CRACK IDENTIFICATION IN A SIMPLE STRUCTURE: A NUMERICAL EXAMPLE AND PHYSICAL IMPLEMENTATION

Flavio Augusto Presezniak, faugusto_baja@yahoo.com.br
Carlos Alberto Bavastri, bavastri@utfpr.edu.br
Universidade Tecnológica Federal do Paraná

Juan Elias Perez Ipiña, pipina@uncoma.edu.ar
Universidad Nacional Del Comahue

Abstract. *The failure in structures can cause many financial and human damages. With the objective to prevent these failures, some techniques have been introduced in the last years. The concepts of fracture mechanics create models able to determine an admissible failure level without the unstable crack propagation. The Concepts of predictive maintenance search to identify damages though structural analysis, creating an acceptable level by previous knowledge. In the last years, the concept of damage prognosis appeared. This methodology uses numerical and experimental structural responses to identify the damage. With the aid of fracture mechanics principles, it's possible to determine with a higher level of precision, the remaining structure life. In this work the fundamentals of the application of this methodology is presented. Wavelet transform concepts are used as a tool for the crack identification through vibration modes. The finite element theory is reviewed to determinate the mass and stiffness matrix of a simple structure which is used to solve the eigenvalue problem and to find the vibration modes. The crack is modeled as a torcional spring connected to the elements near the crack position. A numerical-experimental example is presented to show the fundamentals of this methodology in a simple structure.*

***Keywords**: Damage Prognosis, Failure, Vibration, Modal Analysis, Finite Element Method*

1. INTRODUCTION

The failure in structures can cause several financial and human damages. To prevent these incidents, several studies have been made to identify the crack propagation mechanisms and the critical crack size. This new area was defined as fracture mechanics.

After the incident with the Aloha Airlines airplane in 1988, it was researched through periodic measurements, the load knowledge, metallurgy concepts and failure prediction models, establish some criteria to verify if the structure can keep working safely, preventing unexpected and catastrophic failures due the unstable crack propagation. This new area, actually under development, receives the name of Damage Prognosis (Farrar *et at,* 2005).

Knowing of the load, the material mechanical and fracture properties and the crack size, it is possible to create a failure criteria based on fracture mechanics concepts (Ipiña, 2004).

Several methods are able to perform crack identification, as X-ray, ultrasound, etc. However, methods based on vibration measurements have received special attention recently because of in-service variation of the dynamic characteristics, as naturals frequency or vibration modes (Li et al, 2005). Thus, several works introducing mathematical models to identify cracks in simple structures have been published recently. Another concept recently introduced was the wavelet transform, which can make possible to identify the effect caused on the vibration mode due to a crack. In this way, the crack position through the discontinuity introduced on the vibration mode by the crack is possible to be identified by applying the wavelet transform (Loutridis et al, 2004).

2. WAVELET TRANSFORM

The wavelet transform have some interesting properties when applied to complex functions transform. In certain cases, the Fourier transform needs several numbers of coefficients while the wavelet transform can make the same representation with a reduced number of coefficients (Barbosa, 2001). The most important characteristic of the wavelet system, is that the functions are discrete in time and frequency domains, this means, that the wavelet functions are defined in a restricted time interval. The introduction of the wavelet system concept is necessary to the realization of this transform. A wavelet system is composed by a scale function, represented by ϕ and the wavelet function, represented by ψ, being continuous or discrete functions, which characterizes both types of transforms.

2.1. Daubechies Wavelet System

The generation of the scale functions of the Daubechies wavelet system needs certain restrictions to define them in the integer values of the interval where the function is defined (Daubechies, 1988).

These relations present some coefficients, called filter coefficients, defined by h_k and calculated in order to maintain true the following relation (Stark, 2005):

$$\phi(x) = \sum_{k=0}^{2N-1} 2^{j/2} h_k \phi(2^j x - k) \tag{1}$$

where N means the Daubechies wavelet family (1 to 9), j is a scaling parameter function and k a translation parameter function. In Daubechies (1988), the restrictions used to calculate the filter coefficients value can be found.

