DESCRIBING THE CYCLE ASYMMETRY EFFECTS ON FATIGUE CRACK GROWTH WITH A BIPARAMETRIC EXPONENTIAL EQUATION

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Abstract. *In the literature about fatigue, one can find various papers highlighting the fact that the fatigue crack growth (FCG) data under stationary loading usually show several knees or transitions, which are changes in the slope of the da/dN-ΔK plots. Recently a new exponential equation, named αβ model, was proposed aiming to describe deviations from linearity observed in unalloyed titanium FCG data. On the other hand, the well-known cycle asymmetry effects demonstrate that the mechanical conditions governing FCG are more acurately described by two loading parameters. A method for FCG assessment with biparametric equations was also presented in previous works by our research group. The aim of the present work is to develop the two-parameter exponential model and to compare its performance to a Paris-type biparametric equation and to the conventional Elber's closure model. Two morphologically distinct materials were chosen for this investigation: Commercially pure (ASTM grade II) titanium and aluminum alloy 2524 T3, both of them of great interest for the aeronautic industry. Experimental data from FCG tests carried out under constant amplitude loading and with a wide range of stress ratios (min/max) were employed in the analyses. The superiority of the biparametric exponential model was demonstrated through the calculation of the normalized sum of residuals corresponding to the set of experimental points of each crack growth curve for both materials. The fracture surfaces of the test samples were analyzed through SEM, looking for microscopic non linearity correlation.*

Keywords: *Fatigue crack growth, modeling, cycle asymmetry, titanium, aluminum alloys.*

1. INTRODUCTION

Fatigue crack growth behavior is a main issue in the life prediction and maintenance of aircraft structures. Fracture mechanics-based methods have been used to treat this phenomenon. The description of fatigue crack growth rate (*da/dN*) as function of cyclic stress intensity factor (ΔK) as the main driving force for fatigue was introduced by Paris (1963), as shown in Eq. (1). The Paris potential equation became the canonic model for fatigue crack growth (FCG). By plotting the experimental data *da/dN* vs. ΔK in *log-log* scale the empirical constants C (the power law coefficient) and n (Paris exponent) are obtained from the linear and slope coefficients respectively.

$$\frac{da}{dN} = C(\Delta K)^n \tag{1}$$

During the last tree decades, much research efforts were concentrated in the obtainment of a generalized model to represent satisfactorily the effects of cycle asymmetry on FCG rate. Although it can be understood as the superimposition of a non-zero mean stress to a totally reversed cyclic loading, the cycle asymmetry is more usually expressed in terms of the stress ratio R. The wanted model must then represent the so-called R-effects, where R is taken as K_{min}/K_{max}. Elber (1971) introduced the concepts of crack closure (K_{cl}) and effective cyclic stress intensity factor (ΔK_{eff}) to explain the effects of R-ratio on fatigue crack growth, see Eq. (2). According to Elber's approach, Eq. (3) should represent FCG for all loading spectra.

$$\Delta K_{eff} = (K_{max} - K_{cl}) \tag{2}$$

$$\frac{da}{dN} = C(\Delta K_{eff})^n \tag{3}$$

Crack closure has been the subject of much current investigation. A continuing difficulty has been to quantitatively correlate measurements of closure with fatigue behavior in a consistent method. Furthermore, inconsistencies also have been pointed out in the assessment of the fatigue crack growth rate in terms of ΔK_{eff}. One promising alternative has been to ignore closure measurements and acknowledge that two driving forces are necessary to describe FCG. Sadananda *et al.* (1999) pointed ΔK and K_{max} as these driving forces that govern overall fatigue crack growth behavior of any material. The authors of "Unified Approach" (UA) affirm that the attempts to describe the phenomenon in terms of only one variable (e.g. ΔK) bring an incomplete understanding of the problem. The well-known Walker's approach (1970), as showed in Eq. (4), in which the crack growth rate is given in terms of ΔK and $1-R$, was one of the first models to adopt two loading parameters. However, this model doesn't consider as independent the effects of the loading parameters on the crack growth rate. In a recent work, Kujawski (2004) demonstrated this question and proposed an improvement to Walker's model, but the dependency of the constants was not broken, as shown in a previous work by our research group (Adib *et al., 2006*).

$$\frac{da}{dN} = C\Delta K^{n}\left(1 - R\right)^{n(m-1)} \tag{4}$$

Baptista *et al.* (2006) presented a potential bi-parametric model expressed by Eq. (5). The natural improvement of the Walker model would be a FCG model assuming as independent the exponents of the parameters ΔK and K_{max} or any two other parameters characterizing the driving forces at the crack tip. In this case, the constants *D, f, g* of the model would be determined for a wide range of loading conditions. Another important point is that the proposed model is capable of predicting the FCG behavior under experimental conditions different from those employed in the calculation of its constants.

