Asymptotic analysis of lean premixed-flames in porous inert media

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Abstract. *he structure of lean premixed flames within inert porous media is investigated by using the asymptotic expansion method. This work aims to extend a previous analysis restricted to moderate lean mixtures. As in the previous work, the flame structure is divided in three distinct regions, the solid-phase diffusion region, the gas-phase diffusion region and the reaction region. Since the chemical reaction is very sensitive to temperature, the reaction occurs in a very thin zone inside the gas-phase diffusion region, where the highest gas-phase temperature is found. Discrepancies in the characteristic lengths among the three regions justify the application of asymptotic expansions to determine an approximated (analytical) solution. For this limiting case the flame velocity is low and the interphase heat transfer is hight leading to thermal equilibrium between the phases in the the solid-phase diffusion region. Thermal nonequilibrium is found just in the gas-phase diffusion region that is restricted to a thin region around the flame. As the interphase heat transfer becomes more significant more heat is recirculated to the incoming gases and higher superadiabatic temperatures are found. The influence of the interphasic heat transfer coefficient, the porosity of the medium and the ratio of the solid-phase conductivity to that of the gas-phase are evaluated, revealing the effects of the inner flame structure on the flame stabilization within porous media*

Keywords: *combustion in porous media, asymptotic analysis, lean mixtures*

1. INTRODUCTION

Combustion in porous media is characterized by the existence of conduction heat transfer along the solid matrix from the hot burned gases (downstream from the flame) to the fresh unburned gases (upstream from the flame). This aspect has been called heat recirculation [2] and has received much attention in the last decades as a way of extending flame stability and burning fuel lean mixtures [5].

Many experimental investigations ([9], [14], [6], [8], [15], [11], [7]) and numerical studies ([2], [18], [3], [10], [4], [19], [1]) have been published covering a wide range of aspects of the combustion in porous media. Some analytical solutions were also developed ([20],[21],[13],[17]).

In a previous work [16] the structure of premixed flames within inert porous media is investigated using the asymptotic expansion method. For this, the flame structure is divided in three characteristic length scales. The two innermost length scales, the gas-phase diffusion length scale (l_G) and the reaction length scale (l_R), are the same scales defined in the classical premixed flame structure analysis. The outermost length scale, the solid-phase diffusion length scale (l_S), is related to the heat conduction in the porous matrix. The differences in the characteristic lengths of the three scales result in large thermal non-equilibrium and justify the application of asymptotic expansions to determine an approximate (analytical) solution. The description of the reaction region is obtained using the large activation energy asymptotic expansion and the description of the problem of the order of the gas-phase length scale is obtained using the boundary layer expansion.

The results showed that the influence of the porous medium on the flame is to increase its temperature and velocity. That influence is more pronounced for leaner mixtures, higher solid-phase thermal conductivities, lower porosities and lower fuel Lewis numbers. The thermal affected region (l_S) is about 50 times larger than the gas-phase diffusion length scale (l_G) as a result of the high thermal conductivity of the solid matrix. Maximum gas-phase temperatures up to 38% above the corresponding adiabatic free-flame temperature and flame velocities up to 6.5 times the corresponding adiabatic free-flame velocity are found in the range of analysis. It is also shown that the reaction region is shorter in flames within porous media $l_R < l_R^0$ since higher flame temperatures are found. Due to the simplifications assumed by the model the solution fails for lower equivalence ratios ($\Phi < 0.6$).

The objective of the present study is to extend the previous work to lower equivalence ratios. For this limiting case the flame velocity is low and the interphase heat transfer is hight leading to thermal equilibrium between the phases in the the solid-phase diffusion region. Thermal nonequilibrium is found just in the gas-phase diffusion region that is restricted to a thin region around the flame. As the interphase heat transfer becomes more significant more heat is recirculated to the incoming gases and higher superadiabatic temperatures are found. The influence of the interphasic heat transfer coefficient, the porosity of the medium and the ratio of the solid-phase conductivity to that of the gas-phase are evaluated, revealing the effects of the inner flame structure on the flame stabilization within porous media