Some filter coefficients g_k, are also defined by the wavelet functions by the fowling relation

$$\psi(x) = \sum_{k=0}^{2N-1} g_k \phi(2x - k) \tag{2}$$

The coefficients h_k and g_k are related by

$$g_k = (-1)^k h_{1-k} \tag{3}$$

.

2.2. Multiresolution Analysis

The multiresolution property allows to obtain different solutions with different approximations levels. This concept shows that a solution with a higher approximation level can be found with an addiction to a lower solution level of a "detail" (Stark, 2005). This concept can be mathematically understood by the utilization of sub-spaces composed by the scale and wavelet functions. Starting from a solution in a j level, represented in a V_j subspace, it's possible to verify that the representation of a function in a $L^2(\Re)$ is necessary:

$$V_j \subset V_{j+1} \Leftrightarrow \ldots V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \subset \ldots \tag{4}$$

These sub-spaces must satisfy the following relations:

$$\bigcap_{j=-\infty}^{\infty} V_j = 0 \qquad \bigcup_n V_n = L^2(\Re) \tag{5}$$

In this way, if a function $f(x)$ is defined in a V_j sub-space, a function $f(2x)$ is defined in a V_{j+1} sub-space, this means that the sub-spaces must be related as follows:

$$f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1} \tag{6}$$

The sub-spaces are generated by the translations of the scale functions and the relation

$$f(x) \in V_j \Leftrightarrow f(x+1) \in V_j \tag{7}$$

must be satisfied.

The multiresolution analysis brings to a decomposition of the $L^2(\Re)$ space in V_j sub-spaces which are used to approximate the functions.

These sub-spaces, which compose an orthonormalized base in $L^2(\Re)$ from a given j resolution level (scale j), can be defined as an orthogonal complement (V_{jc}) of V_j in V_{j+1}, that means, they are the necessary complement to pass from a j resolution level to a $j+1$ resolution level, being mathematically defined as:

$$V_{j+1} = V_j \oplus W_j \tag{8}$$

This means that the sub-space W_j have the "detail" or "information" necessary to pass from an approximation in a j resolution to a $j+1$. This can be represented graphically in Fig. 1.

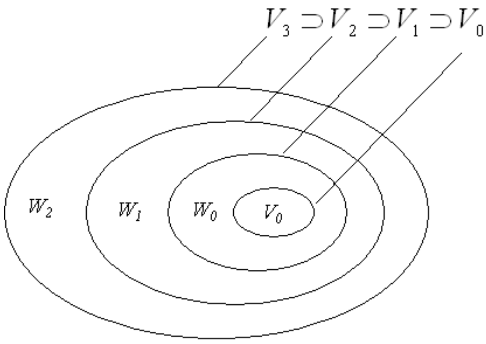

Figure 1: Graphic representation of multiresolution analysis

2.3. Wavelet Transform and Filters

The discrete wavelet transform can be improved as a signal processing analysis by using a filter bank, with the coefficients given by the h_k and g_k values, calculated from the scale and wavelet function definitions (Burrus *et al*, 1997). The derivation of this property can be found in Burrus *et al* (1997). Esquematically, this analysis can be performed as represented in Fig. 2.

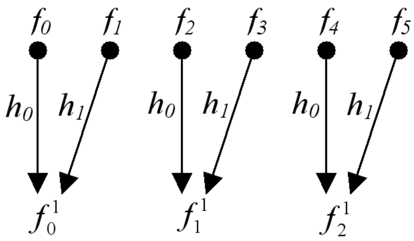

Figure 2: Signal filtering using scale function filters coefficients.

3. STRUCTURAL MATHEMATICAL MODEL

The model of a simple cracked structure, by finite elements, has to be made to allow the wavelet transform application, given the possibility to identify the discontinuity caused by the crack. In this way, the model must consider the local stiffness variation caused by the crack presence, and allows the stiffness variation during the optimization procedure for the crack parameters identification.

3.1. Finite Element Model

The modeling is made by using the two most commons beam elements. The Timoshenko beam element is used in the crack non-adjacent elements. In these elements the Euler-Bernoulli beam elements are used. This model seeks to minimize the effects of the shear stress, considered in the Timoshenko element although not applied in the crack model, which uses a spring to simulate the crack.