$$\frac{da}{dN} = D(\Delta K)^{f}(K_{max})^{g} \tag{5}$$

1.1. Non-linear FCG behavior

For a given *R*, all of the conventional Paris-based FCG models are represented by a straight line in a *log-log* plot. However, it was observed that some metals and alloys do not show a high degree of linearity in the region of intermediate crack growth rates. Instead of a straight line the experimental points present soft transitions (non linear behavior) or hard transitions (called "knees"). Newman (1999), in his evaluation of aluminum alloy 7075-T6, argued that the transitions are consequence of material microstructure and probably they are related with grain contour crack proximity. Wanhill (1989) published a paper in which these transitions are correlated with fracture mode changes in Ti-6Al-4V alloy. Overthrowing the potential models domain, Adib (2006) proposed an empirical model initially named "αβ - Model" based on exponential function, as presented in Eq. (6). In a recent study, Adib *et al.* (2007) published a paper showing αβ-Model applied to unalloyed titanium FCG data. This approach has not only the intention to represent the linearity deviation but also FCG behavior for a wide range of loading conditions.

$$\frac{da}{dN} = e^{Y/\Delta K} = e^{\alpha}.e^{\beta/\Delta K} \tag{6}$$

The aim of the present paper is to develop a bi-parametric exponential model (αβ) and compare its performance with two other approaches, bi-parametric potential and Elber's model.

2. BI-PARAMETRIC αβ MODEL DEVELOPMENT

The use of an Arrhenius type exponential equation for FCG description was originally proposed in a master thesis by Adib (2006). This widely employed equation had not been used for FCG description. Initially the αβ Model was developed to consider the relationship between *da/dN* and ΔK as shown in Eq. (6), where the parameters α and β can be found from experimental data through the *Y* parameter method presented in Eq. (7).

$$Y = ln\frac{da}{dN}.\Delta K = \alpha\Delta K + \beta \tag{7}$$

By plotting $Y(\Delta K)$ for different *R*-ratios (a group of experimental curves), it was observed that all curves can be represented by decreasing straight lines in 4th quadrant of Cartesian plane. So, *Y* parameter can be written by a first order linear function, as in Eq. (7), in which α and β are the slope and linear coefficients respectively. At $0 < da/dN < 1$ interval, the constants are always negative. Another important observation is that all of the curves are practically parallel each other, as showed in Fig. 1. It means that a unique α value may be adopted for all of them.

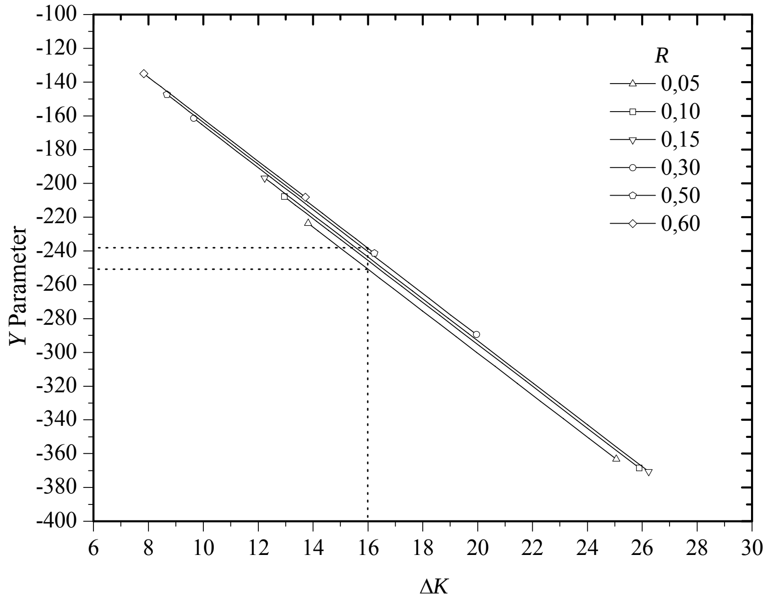


Figure 1. Graphic representation of *Y* (Δ*K*) for a group of curves

2.1. αβ model generalization

Considering all *Y* (Δ*K*) curves as parallel straight lines, a unique slope coefficient can be adopted for all *R*-ratios. If α is the same for all experiments, β is responsible for *R*-effect representation. The next step is to investigate the relationship between β and *R*. A good accuracy is achieved in linear fit by plotting β (*logR)* as showed in Fig. 2 and represented by Eq. (8).