2. MATHEMATICAL FORMULATION

The steady state, volume-averaged energy and species conservation equations (omitting for simplicity the volume-averaging notation) for the combustion within porous media are written following [18]

$$\varepsilon\rho u = \varepsilon\rho_n u_n \tag{1}$$

$$\varepsilon\rho_n u_n \frac{dY_F}{dx} = \varepsilon\rho D_F \frac{d^2 Y_F}{dx^2} - \varepsilon A\rho^2 Y_O Y_F\, T_g^a e^{-E_a/R_u T_g} \tag{2}$$

$$\varepsilon\rho_n u_n \frac{dY_O}{dx} = \varepsilon\rho D_O \frac{d^2 Y_O}{dx^2} - \varepsilon\nu A\rho^2 Y_O Y_F\, T_g^a e^{-E_a/R_u T_g} \tag{3}$$

$$\varepsilon\rho_n u_n c_p \frac{dT_g}{dx} = \varepsilon\lambda_g \frac{d^2 T_g}{dx^2} + \varepsilon Q A\rho^2 Y_O Y_F\, e^{-Ea/R_u T_g} + h_v(T_s - T_g) \tag{4}$$

$$0 = (1-\varepsilon)\lambda_s \frac{d^2 T_s}{dx^2} - h_v(T_s - T_g) \tag{5}$$

where Q is the fuel mass based heat of reaction, h_v is the volumetric convection coefficient, E_a is the activation energy and R_u is the universal gas constant.

2.1 Nondimensionalization

Defining the nondimensional variables [**?**]

$$y_F = \frac{Y_F}{Y_{Fn}},\ y_O = \frac{Y_O}{Y_{On}},\ \theta = \frac{c_p(T-T_n)}{Y_{Fn}\,Q} = \frac{T-T_n}{T_r-T_n},\ \zeta = \int_0^x \frac{\rho_n u_n}{\lambda_s/c_p}dx \tag{6}$$

the premixed-flame within a porous medium is described by the following conservation equations

$$\varepsilon\frac{dy_F}{d\zeta} = \frac{\varepsilon}{Le_F\,\Gamma}\frac{d^2 y_F}{d\zeta^2} - \varepsilon\widehat{D}a\; y_O y_F \exp\left[-\frac{\beta(1-\theta_g)}{1-\alpha(1-\theta_g)}\right] \tag{7}$$

$$\varepsilon\frac{dy_O}{d\zeta} = \frac{\varepsilon}{Le_O\,\Gamma}\frac{d^2 y_O}{d\zeta^2} - \varepsilon\Phi\widehat{D}a\; y_O y_F \exp\left[-\frac{\beta(1-\theta_g)}{1-\alpha(1-\theta_g)}\right] \tag{8}$$

$$\varepsilon\frac{d\theta_g}{d\zeta} = \frac{\varepsilon}{\Gamma}\frac{d^2\theta_g}{d\zeta^2} + \varepsilon\widehat{D}a\; y_O y_F \exp\left[-\frac{\beta(1-\theta_g)}{1-\alpha(1-\theta_g)}\right] + N(\theta_s - \theta_g) \tag{9}$$

$$0 = (1-\varepsilon)\frac{d^2\theta_s}{d\zeta^2} - N(\theta_s - \theta_g) \tag{10}$$

where

$$\Gamma \equiv \frac{\lambda_s}{\lambda_g},\quad \Phi \equiv \frac{Y_{Fn}\nu}{Y_{On}},\quad \alpha \equiv \frac{(T_r - T_n)}{T_r},\quad \beta \equiv \frac{E_a(T_r-T_n)}{R_u T_r^2},$$

$$N \equiv \frac{\lambda_s h_v}{(\rho_n\; u_n\; cp)^2},\quad \widehat{D}a \equiv \frac{A\,\rho^2\,\lambda_s\,Y_{On}\,T_g^a\quad exp(-\beta/\alpha)}{(\rho_n^2\;u_n^2\;c_p)}.$$