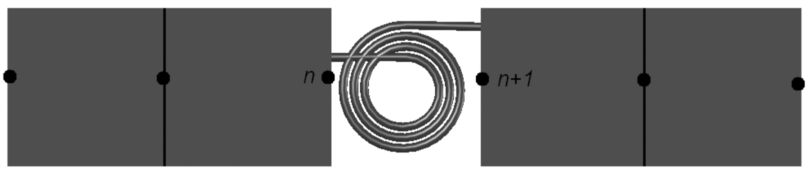

Figure 3: Beam model

Because of a torcional spring is employed to model the crack, some considerations must be made in the global mass and stiffness matrix assembly. The connection between the n and $n+1$ elements (Fig. 3) is made only by their displacements, that is, they have the same displacement although different rotations. Thus, the stiffness and mass matrices near the crack are given by:

$$[K_t]=\begin{bmatrix} \ddots & & & & & & & \\ & k_{11}^n & k_{12}^n & k_{12}^n & k_{14}^n & 0 & 0 & 0 \\ & k_{21}^n & k_{22}^n & k_{23}^n & k_{24}^n & 0 & 0 & 0 \\ & k_{31}^n & k_{32}^n & k_{33}^n + k_{11}^{n+1} & k_{34}^n & k_{12}^{n+1} & k_{13}^{n+1} & k_{14}^{n+1} \\ & k_{41}^n & k_{42}^n & k_{43}^n & k_{44}^n + K_t & -K_t & 0 & 0 \\ & 0 & 0 & k_{21}^{n+1} & -K_t & k_{22}^{n+1} + K_t & k_{23}^{n+1} & k_{24}^{n+1} \\ & 0 & 0 & k_{31}^{n+1} & 0 & k_{32}^{n+1} & k_{33}^{n+1} & k_{34}^{n+1} \\ & 0 & 0 & k_{41}^{n+1} & 0 & k_{42}^{n+1} & k_{43}^{n+1} & k_{44}^{n+1} \\ & & & & & & & \ddots \end{bmatrix} \quad \begin{matrix} v_1 & \theta_1 & v_2 & \theta_2 & \theta_3 & v_4 & \theta_4 \end{matrix} \tag{9}$$

$$[M_t]=\begin{bmatrix} \ddots & & & & & & & \\ & m_{11}^n & m_{12}^n & m_{12}^n & m_{14}^n & 0 & 0 & 0 \\ & m_{21}^n & m_{22}^n & m_{23}^n & m_{24}^n & 0 & 0 & 0 \\ & m_{31}^n & m_{32}^n & m_{33}^n + m_{11}^{n+1} & m_{34}^n & m_{12}^{n+1} & m_{13}^{n+1} & m_{14}^{n+1} \\ & m_{41}^n & m_{42}^n & m_{43}^n & m_{44}^n & 0 & 0 & 0 \\ & 0 & 0 & m_{21}^{n+1} & 0 & m_{22}^{n+1} & m_{23}^{n+1} & m_{24}^{n+1} \\ & 0 & 0 & m_{31}^{n+1} & 0 & m_{32}^{n+1} & m_{33}^{n+1} & m_{34}^{n+1} \\ & 0 & 0 & m_{41}^{n+1} & 0 & m_{42}^{n+1} & m_{43}^{n+1} & m_{44}^{n+1} \\ & & & & & & & \ddots \end{bmatrix} \quad \begin{matrix} v_1 & \theta_1 & v_2 & \theta_2 & \theta_3 & v_4 & \theta_4 \end{matrix} \tag{10}$$

being the values of the matrix elements given by the Euler-Bernoulli model. For the other element of the global matrix the Timoshenko model is used.