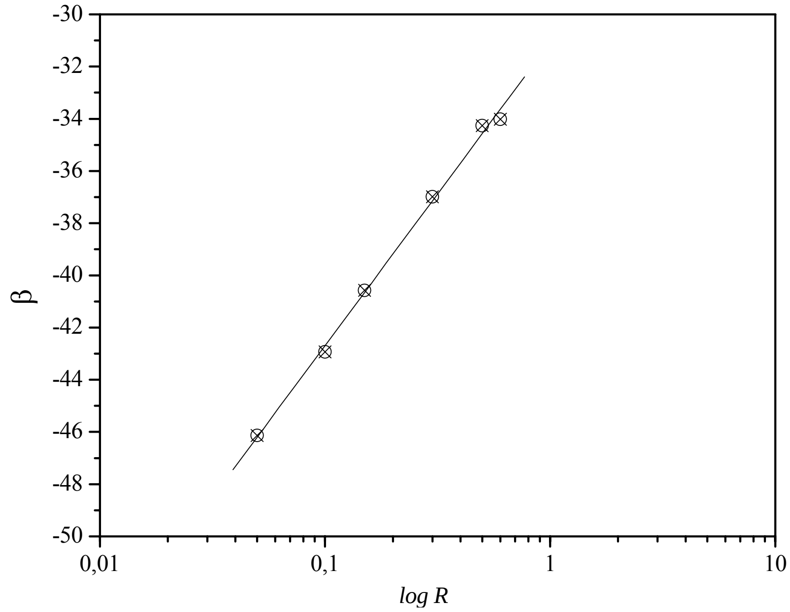


Figure 2. β as function of *log R*

$$\beta = \delta log(R) + \gamma \tag{8}$$

By substituting β in Eq. (7), the *Y* parameter can be described by a plane according to Eq. (9). The three-dimensional representation of FCG plane is shown in Fig. 3.

$$Y = \alpha\Delta K + \delta log(R) + \gamma \tag{9}$$

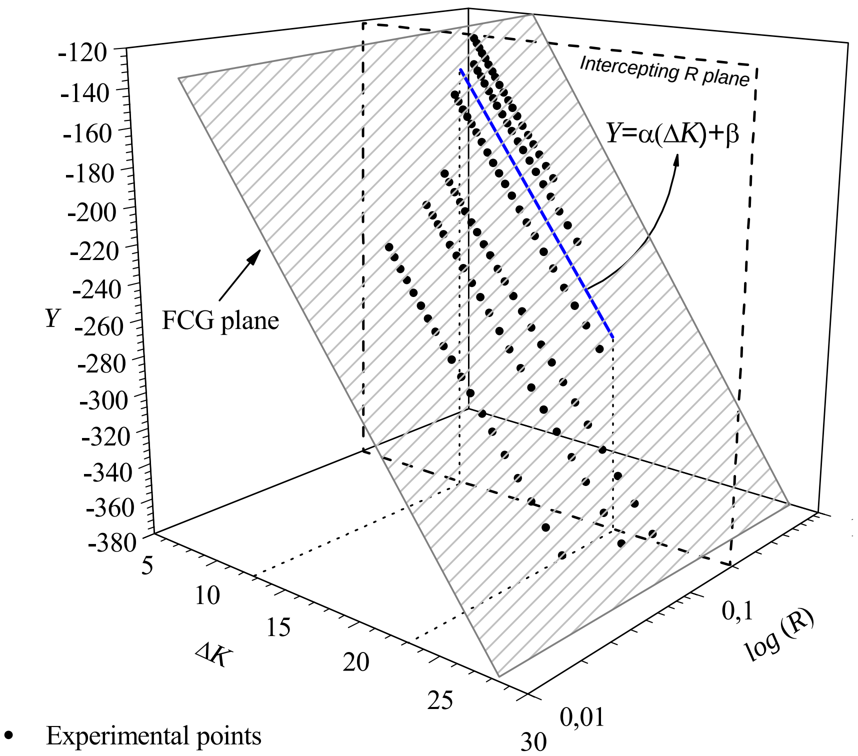


Figure 3. FCG plane formed by experimental points

The plane equation coefficients α, δ, γ are easily found through the first order linear regression in R^3 by the least square method. Finally, the substitution of *Y* parameter in Eq. (5) leads to the bi-parametric αβ Model, as represented by Eq. (10).

$$\frac{da}{dN} = e^{\alpha}.e^{\frac{\delta log(R)+\gamma}{\Delta K}} \tag{10}$$

The bi-parametric αβ Model describes FCG for a given material in function of two independent loading parameters, Δ*K* and *R*.

3. EXPERIMENTAL DETAILS

Fatigue crack growth data were collected from 2 distinct metallic materials commonly used in aerospace engineering applications: 2524-T3 aluminum alloy and ASTM grade II commercially pure titanium. Table 1 gives the materials properties and the specimen thickness for each material. All of the specimens were of Compact Tension type and cut in the LT orientation. The tests were conducted at room temperature in laboratory air and followed the standard practice for measurement of fatigue crack growth rates (ASTM E647-95a). The adopted load ratios were R=0.05, 0.10, 0.15, 0.30, 0.50 and 0.60.