2.2 Outer zone: problem of the order of unity

In the characteristic length scale $\zeta - \zeta_f \sim O(1)$, the diffusive terms are of the order of Γ^{-1}, the interphase heat transfer parameter N is of the order of Γ and the reaction is exponentially small. Thus, (7) to (10) take the form

$$\varepsilon\frac{dy_F}{d\zeta} = \frac{\varepsilon}{Le_F\,\Gamma}\frac{d^2 y_F}{d\zeta^2} \tag{11}$$

$$\varepsilon\frac{dy_O}{d\zeta} = \frac{\varepsilon}{Le_O\,\Gamma}\frac{d^2 y_O}{d\zeta^2} \tag{12}$$

$$\varepsilon\frac{d\theta_g}{d\zeta} = \frac{\varepsilon}{\Gamma}\frac{d^2\theta}{d\zeta^2} + N(\theta_s - \theta_g) \tag{13}$$

$$0 = (1-\varepsilon)\frac{d^2\theta_s}{d\zeta^2} - N(\theta_s - \theta_g) \tag{14}$$

The solution of Eqs. (11) to (14) can be written as

$$\begin{array}{lllllll} \theta_s & = & \theta_s^{(0)} & + & \Gamma^{-1}\theta_s^{(0)(1)} & + & o(\Gamma^{-1}) \\ \theta_g & = & \theta_g^{(0)} & + & \Gamma^{-1}\theta_g^{(0)(1)} & + & o(\Gamma^{-1}) \\ y_O & = & y_0^{(0)} & + & \Gamma^{-1}y_0^{(0)(1)} & + & o(\Gamma^{-1}) \\ y_F & = & y_F^{(0)} & + & \Gamma^{-1}y_F^{(0)(1)} & + & o(\Gamma^{-1}) \end{array} \tag{15}$$

Substituting (15) into (11) to (14), applying the limits $\Gamma \to \infty$, and $N \to \infty$ (for lean mixtures), the first approximation for the set of equations of the order of unity, $O(1)$, is

$$\frac{dy_F^{(0)}}{d\zeta} = 0 \tag{16}$$

$$\frac{dy_O^{(0)}}{d\zeta} = 0 \tag{17}$$

$$\theta_s^{(0)} = \theta_g^{(0)} = \theta^{(0)} \tag{18}$$

The boundary conditions for $\zeta \to -\infty$ are $\theta^{(0)} = 0$ and $y_F^{(0)} = y_O^{(0)} = 1$ and for $\zeta \to +\infty$ are $\theta^{(0)} = 1$ and $y_F^{(0)} = y_O^{(0)} = 0$. The solution for Eqs. (16) and (17) are $y_F^{(0)} = y_O^{(0)} = 1$ for $\zeta < \zeta_f$ and $y_F^{(0)} = 0$ and $y_O^{(0)} = 1 - \Phi$ for $\zeta > \zeta_f$.

Summing up (13) and (14) with thermal equilibrium (18) one finds

$$\varepsilon\frac{d\theta^{(0)}}{d\zeta} = \frac{1}{\Gamma}\frac{d^2\theta^{(0)}}{d\zeta^2} + (1-\varepsilon)\frac{d^2\theta^{(0)}}{d\zeta^2} \tag{19}$$

Integrating (19) and applying the proper boundary conditions one finds

$$\theta^{(0)} = \begin{cases} exp\left[(\zeta - \zeta_f)\left(1/\Gamma + (1-\varepsilon)/\varepsilon\right)^{-1}\right], & \text{for} \quad \zeta < \zeta_f \\ 1, & \text{for} \quad \zeta > \zeta_f \end{cases} \tag{20}$$

2.3 Inner zone: problem of the order of Γ^{-1}

In this zone, the variation of the nondimensional variables is of the order of unity along a characteristic length of the order of Γ^{-1} around the flame, exept for the solid-phase temperature that presents just a small variation. The variables in this thin zone are denoted by $y_F^{(*)}$, $y_O^{(*)}$, $\theta_s^{(*)}$ and $\theta_g^{(*)}$.

