interacts with bearings that send it back to the rotor. This dynamical interaction with the foundation affects the behaviour of the complete system (Rotor-Bearings-Foundation), and because of that it is important to identify its characteristics to correct modelling.

Several methods were employed to model the dynamic behaviour of foundations, from simplified models considering mass-spring systems or beams, to complex finite element models. Nevertheless the models that showed better results are the experimental ones, built from the modal analysis of the frequency response function (FRF) measured in foundation structure.

Weber (1961) simulated the behaviour of a foundation structure of rotating machinery through an extension of the transfer matrix method, applying it to a model composed by two beams, representing a rotor and a table foundation. Gasch (1976) proposed the inclusion of the foundation effects in a rotor-bearing system through its dynamical stiffness matrices, obtained by inverting the sum of the experimental receptance matrices of a foundation structure, and receptance matrices of the fluid film, obtained from the stiffness and damping coefficients of the bearings.

Bachschmid et al. (1982) took several experimental tests in the foundation structure of a 660 MW turbo-generator, composed by concrete reinforced structure embedded in sand by a series of long pillars. The experimental data of the foundation were compared with the theoretical model one, composed by beam elements, and to represent the soil influence, it was added a stiffness coefficient to the connection nodes between the structure and the soil. They concluded that the mathematical model did not simulate the dynamic behaviour of the actual structure, and the closest results were obtained through several adjustments of the parameters.

Curami and Vania (1985) applied several modal analysis techniques to estimate the modal parameters of a foundation structure of rotating machinery. They concluded the methods are sensitive to the transfer function resolution, and it generates large difference between the calculated values. The methods are also sensitive to noise, and the application of each method strongly depends on the characteristics of the transfer function. Diana et al. (1988) proposed a method to identify the modal parameters through the run-up and run-down of a rotating machine.

Cavalca (1993) used the Mixed Co-ordinates method to include foundation effects in a turbo-generator consisting of three rotors and seven bearings. The Mixed Co-ordinates method uses physical co-ordinates to the rotor and modal co-ordinates to the foundation structure. Cavalca et al. (2002) developed a model of a Rotor-Bearings-Foundation system, and they studied the effect of varying the number of foundation modes, and concluded that it is only necessary to include the most significant modes to obtain the dynamical behaviour of the complete system.

Cavalca et al. (2005) extended the study reported in 2002, and performed a more detailed investigation of the foundation structure, and compared it to the experimental data. They achieved a good correlation in the calculation of natural frequencies, the amplitudes presented quite different results comparing the experimental and theoretical models, because of a miscalculation of the damping coefficients of the structure.

To improve the damping calculation in the modal analysis of the foundation structure this work proposes a comparison between four modal analysis methods, and their effect on the Rotor-Bearing system, using two different optimization methods to increase the convergence between the experimental data and the model built from the modal analysis.

2. MODAL ANALYSIS

The foundation structure of rotating machinery is usually a complex system composed by several steel structures attached to large block of concrete embedded in soil. The theoretical modeling of this structure demands a very long time and the results do not fulfill the requirements of the simulation of the complete system, as demonstrated by Bachschmid et al. (1982). The modal analysis of a foundation structure can provide more information about it, and can be easily joined to the modeling of the complete system.

Modal analysis is widely applied to identify the dynamic behaviour of stationary structures, and there are several techniques to calculate the modal parameters, which can be readily integrated to rotor-bearings systems through the Mixed Co-ordinates method, to obtain the response of the complete system. The modal analysis techniques are influenced by the noise of the experimental measurement of the structure, which can lead the analysis to an incorrect model, to overcome this problem two optimization algorithms, Levenberg-Marquardt and Golden Search with Least Squares, were used to refine the modelled data.

A modal model of a foundation structure can be represented by the frequency response function for multiple modes (Craig, 1981):

$$h_{kj}\left(\Omega_e, p\right) = \sum_{i=1}^{N} \frac{X_k^i X_j^i}{m_i\left(\omega_i^2 - \Omega_e^2 + 2i\zeta_i\omega_i\Omega_e\right)} \tag{1}$$

Where $h_{kj}\left(\Omega_e, p\right)$ is the frequency response function measured at co-ordinate *k* and excited at co-ordinate *j*; X_k^i is the kth element of the eigenvector of the ith mode; m_i is the modal mass of the ith mode; ω_i is the natural frequency of the ith mode; ζ_i is the damping coefficient of the ith mode; Ω_e is the excitation frequency.