Table 1. Materials properties

Material	Yield Str. (MPa)	Tensile Str. (MPa)	Elongation (%)	Thickness (mm)
Al 2524 - T3	340	450	21	6.35
ASTM grade II Ti	325	463	34	7.00

Crack closure measurements were performed for both tested materials order to allow ΔK_{eff} calculations to be used in FCG modeling according to Elber's approach. In this case, a COD (crack opening displacement) clip-on gage was attached to the specimens, as shown in Fig. 4. The linear-quadratic spline method was employed in closure calculations. In this method, the "load vs. COD" plots are modeled using two-section least square fit curves.

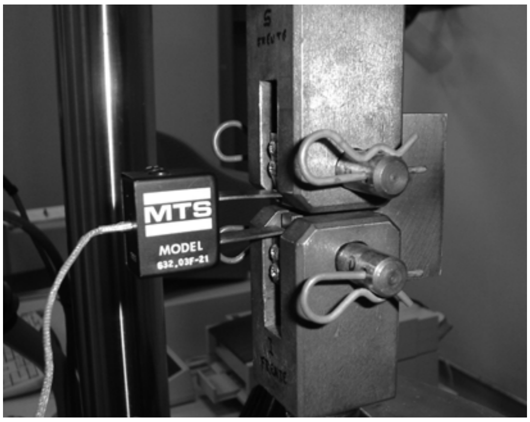


Figure 4. Displacement gage mounted in specimen fixed in fracture mechanics grips.

4. RESULTS AND DISCUSSION

This section is divided into four parts: Elber model application in which the Elber's approach is developed for both of the studied materials; bi-parametric model application in which the two-parameter potential equation is applied to the two available data sets; bi-parametric αβ Model application in which the two-parameter exponential equation is applied for both of the studied materials; and a comparative analysis between the results of these three models.

4.1. Elber's model application

From the closure measurements, the values of ΔK_{eff} were obtained according to Elber`s approach. In Figures 5 and 6 respectively the experimental results "FCG rate vs. ΔK_{eff}" of the ASTM grade II Ti and Al 2524-T3 tests are plotted in *log-log* scale. It is easily seen that the six load ratios become closer to each other when ΔK_{eff} is considered. However, they don't represent one single curve, as expected by the crack closure theory (some inevitable noise in closure calculations may contribute to this result). Table 2 gives the Elber's constants for both materials.

Table 2. Elber's model constants

Material	$C \times 10^{10}$	*n*	*Corr*
ASTM grade II Ti	5,17869	2,22614	0,96796
Al 2524 -T3	4,49686	2,61703	0,97513