The solution of $\theta_s^{(*)}$ and $\theta_g^{(*)}$ can be written as

$$\begin{array}{lllllll} \theta_s^{(*)} & = & 1 & - & \Gamma^{-1}\theta_s^{(*)(1)} & + & o(\Gamma^{-1}) \\ \theta_g^{(*)} & = & \theta_g^{(*)(0)} & + & \Gamma^{-1}\theta_g^{(*)(1)} & + & o(\Gamma^{-1}) \end{array} \tag{21}$$

By rescaling the spatial coordinate ($\Gamma(z - \zeta_f) = \xi$), defining $N \equiv \gamma\Gamma$, substituting the asymptotic expansions (21) and collecting the higher order terms, the governing equations become

$$\varepsilon\frac{dy_F^{(*)}}{d\xi} = \frac{\varepsilon}{Le_F}\frac{d^2y_F^{(*)}}{d\xi^2} \tag{22}$$

$$\varepsilon\frac{dy_O^{(*)}}{d\xi} = \frac{\varepsilon}{Le_O}\frac{d^2y_O^{(*)}}{d\xi^2} \tag{23}$$

$$\varepsilon\frac{d\theta_g^{(*)(0)}}{d\xi} = \varepsilon\frac{d^2\theta_g^{(*)(0)}}{d\xi^2} + \gamma(1 - \theta_g^{(*)(0)}) \tag{24}$$

$$0 = (1-\varepsilon)\frac{d^2\theta_s^{(*)(1)}}{d\xi^2} + \gamma(1 - \theta_g^{(*)(0)}) \tag{25}$$

The boundary conditions are determined when the solution corresponding to the problem of the order of unity is matched with the problem of the order of Γ^{-1}. Thus, in the unburned region (upstream from the flame), for $\xi \to -\infty$, $\theta_g^{(*)} \to \theta_g^{(0)} = \theta^{(0)}$, $\theta_s^{(*)} \to \theta_s^{(0)} = \theta^{(0)}$, $y_F^{(*)} \to y_F^{(0)} = 1$, $y_O^{(*)} \to y_O^{(0)} = 1$. In the burned region (downstream the flame), for $\xi \to \infty$, $\theta_g^{(*)} \to \theta_g^{(0)} = 1$, $\theta_s^{(*)} \to \theta_s^{(0)} = 1$, $y_F^{(*)} \to y_F^{(0)} = 0$ and $y_O^{(*)} \to y_O^{(0)} = (1 - \Phi)$. At the flame, the gas- and solid-phase temperatures, $\theta_{gf}^{(*)}$ and $\theta_{sf}^{(*)}$, are unknowns to be determined.

The solution of Eqs. (21) and (22) are $y_F^{(*)} = 1 - e^{Le_F(\xi-\xi_f)}$ and $y_O^{(*)} = 1 - \Phi e^{Le_O(\xi-\xi_f)}$ for $-\infty < \xi < \xi_f$, and $y_F^{(*)} = 0$ and $y_O^{(*)} = 1 - \Phi$ for $\xi_f < \xi < \infty$.

Equation (24) can be written as

$$\frac{d^2\overline{\theta}}{d\xi^2} - \frac{d\overline{\theta}}{d\xi} - \frac{\gamma}{\varepsilon}\overline{\theta} = 0 \tag{26}$$

were $\overline{\theta} = \left(\theta_g^{(*)(0)} - 1\right)$. The solution of (26) is $\overline{\theta} = C_1 e^{r_1\xi} + C_2 e^{r_2\xi}$. Applying the boundary conditions one finds

$$\theta_g^{(*)} = \begin{cases} 1 + \left(\theta_{gf}^{(*)} - 1\right) e^{r_1\xi}, & \text{for} \quad \xi < \xi_f \\ 1 + \left(\theta_{gf}^{(*)} - 1\right) e^{r_2\xi}, & \text{for} \quad \xi > \xi_f \end{cases} \tag{27}$$

in which

$$r_1 = \frac{1}{2} + \frac{1}{2}\left[1 + 4\frac{\gamma}{\varepsilon}\right]^{1/2}$$

and

$$r_2 = \frac{1}{2} - \frac{1}{2}\left[1 + 4\frac{\gamma}{\varepsilon}\right]^{1/2}$$