The modal analysis techniques are used to calculate the modal parameters of Eq. 1, and these parameters are included in the Rotor-Bearing-System to the simulation of the complete system.

This section gives an overview of the modal analysis techniques, the reader is invited to read the works cited below to get a deeper insight of this subject.

2.1. Peak Picking

The "peak-picking" or "peak-amplitude" method is one of the simplest methods of modal analysis (Bishop, 1963). It is a method that can be applied to experimental data from structures with well separated modes that have an accurate measurement of the resonance, and are not influenced by other modes. This seems to be quite limited; however it can be useful to obtain initial estimations of the modal parameters, speeding up the parameter calculation of other methods.

The method has these steps:

a) Individual resonance peaks are detected on the FRF ($H\left(\omega\right)$), and the frequency related to a peak is considered the natural frequency of this particular mode.

b) The maximum amplitude at the peaks ($\hat{H}\left(\omega\right)$) are used to calculate the frequencies ω_a and ω_b when the FRF amplitude is equal to $\hat{H}(\omega)/\sqrt{2}$.

c) The damping of the mode can be estimated using the Eq. 2:

$$\zeta_r = \frac{\omega_a^2 - \omega_b^2}{2\omega_r\left(\omega_a + \omega_b\right)} \tag{2}$$

Where ζ_r is viscous damping coefficient of the r[th] mode, ω_r is the natural frequency of the r[th] mode, ω_a is the frequency above ω_r where $H(\omega)=\hat{H}(\omega)/\sqrt{2}$, ω_b is the frequency below ω_r where $H(\omega)=\hat{H}(\omega)/\sqrt{2}$.

d) The modal constant A_r is calculated using Eq. 3, assuming that the total response in the resonance region is given by a single term of the FRF series:

$$A_r = \left|\hat{H}(\omega)\right| 2i\omega_r^2\zeta_r \quad (3)$$

This method relies on the accuracy of the FRF resonance peaks, which is particularly difficult to measure, because the majority of the errors in measurement are concentrated around the resonance region.

2.2. Circle Fit

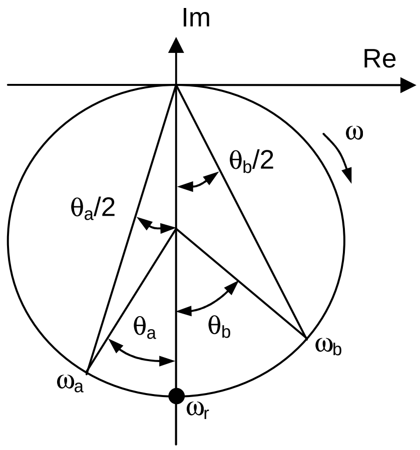


Figure 1: Properties of modal circle (Mobility FRF).

The Circle-Fit method is based on the fact that the FRF produces circle-like curves in the Nyquist mobility diagram (Ewins, 1995). This method uses the mobility FRF $Y(\omega)$, which is the ratio between the harmonic velocity response and the harmonic force, and it has the following sequence of steps:

a) Select points to be used in the circle fitting procedure, which can be selected in an automatic way;
b) Fit the circle, which can be performed by a Least Squares routine, and calculate the quality of the fitting;
c) The location of the maximum sweep rate is given by Eq. 4, and it is also the location of the natural frequency;

$$\frac{d}{d\omega}\left(\frac{d\omega^2}{d\theta}\right) = 0 \quad (4)$$

Where ω is the frequency, and θ is the angle in respect to the modal circle center.

d) Calculate multiple damping estimates using the following equation (see Fig. 1):

$$\zeta_r = \frac{\omega_a^2 - \omega_b^2}{2\omega_r\left(\omega_a \tan(\theta_a/2) + \omega_b \tan(\theta_b/2)\right)} \quad (5)$$

Where ζ_r is the modal damping coefficient of the r[th] mode, ω_r is the natural frequency of the r[th] mode, ω_a is the frequency above ω_r, ω_b is the frequency below ω_r, θ_a is the angle in respect to the frequency ω_a, θ_b is the angle in relation to the frequency ω_b.

e) Determine the modal constant using Eq. 6.:

$$ {}_rD_{jk} = \frac{\left|{}_rA_{jk}\right|}{i\omega_r^2\zeta_r} \tag{6} $$

Where ${}_rD_{jk}$ is the diameter of modal circle (mode *r*) of the FRF measured at co-ordinate *k* and excited at co-ordinate *j*; ${}_rA_{jk}$ is the modal constant (mode *r*) of the FRF measured at co-ordinate *k* and excited at co-ordinate *j*.