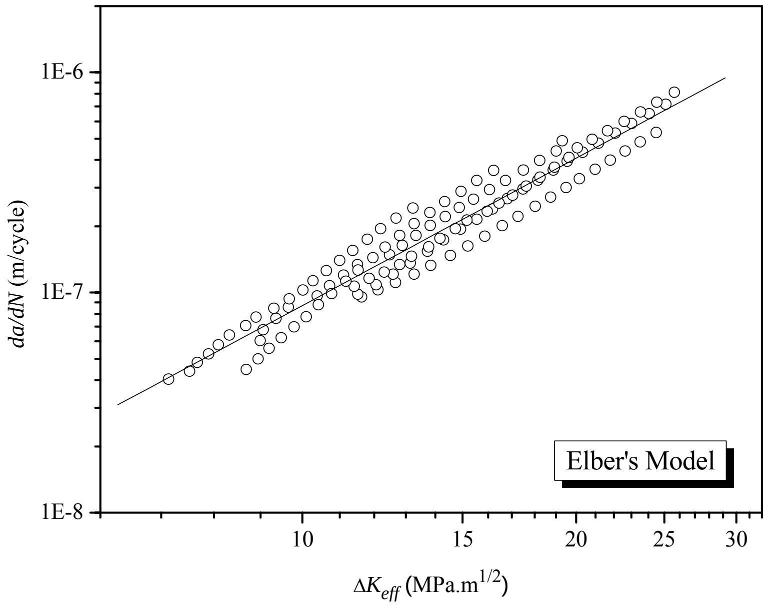


Figure 5. *da/dN* vs. ΔK_{eff} , ASTM grade II Ti

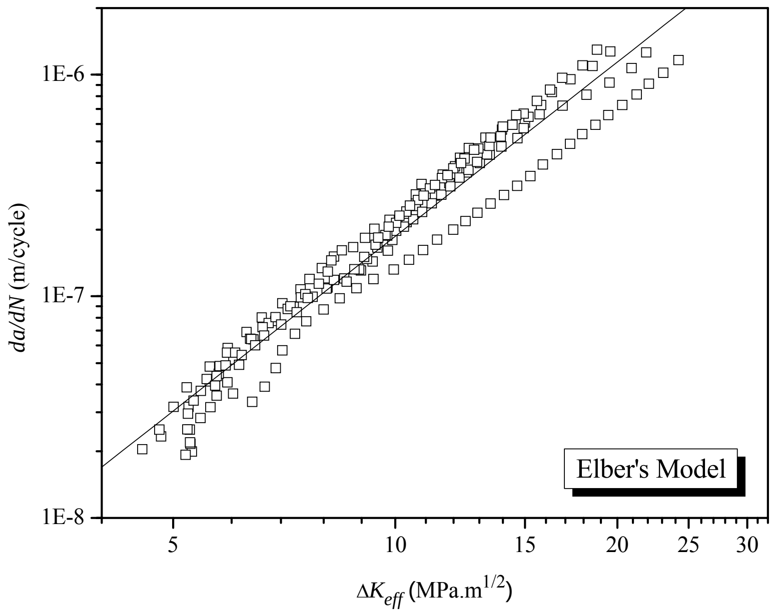


Figure 6. *da/dN* vs. ΔK_{eff} , Al 2524-T3

4.2. Bi-parametric potential model application

Equation (4) was applied for both materials adopted for this work, using for each of them six loading ratios simultaneously (see experimental details). The results are shown in terms of *da/dN*-Δ*K* in *log-log* scale plots for the ASTM grade II Ti and Al 2524-T3, see Figures 7 and 8 respectively. The bi-parametric potential equation gives a set of parallel lines which, if don't superimpose the experimental data, give a reasonable description of the *R*-effects for each of the evaluated materials. The sets of obtained model constants are given in Tab. 3. It must be clear that they are not material's constants, but the result of error minimization for the available data set. Different data can lead to different groups of constants for the same material, each group leading to approximately similar results of crack growth behavior.

Table 3. Potential model constants

Material	$D \times 10^{11}$	*f*	*g*
ASTM grade II Ti	6,49	2,15	0,729
Al 2524 -T3	9,01	2,10	0,946