With the knowledge of the leading order term of the gas solution, $\theta_g^{(*)(0)}$, equation (25) can be integrated giving

$$\theta_g^{(*)} = \begin{cases} \frac{\gamma(1-\theta_{gf}^{(*)(0)})}{r_1^2(1-\varepsilon)}(1 - e^{r_1\xi}) + \theta_{sf}^{(*)(1)} - [\varepsilon/(1-\varepsilon)]\xi, & \text{for} \quad \xi < \xi_f \\ \frac{\gamma(1-\theta_{gf}^{(*)(0)})}{r_2^2(1-\varepsilon)}(1 - e^{r_2\xi}) + \theta_{sf}^{(*)(1)}, & \text{for} \quad \xi > \xi_f \end{cases} \tag{28}$$

Applying the continuity of the heat flux in the solid-phase at the flame and the remaining boundary condition one finds

$$\theta_{gf}^{(*)(0)} = 1 + (\varepsilon/\gamma)[r_1 r_2/(r_2 - r_1)] \tag{29}$$

and

$$\theta_{sf}^{(*)(1)} = \frac{\gamma(\theta_{gf}^{(*)(0)} - 1)}{r_2^2(1-\varepsilon)} \tag{30}$$

Now, the first correction for the gas-phase can be found to compleat the description of the flame structure. Nevertheless, the heat fluxes at the flame are already determined, then this correction is not necessary to find the expression for flame velocity.

3. Inner zone: reaction region $O(\delta\Gamma^{-1})$

In a region of the order of $\delta\Gamma^{-1}$ around the flame, the variables present a variation of the order of δ. The solution follows the same steps already discussed in the previous solution for higher equivalence ratios [16], more details about the asymptotic solution at the reaction region can be found in [12].

The expression relating the flame velocity with the problem parameters is

$$\frac{2A\rho_f^2\lambda_g Y_{On} T_{gf}^a \, exp(-\beta/\alpha)}{(\rho_n^2 \, s_f^2 \, c_p)} \left(\delta^2 Le_F(1-\Phi)\right) \times$$

$$\exp\left\{\frac{-\beta(1-\theta_{gf}^{(*)})}{1-\alpha(1-\theta_{gf}^{(*)})}\right\} = e^{-m\,n} \tag{31}$$

in which [12]

$$m\,n = 1.344m - 4m^2(1-m)/(1-2m)+ \\ 3m^3 - ln(1-4m^2), \quad \text{for} \quad -0.2 < m < 0.5 \tag{32}$$

and

$$m = \frac{r_2}{r_2 - r_1} \tag{33}$$

4. DISCUSSION

The above model is currently being implemented. Results and discussion will be presented in the next opportunity.

4.1 Figures and tables

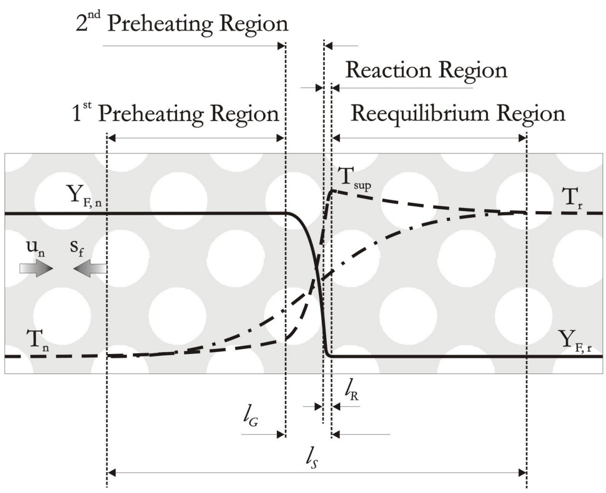

Figure 1. Rendering of the temperatures and fuel mass fraction distributions for the different characteristic length scales of the problem.

5. CONCLUSIONS

6. ACKNOWLEDGEMENTS

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