2.3. Levenberg-Marquardt optimization.

The modal parameters calculated through the Circle-Fit method can be influenced by noise in the measurements; to overcome this problem and improve the curve fitting, a non-linear least squares method, known as Levenberg-Marquardt is used to minimize the error between the experimental and the modelled data, using the following equation:

$$ f_e(\omega_r, \eta_r, A_r) = \sqrt{\frac{\sum_{i=1}^{N} (\mathrm{Re}(Ha_i) - \mathrm{Re}(He_i))^2 + (\mathrm{Im}(Ha_i) - \mathrm{Im}(He_i))^2}{\sum_{i=1}^{N} \mathrm{Re}(He_i)^2 + \mathrm{Im}(He_i)^2}} \tag{7} $$

Where *Ha* is the modelled FRF, *He* is the experimental FRF, Re(*f*) is the real part of the FRF, Im(*f*) is the imaginary part of the FRF and *N* is the number of frequencies.

Marquardt (1963) presented a method of curve fitting, from the technique proposed by Levenberg (1944), which would move smoothly between the method of the inverse Hessian (Newton) and method of the steepest descent. The last one is used far from the minimum, exchanging continuously the previous calculated value, as it reaches the minimum. The Levenberg-Marquardt method showed a good practical application, and it became a standard in non-linear least squares. Like the quasi-Newton methods, the Levenberg-Marquardt algorithm was developed for a second order approximation, without calculating the Hessian matrix. When the objective function is composed by a sum of squares, the Hessian matrix can be represented in a simpler form and easily calculated.

2.4. Golden Section Search and Least Squares.

The results obtained from the modal analysis, do not always present a good correlation between the model and the experimental data of the structure, to solve this issue, an optimization method was developed by Okabe and Cavalca (2005). The initial values of the modal parameters are calculated through the “circle-fit” method.

Then the damping coefficients and the phase angles are optimized by a direct search method known as Golden Section Search (Bunday, 1984). This method does not require a predefined number of calculations of the objective function, like the Fibonacci search, to reach a determined precision. The denomination of this method, Golden Search, comes from the golden ratio, which is the constant ratio obtained when the number of divisions of a segment is taken to the infinite.

In each step of the algorithm, the Golden Section Search is used several times to optimize the phase angle and the damping values, then the generalized mass is refined through the application of the Least Squares method (Ruggiero et al, 1988), which minimizes the error function (Eq. 6). This method is fast and reliable; however its application is restricted, because the derivatives of the objective function should be linear.

3. ROTOR-BEARINGS-FOUNDATION SYSTEM

The mathematical modelling of the complete Rotor-Bearings-Foundation system can be split in two subsystems: Rotor-Bearings and Foundation Structure. Each subsystem is analysed, and the response of the complete system is calculated joining the dynamical matrices of these two subsystems. The foundation effect can be described by its mechanical impedance or its most significant modes (Cavalca et al., 2005).

The shaft can be modelled through the Finite Element method, using a direct assembly approach. The mass and stiffness matrices were proposed by Nelson (1980), who formulated them based on the Timoshenko beam theory. It includes the effects of the rotatory inertia, gyroscopic moments, axial load and transversal shearing. The model of the shaft used in this work can be seen in Fig. 2, and it is a Jeffcott/Laval rotor, with a disk positioned in the middle of the shaft.

The hydrodynamic bearings are represented by their linear stiffness and damping coefficients, calculated through the finite difference solution of a finite length bearing (Pinkus, 1958). The finite difference determines the pressure distribution generated by the fluid film over the surface of the bearing, the hydrodynamic forces are obtained through the integration of the pressure by the area of the bearing, and the derivatives of these forces with respect to the shaft displacements and velocities yield the coefficients of stiffness and damping.