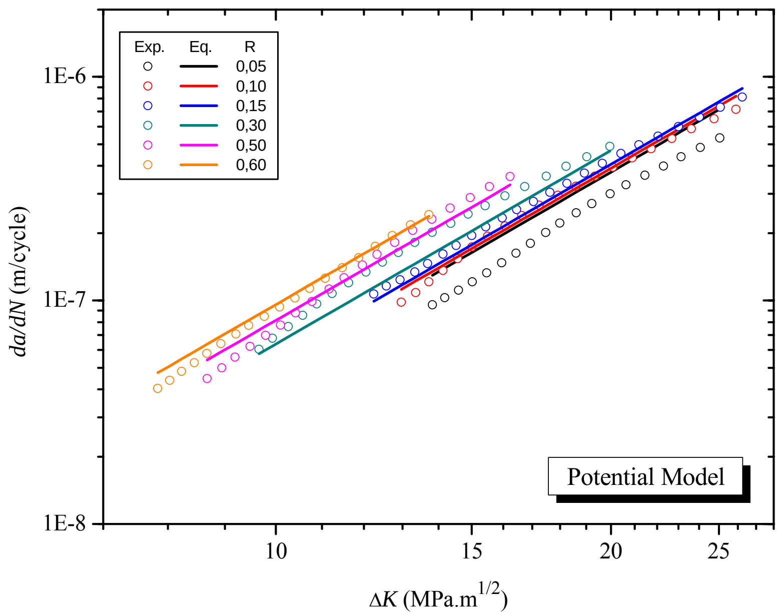


Figure 7. *da/dN* vs. Δ*K*, ASTM grade II Ti

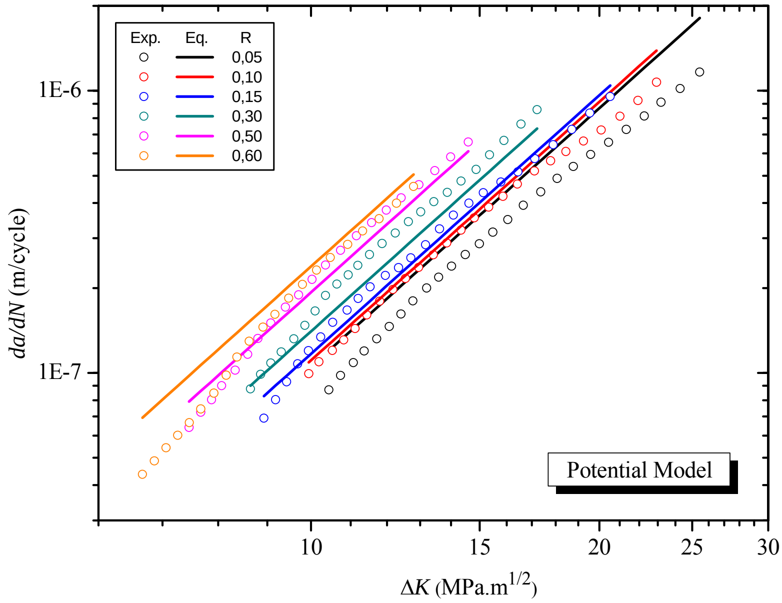
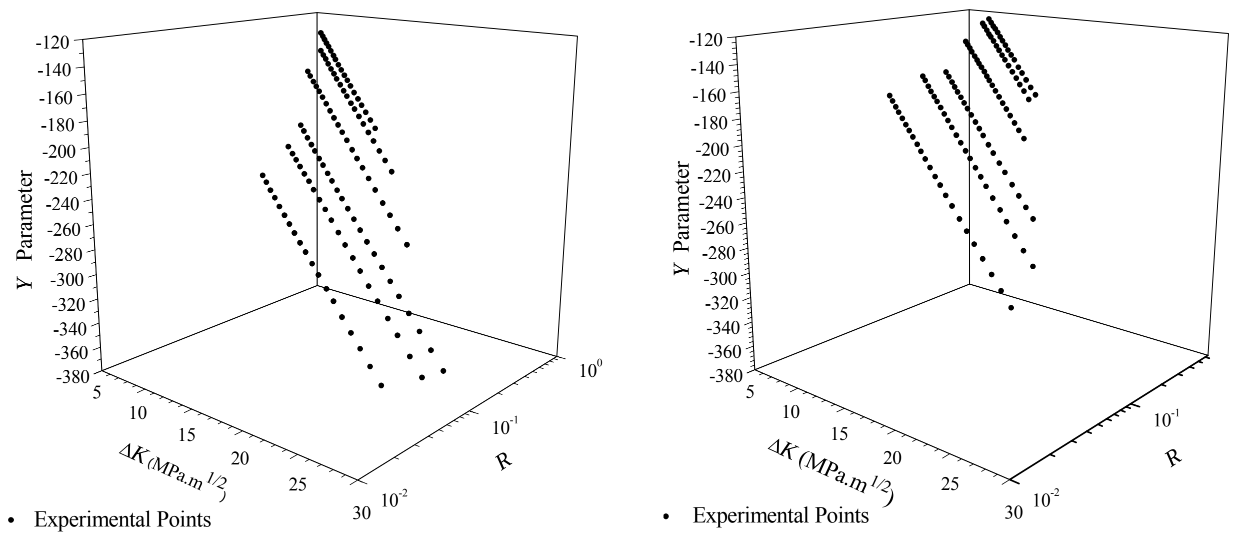


Figure 8. *da/dN* vs. Δ*K*, Al 2524-T3

4.3. Bi-parametric αβ – Model application

Three-dimensional plots in terms of *Y*, Δ*K* and *R* were built from ASTM grade II Ti and Al 2524-T3 experimental data, as shown in Figure 9 (a) and (b).



(a) (b)

Figure 9. *Y* (Δ*K, R*), (a) ASTM grade II Ti, (b) Al 2524-T3.

The plane equation coefficients α, δ, γ were independently calculated for ASTM grade II Ti and Al 2524-T3 through the linear regression method. They are presented in Equations (11) and (12) respectively.

$$Y = -12{,}310(\Delta K) + 13{,}282 \, log(R) - 36{,}145 \quad (11)$$

$$Y = -11{,}960(\Delta K) + 11{,}500 \, log(R) - 30{,}878 \quad (12)$$

By substituting *R*-ratios values on such equations the "α" and "β" constants were obtained for each material as shown in Tab. 4. The FCG curves reconstitutions were performed for ASTM grade II Ti and Al 2524-T3 through the Δ*K* substitution in all equations presented in the Tab. 4, as shown in Fig. 10 and 11.

Table 4. The "α" and "β" constants

R-ratios	Material	$Y = \alpha(\Delta K)+\beta$	Material	$Y = \alpha(\Delta K)+\beta$
0,05	ASTM grade II Ti	Y=-12,310(ΔK)-53,425	Al 2524-T3	Y=-11,960(ΔK)-45,840
0,10		Y=-12,310(ΔK)-49,427		Y=-11,960(ΔK)-42,379
0,15		Y=-12,310(ΔK)-47,088		Y=-11,960(ΔK)-40,354
0,30		Y=-12,310(ΔK)-43,090		Y=-11,960(ΔK)-36,892
0,50		Y=-12,310(ΔK)-40,143		Y=-11,960(ΔK)-34,340
0,60		Y=-12,310(ΔK)-39,091		Y=-11,960(ΔK)-33,430