3.2. Crack Stiffness Determination

The stiffness of the torcional spring was obtained from several empiric models, through several different cross sections by fitting a flexibility curve as a function of the crack characteristics. Dimarogonas (1997), presents the model used in a rectangular cross section beam, being this model given by:

$$c=\frac{6\pi h}{bEI}F\left(\frac{a}{h}\right) \tag{11}$$

where a is the crack size, b the height, h the width, E the young modulus and I the second order inertia moment. The F function is given by:

$$F\left(\frac{a}{h}\right)=1{,}86\left(\frac{a}{h}\right)^2-3{,}95\left(\frac{a}{h}\right)^3+16{,}37\left(\frac{a}{h}\right)^4+37{,}22\left(\frac{a}{h}\right)^5+76{,}81\left(\frac{a}{h}\right)^6+ \\ 126{,}9\left(\frac{a}{h}\right)^7+172{,}5\left(\frac{a}{h}\right)^8-144\left(\frac{a}{h}\right)^9+66{,}6\left(\frac{a}{h}\right)^{10} \tag{12}$$

And the stiffness is given by:

$$K_t=\frac{1}{c} \tag{13}$$

4. VIBRATION MODES DETERMINATION

The vibration modes are calculated by the not-damping-eigenvalues problem given by:

$$\left[s^2 M + K\right]\phi = 0 \tag{14}$$

The eigenvalues of this equation are obtained in pairs, pure and conjugates imaginaries, from the following equation:

$$\begin{aligned} s_j &= i\,\Omega_j \\ s_j^{\,*} &= -i\,\Omega_j \end{aligned} \tag{15}$$

This allows that equation (14) can be rewritten as follows:

$$K\,\phi = \Omega^2 M\,\phi \tag{16}$$

The solution of this problem gives directly to Ω_j^2 and ϕ_j. In a matrix form, this solution can be written as:

$$\Lambda = diag\left(\Omega_j^2\right),\ \Phi = \left[\phi_1, \phi_2, \phi_3, \ldots, \phi_n\right] \tag{17}$$

5. EXAMPLE

In order to validate the methodology presented in this work, a numerical-experimental example was proposed. This example seeks to identify the location of a crack in a structure from the discontinuity induced in the vibration mode.

The beam characteristics are presented in Tab. 1.

Table 1: Beam characteristics

Dimension	
Length	0.51 m
Heigth	0.013 m
width	0.025
Material	
Young modulus	207 Mpa
Density	7850 kg/m2
Poison's ratio	0.29
Crack	
Size	0.013 m
Location	0.17 m

The measurements have been made in a free-free condition.

5.1. Numerical Analysis

Considering the models presented in this work, a finite element model was developed in MATLAB to obtain the mass and stiffness matrices, necessary to solve the eigenvalues problem and wavelet transform application.

The finite element model was developed with 250 elements, being the crack located in the 80th element. The crack was modeled as a torcional spring, as presented in the section 3.2 of this work.

The first two vibration modes were calculated, and the wavelet transform applied to each one of this modes. The results are presented in Fig. 4.

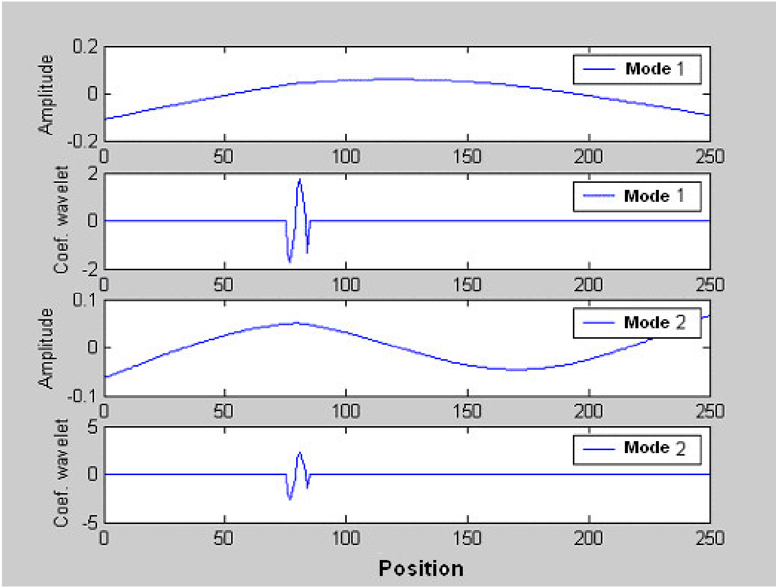

Figure 4: Wavelet transform applied to the first two vibration modes

It is possible to verify the identification of the crack location though the wavelets coefficients. The values of these coefficients have been improved in the position where the crack has been introduced. In a general way, the values of these coefficients vary with the crack size, which allows to identify the crack size.