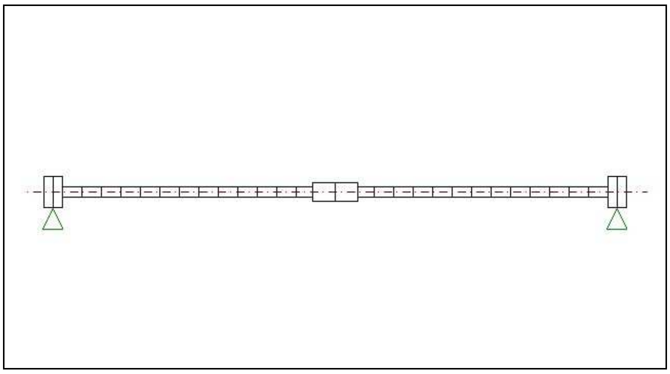


Figure 2. Finite element model of the shaft.

The finite element model of the shaft, the hydrodynamic bearing coefficients and modal parameters of the foundation structure are assembled together in the following equation (Cavalca et al., 2005):

$$\begin{bmatrix} [M_{RR}] & 0 \\ 0 & \lfloor m_f \rfloor \end{bmatrix} \begin{Bmatrix} \{\ddot{x}_R\} \\ \{\ddot{q}\} \end{Bmatrix} + \begin{bmatrix} [R_{RR}] & [R_{RF}][\Phi] \\ [\Phi]^T [R_{FR}] & \lfloor r_f \rfloor + [\Phi]^T [R_{FF}][\Phi] \end{bmatrix} \begin{Bmatrix} \{\dot{x}_R\} \\ \{\dot{q}\} \end{Bmatrix} + \begin{bmatrix} [K_{RR}] & [K_{RF}][\Phi] \\ [\Phi]^T [K_{FR}] & \lfloor k_f \rfloor + [\Phi]^T [K_{FF}][\Phi] \end{bmatrix} \begin{Bmatrix} \{x_R\} \\ \{q\} \end{Bmatrix} = \begin{Bmatrix} \{F_0\} \\ 0 \end{Bmatrix} \tag{8}$$

Where $[M_{RR}]$, $[R_{RR}]$, $[K_{RR}]$ are the mass, damping and stiffness matrices of the subsystem Rotor-Bearings; $\lfloor m_f \rfloor$, $\lfloor r_f \rfloor$, $\lfloor k_f \rfloor$ are the mass, damping and stiffness matrices of the foundation structure; $[R_{RF}]$, $[R_{FR}]$, $[R_{FF}]$ are the bearing damping coefficient matrices connecting the subsystems; $[K_{RF}]$, $[K_{FR}]$, $[K_{FF}]$ are the bearing stiffness coefficient matrices connecting the subsystems; $\{x_R\}$ is the physical co-ordinates vector of the rotor; $\{q\}$ is the modal co-ordinates vector of the foundation; $\{F_0\}$ is the excitation force vector; $[\Phi]$ is the modal matrix.