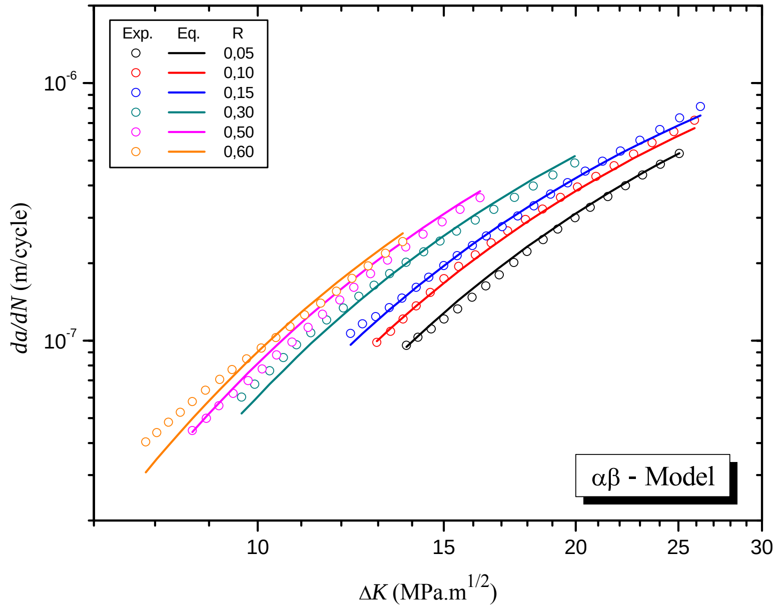


Figure 10. *da/dN* vs. ΔK, ASTM grade II Ti

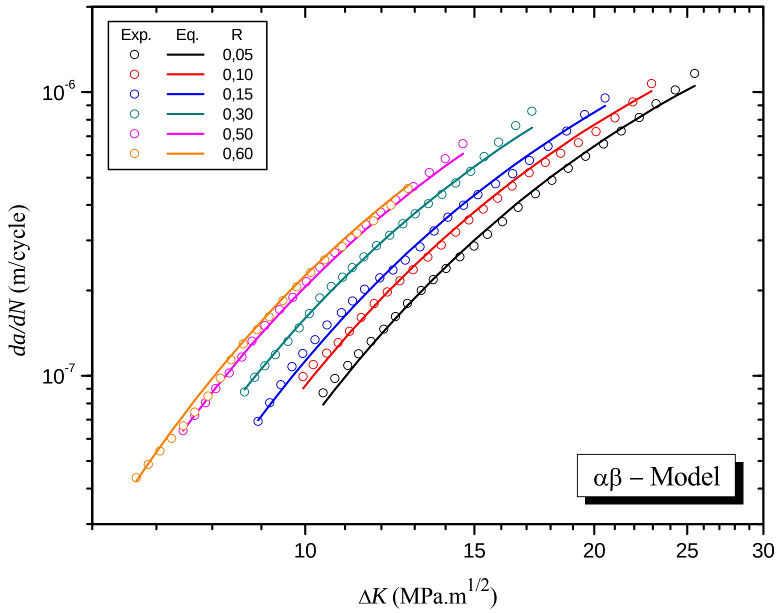


Figure 11. *da/dN* vs. ΔK, Al 2524-T3

4.4. Model comparisons

In order to compare the models quantitatively, it was adopted the criterion of normalized sum of residuals, as shown in Eq. (13). Table 5 summarizes the results for both materials. These results in terms of residual percentages represent the models capacity in reconstitute FCG curves. In other words, the lower is the residue, the higher is the accuracy.

$$\%\text{Residue} = \sum_{i=1}^{q} \sqrt{\left(\frac{da/dN_{exp} - da/dN_{calc}}{da/dN_{exp}} \right)^2} \Bigg/ q \tag{13}$$

Table 5. Normalized sum of residuals

Material	Model ⇔ % Residue	***R*=0,05**	***R*=0,10**	***R*=0,15**	***R*=0,30**	***R*=0,50**	***R*=0,60**
ASTM grade II Ti	Elber	2,68	5,27	2,87	4,90	2,96	1,04
	bi-parametric Potential	0,299	0,056	0,065	0,141	0,093	0,063
	bi-parametric αβ − Model	0,036	0,033	0,028	0,069	0,062	0,106
Al 2524-T3	Elber	4,51	4,79	6,66	4,77	5,81	5,71
	bi-parametric Potential	0,33	0,170	0,073	0,124	0,100	0,25
	bi-parametric αβ − Model	0,039	0,040	0,042	0,028	0,030	0,043

4.5. Fractographic Analysis

The fracture surfaces of the specimens were observed via SEM (scanning electron microscope) operating in the secondary electrons mode, revealing that the fracture mechanisms gradually changed during crack propagation for ASTM grade II Ti, as much as, Al 2524-T3. It can be seen in Fig. 12 (a) and (c) that, for lower ΔK values, the fracture is of crystallographic type, with the predominance of facets for both materials. As ΔK increases, the fracture surface remains rough (reflecting the influence of microstructure), but the presence of fatigue striations is intensified, until they are sprayed for almost all of the grains, see Fig. 12 (b) and (d). It is believed that the transition from facets to striation formation (a more energy-consuming process) is behind the FCG no linear behavior, but a mathematical correlation between the fracture mechanisms and crack growth rate demands further research.