5.2 Experimental Analysis

To verify the methodology presented, an experimental analysis was performed. A beam with the same characteristics of the used in the numerical analysis was divided in 50 elements and the crack was located between the 16^{th} and the 17^{th} elements. The first vibration mode was measured with the beam supported over a foam layer to simulate the free-free condition. The measurements were done by using an impulse force hammer 086C04 and a shear accelerometer 352C68.

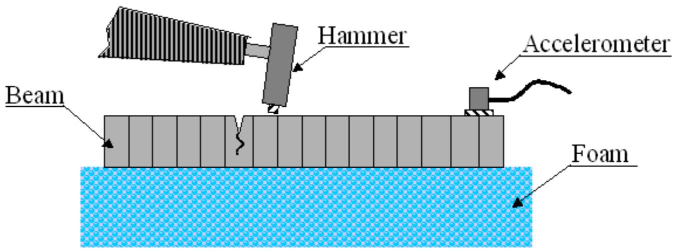

Figure 5: Experimental procedure

The modal analysis was performed by using the software ICATS. The first mode can bee seen in Fig. 6.

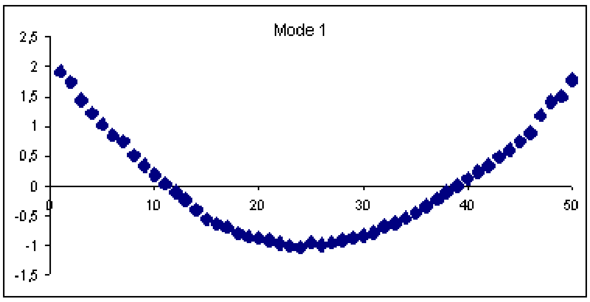

Figure 6: Experimental first vibration mode

Some discontinuities in the first mode can be observed, they are induced by experimental errors. The application of the wavelet transform filtered this information, not allowing, in a first approximation, the crack identification

Therefore the multiresolution analysis concept was used. In the first two levels, the wavelet transform filtered the signal discontinuities induced in the measurement procedure, which correspond to a high frequency noise. The noise eliminated is presented in Fig. 7.

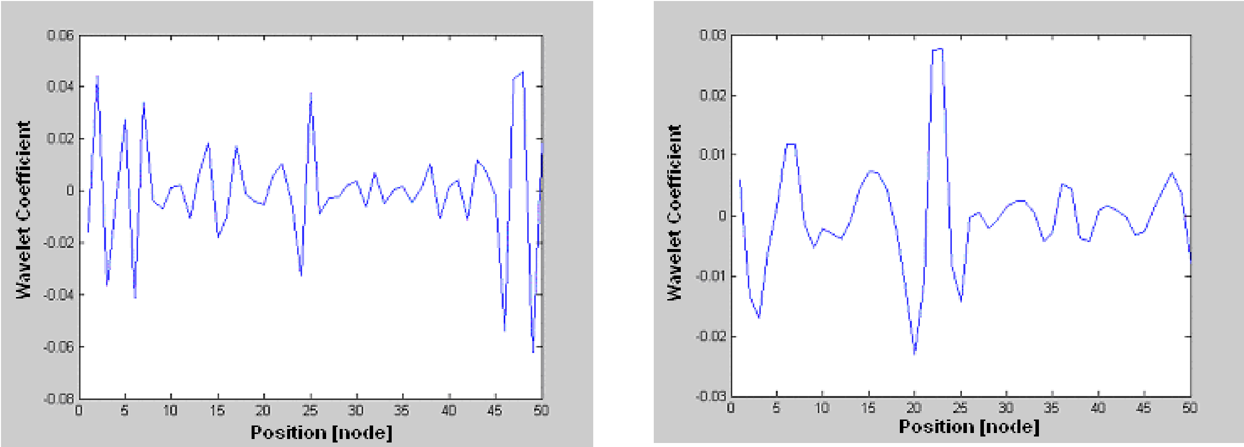

Figure 7: Eliminated noise by wavelet transforms

Applying a level three of wavelet transform, it was possible to verify the amplification of the wavelet detail coefficients in the crack region (Fig. 8).