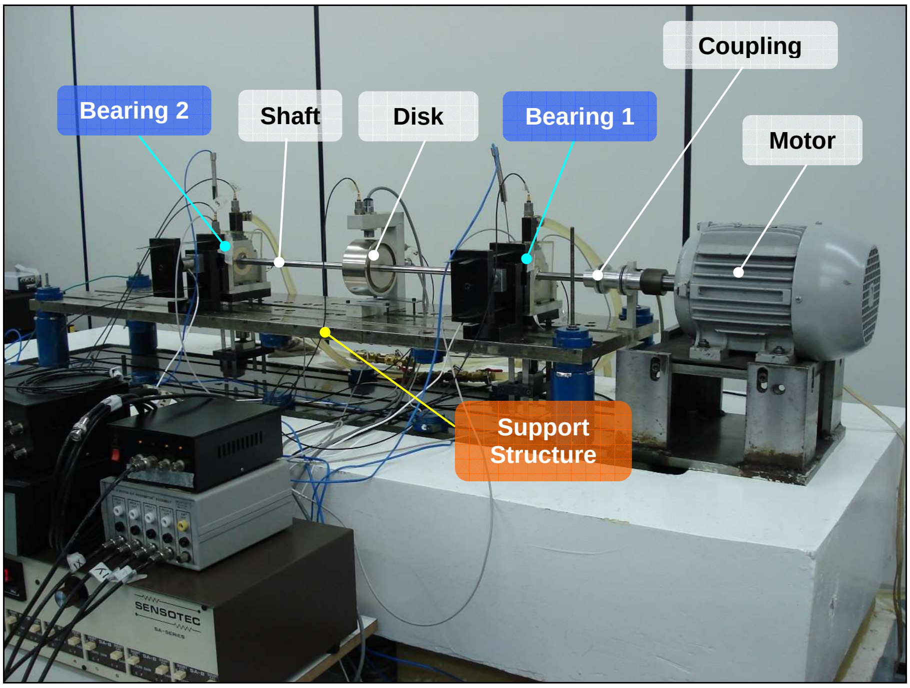


Figure 3. Test bench in the LAMAR.

The model of the complete system was based on the rotor of the test bench (Fig. 3) assembled in the Laboratory of Rotating Machinery (LAMAR) of the Mechanical Engineering Faculty of UNICAMP, and it has complete description reported by Cavalca et al. (2002).

4. RESULTS

The modal parameters were extracted from the experimental data measured on the foundation structure of the LAMAR test bench (Okabe, 2007), which can be observed in Fig. 4. The structure was excited with a white random noise through a shaker, in both positions, vertical and horizontal. The response was measured by a group of accelerometers attached to the surface of the structure, the signal was processed by the conditioning amplifiers and sent to the data acquision board to be analysed inside of the Labview ® software.

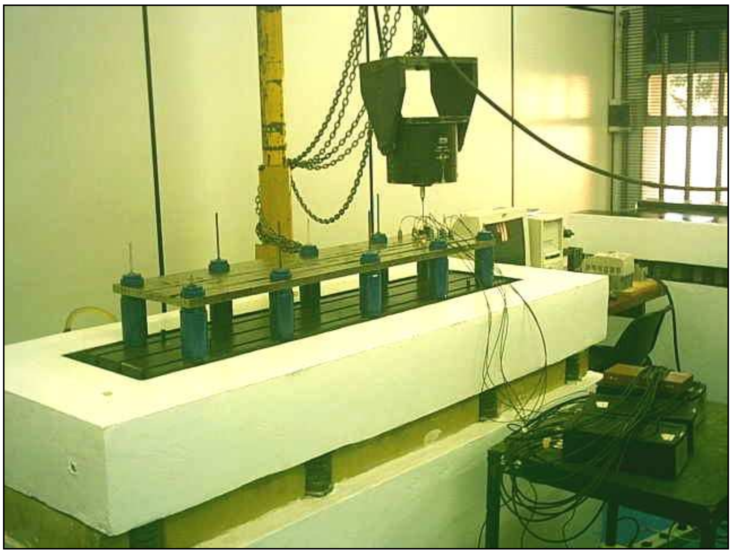


Figure 4. Experimental test of the foundation structure.

4.1. Modal Analysis

The modal analysis was carried out using the experimental FRF of the foundation structure of the test bench. The four methods were applied, and the modelled data compared to the experimental. The number of identified modes was kept constant, to improve the comparison between the methods, because the "peak-picking" demands several characteristics of the FRF, and not all of the detected modes could accomplish them.