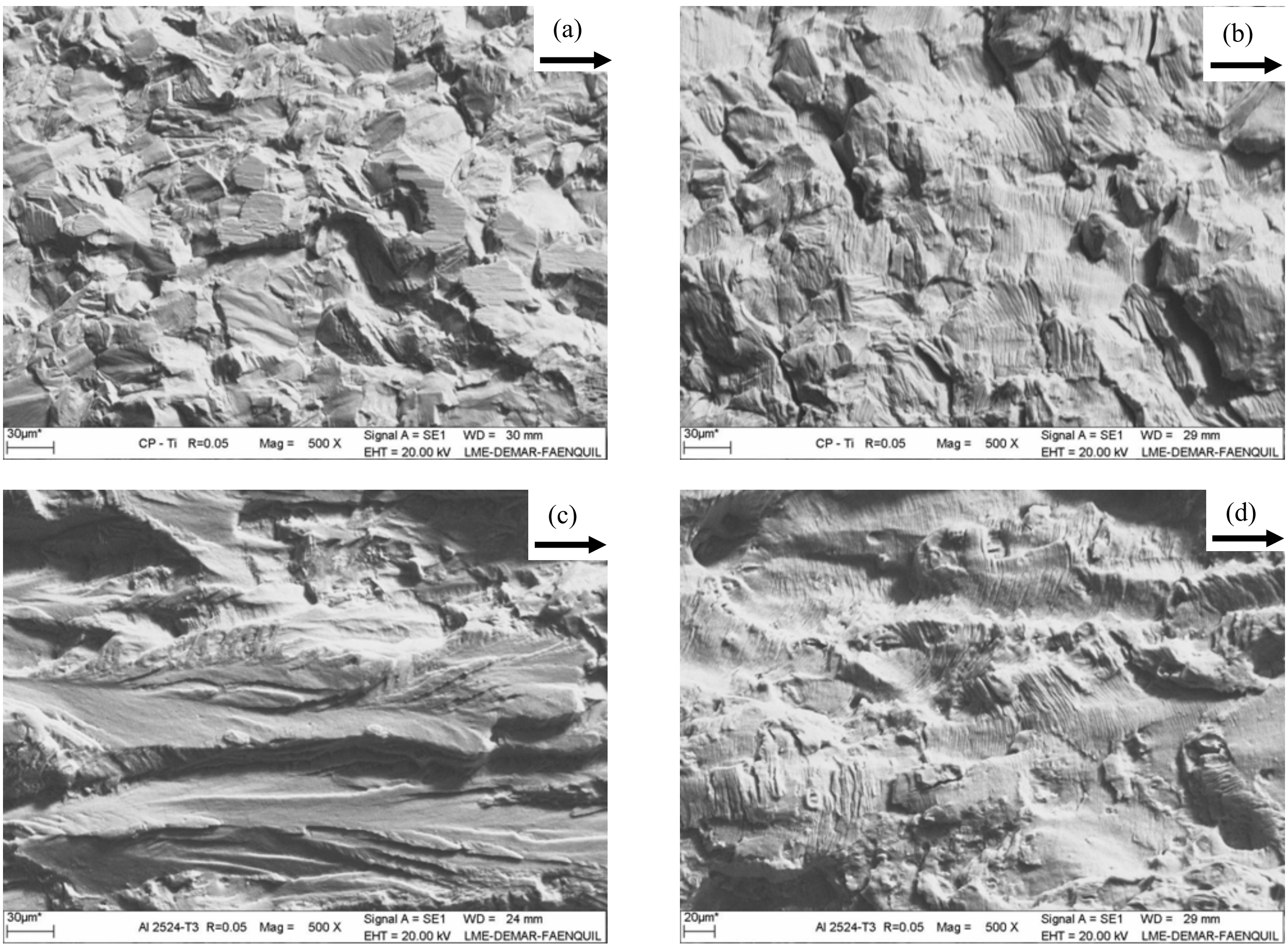


Figure 12 – SEM fractographs, (a) ASTM grade II Ti R=0,05 $\Delta K \approx 14$ MPa.m$^{1/2}$; (b) ASTM grade II Ti R=0,05 $\Delta K \approx 24$ MPa.m$^{1/2}$; (c) Al 2524-T3 R=0,05 $\Delta K \approx 14$ MPa.m$^{1/2}$; (d) Al 2524-T3 R=0,05 $\Delta K \approx 24$ MPa.m$^{1/2}$.

5. CONCLUSION

The experimental results presented in this work demonstrated that the FCG curves of ASTM grade II titanium and aluminum alloy 2524-T3 deviate from the linear relationship predicted by the Paris equation. It was shown that the bi-parametric αβ Model, defined in terms of an exponential equation, describes more adequately the FCG behavior of these materials. Besides, in the majority of cases, bi-parametric αβ Model presented residues lower than the presented by potential model and much lower than the presented by Elber's model. This new generalized model can easily describe FCG for a range of stress ratios and allow for defining a 3D plane of sub-critical crack propagation. Bi-parametric αβ Model is more versatile than the bi-parametric Paris-based potential model and Elber's approach. It qualifies as a promising tool to describe the FCG curves.

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7. RESPONSIBILITY NOTICE