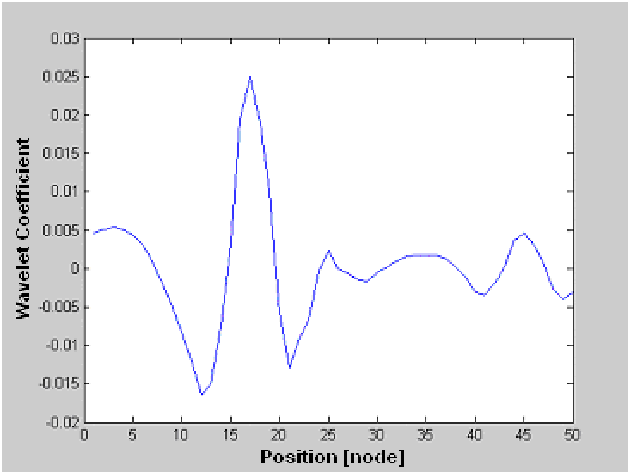

Figure 8: Detail of level 3 of the Wavelet transform

6. CONCLUSIONS

A methodology using a torcional spring as crack model was reproduced. To improve this methodology, a combined Timoshenko and Euler-Bernoulli finite element model for the simple structure with a crack was presented.

A modal analysis of a free-free cracked beam was performed and the first vibration mode was experimentally measured.

The concept of wavelet transform was reviewed. A methodology to detected the effect of the crack using this technique was presented. This procedure showed capable to identify small discontinuities from a signal, and it can be applied to determine small shape variations caused by the crack in the first vibration mode.

The numerical-experimental example of a free-free beam showed the difficulty to identify the effect of a crack in the response (vibration mode). The mode presented discontinuities as a consequence of the numerical-experimental analysis, that made difficult its identification. Nevertheless, the wavelet transform made possible the identification of the crack position in a higher level of resolution, filtering the noise generated during the measurements.

7. AKNOWLEDGEMENTS

This work was supported by the National Council for Scientific and Technological Development, CNPq, Brazil. One of the authors received support from CONICET, Argentina.

9. REFERENCES

Barbosa, A.R., 2001, *Wavelets no Intervalo em Elementos Finitos.* Dissertação de Mestrado em Engenharia de Estruturas, Instituto Superior Técnico, Lisboa.

Burrus, C. S., Gopinath, R. A., Guo, H., 1997, *Introduction to Wavelets and Wavelet Transforms*. 1ª Ed. New Jersey, Ed. Prentice Hall.

Daubechies, I., 1988, "Orthonormal bases of compactly supported wavelets". *Comm. Pure and Appl. Math*. Vol. **41**, pp. 909–996.

Dimarogonas, A.D., 1996, "Vibration of cracked structures: a state of the art review". *Engineering of Fracture Mechanics*, Vol. **55** n. 5 pp. 831-857.

Farrar, C. R., Lieven, N. A. J., Bement, M. T., 2005, "An Introduction to Damage Prognosis", Chaapter 1, Damage Prognosis for Aerospce, Civil and Mechanical Systems, Ed vy D. J. Inman et al, J. Wiley & Sons.

Ipiña, J. E. P., 2004. *Mecánica de Fractura*, 1ª ed. Buenos Aires, Librería y Editorial Alsina.

Li, B., Chen, X. F., Ma, J. X., He, Z. J., 2005, "Detection of crack localization and size in structures using wavelet finite element methods". *Journal of Sound and Vibration*, Vol. **285**, pp. 767-782.

Loutridis, S., Douka, E., Trochidis, A., 2004, "Crack identification in double-cracked beams using wavelet analysis". *Journal of Sound and Vibration*, Vol **277**, pp. 1025-1039.

Stark, H. G., 2005, *Wavelets and Signal Processing*. 1ª Ed. Springer

10. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.