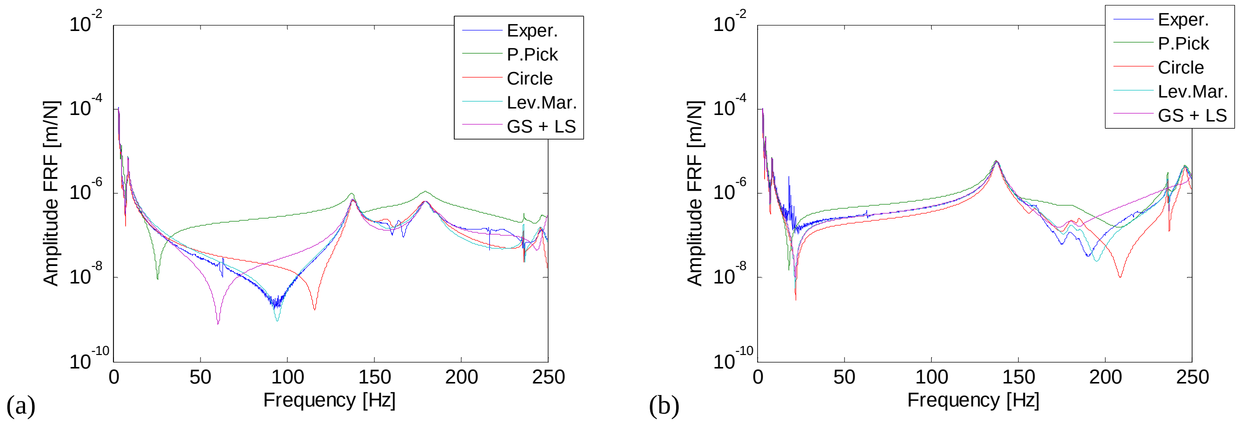


Figure 5. Experimental and modelled FRF of the bearing support points (a – bearing 1 and b – bearing 2) on the foundation structure under horizontal excitation.

Figure 5 shows the FRF of the bearing support points on the foundation structure, and they clearly show different behaviours. The "peak-picking" method shows the worst fitting of the FRF in both points, while the "circle-fit" presents a better approach. The optimization algorithms, applied to the "circle-fit" method, took the modelled FRF even closer to the experimental.

Table 1 present the damping coefficient of the ten identified modes through the modal analysis methods, and it reveals a certain correlation between the "circle-fit" and the optimization processes, while the "peak-picking" is less accurate than the others.

Table 1. Damping coefficients of the modes calculated by the modal analysis methods with the experimental data of the horizontal excitation test.

Frequency [Hz]	Peak Picking	Circle Fit	Levenberg Marquardt	G. Search + L. Squares
3.4	0.040	0.024	0.031	0.031
4.7	0.032	0.021	0.024	0.024
8.6	0.026	0.022	0.032	0.033
138.0	0.016	0.013	0.016	0.014
246.2	0.009	0.004	0.009	0.002
236.2	0.001	0.001	0.001	0.001
236.7	0.014	0.000	0.000	0.000
179.9	0.025	0.022	0.017	0.017
185.0	0.134	0.006	0.008	0.012
158.9	0.139	0.015	0.026	0.030

Figure 6 illustrates the experimental and modelled FRF of the bearing support points subjected to a vertical excitation, and like the result obtained through the horizontal excitation, the support points presented different FRFs from each other. It is possible to observe that the "peak-picking" method again presented the worst fitting compared to the others.

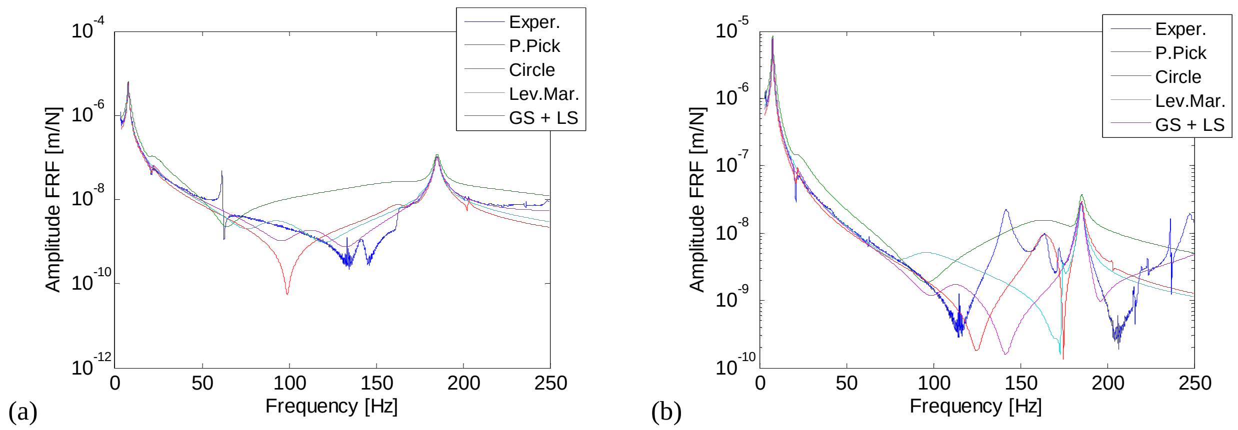


Figure 6. Experimental and modelled FRF of the bearing support points (a – bearing 1 and b – bearing 2) on the foundation structure under vertical excitation.

Table 2. Damping coefficients of the modes calculated by the modal analysis methods with the experimental data of the vertical excitation test.

Frequency [Hz]	Peak Picking	Circle Fit	Levenberg Marquardt	G. Search + L. Squares
7.7	0.039	0.029	0.033	0.036
8.6	0.069	0.017	0.389	0.042
185.2	0.008	0.006	0.006	0.006
21.7	0.193	0.021	0.408	0.053
164.4	0.114	0.033	0.171	0.124
172.2	0.193	0.023	0.084	0.055
203.1	0.237	0.002	0.002	0.001

The damping coefficients of the seven modes identified were shown in Tab. 2. It can be noticed that there is no clear correlation among the coefficients, due to the different nature of each method. When the peak of resonance was well defined (7.7 and 185.2 Hz), all methods presented the similar results.

The difference between the experimental FRF and those calculated by the modal analysis methods is shown in Tab. 3. The "peak-picking" method presented the biggest error in both sets of data (vertical and horizontal), the circle-fit produced a better result, particularly in the vertical excitation test. The optimization methods applied to the circle-fit method, improved the curve fitting in both cases, and the Levenberg Marquardt presented a more constant result with 15% of error.

Table 3. Difference between experimental and calculated FRFs using modal analysis.

Excitation	Peak Picking	Circle-Fit	Levenberg Marquardt	Golden Search - Least Squares
Vertical	61.5%	24.0%	15.5%	12.6%
Horizontal	42.3%	31.9%	15.1%	29.8%

The modal parameters calculated from the experimental data of the foundation structure were added to a rotor-bearing system, and the effect of these FRF fitting errors can be observed in the behaviour of rotating machinery.

4.1. Theoretical-Experimental Model of a Rotor-Bearings-Foundation System.

The model of the complete system is achieved integrating the theoretical models of the shaft and bearings to the experimental model of foundation. The experimental foundations are compared to a rigid foundation, where the support structure does not have any movement.

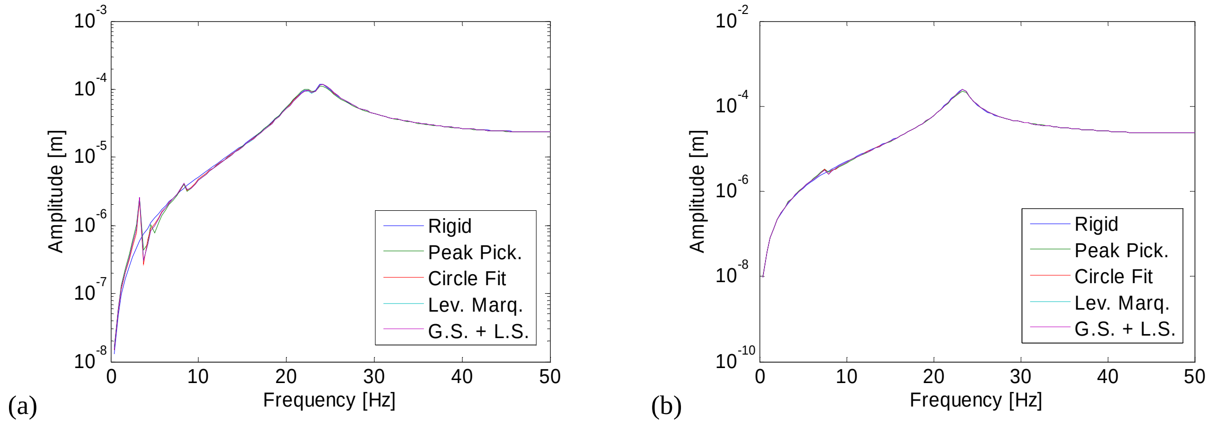
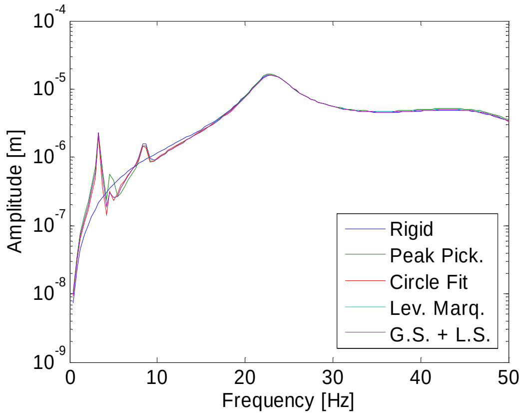
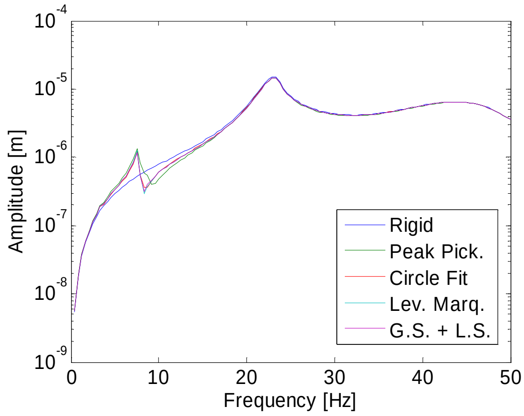


Figure 7. Amplitude of vibration of the disk (a – horizontal and b – vertical) using a rigid foundation and a flexible foundation modelled through different modal analysis methods.

The amplitude of vibration of the disk is shown on Fig. 7a, in the horizontal axis. Comparing the rigid foundation model to the experimental ones, it is easily noticed few peaks in the range from 2 to 10 Hz, and the peak in 3.4 Hz can be clearly seen and it corresponds to the horizontal response of the foundation. Figure 7b shows the vertical vibration of the disk, and the responses of the different models of foundation are overlaid. It is only noticeable a small peak between 7 and 8 Hz.



(a)



(b)

Figure 8. Amplitude of vibration of the shaft (a – horizontal and b – vertical) inside the bearing 1 (journal) using a rigid foundation and a flexible foundation modelled through different modal analysis methods.

Figure 8a shows the horizontal amplitude of vibration of the shaft inside the bearing 1 (journal), and it can be noticed that the amplitude of vibration in the journal is lower than in the disk, and it highlights the foundation effect as can be seen in the range from 2 to 10 Hz. The vertical vibration of the journal, showed in Fig. 8b, points out a peak between 7 and 8 Hz, which corresponds to one of the vibration modes of the foundation.

Table 4. Comparison of the amplitude of vibration of the shaft with a rigid foundation and the experimental models of foundation.

Method	Disk	Bearing 1	Bearing 2
Peak Picking	3,8%	6,8%	7,6%
Circle Fit	1,8%	3,5%	3,3%
Lev. Marquardt	2,2%	4,3%	4,3%
G.S. + L. Squares	2,1%	4,3%	3,8%

Table 4 shows the difference of amplitude between the rigid foundation model and the flexible experimental ones, in three positions of the shaft. The influence of the foundation is more noticeable in the journals (inside the bearings), because they are the elements connecting the foundation to the shaft, however the influence is still small considering that it alters the amplitude in just three to four percent. It was observed, comparing Fig. 7 and Fig. 8, that the disk has a larger amplitude than the journal, and because of that the foundation effect is relatively lower, changing the amplitude in approximately two percent.

5. CONCLUSIONS

This work presents a simulation of a Rotor-Bearings-Foundation system with a theoretical-experimental approach. The rotor-bearings subsystem is modelled using finite elements and finite differences, and the foundation is represented by modal parameters calculated through modal analysis. A comparison between different modal analysis methods was performed, and the influence of a flexible foundation was compared to a theoretical rigid foundation.

The "peak-picking" method just gave rough estimations of the modal parameters of the foundation structure, with a lower correlation with the experimental data. The "circle-fit" method is less influenced by the accuracy of the resonance region, and it showed better correlation with the experimental data. The optimization methods, used after the circle-fit procedure, refined the model getting even closer to the experimental FRF, and the Levenberg-Marquardt method demonstrated a more regular behaviour comparing both tests (vertical and horizontal excitation) used to extract the modal parameters.

The simulation of the Rotor-Bearings-Foundation system showed that the flexible foundation structure has a small and limited effect on the analyzed rotor. It changes from two to three percent the mean amplitude of vibration of the disk, and from three to seven percent the mean amplitude of shaft inside the bearings (journal). These changes are concentrated in the resonance regions of the foundation structure, not affecting the resonance of the shaft (around 23 Hz). These results demontrate that the hydrodynamic bearings can absorb the vibrations coming from the foundation structure, and the highest amplitudes modes of the foundation, in the operating range of the rotor, have a greater influence on the behaviour of the complete system.

6. ACKNOWLEDGEMENTS

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