OPTIMIZATION OF THE WORKSPACE VOLUME OF 3R MANIPULATORS USING A HYBRID METHODOLOGY

Sezimária de Fátima Pereira Saramago
Giovana Trindade da Silva Oliveira
College of Mathematics, Federal University of Uberlândia, Santa Mônica, CEP: 38408-100, Uberlândia, MG, Brazil.
saramago@ufu.br

Paulo Roberto Bergamaschi
Federal University of Goiás – Campus Catalão
Av. Dr. Lamartine Pinto de Avelar, 1120, Setor Universitário, CEP: 75.704-020, Caixa Postal 56, Catalão, GO, Brazil.
prbergamaschi@gmail.com

Abstract. *A fundamental feature of a manipulator is the capacity of its workspace because it influences on the manipulator design, in the manipulator position in the work environment and in the its dexterity. In the case of 3R manipulators, it is represented by a revolution solid. Thus, in this paper the workspace volume is calculated based on the area of its radial section. An optimization problem is formulated with the objective of determining the geometric parameters of 3R manipulators so that the maximum workspace volume is obtained. The maximization is accomplished forcing the workspace to occupy the largest set of points inside of a pre-established region, which in this case is a cylinder whose axis coincides with the axis of rotation of the workspace. In the optimization procedure a function that establishes the difference between the volume of the pre-established region and the volume of the workspace is minimized. An optimization strategy that considers a hybrid methodology employing two techniques: sequential quadratic programming (SQP) and differential evolution (DE) is proposed. Additional constraints are included to obtain manipulator dimensions within practical values, and to specify limits at the workspace. Application examples are presented to validate the proposal methodology.*

Keywords*: Manipulator Design, Optimization, Hybrid Methodology, Workspace.*

1. INTRODUCTION

The manipulator workspace is defined as the region of reachable points by a reference point *H* on the extremity of a manipulator chain (Kumar and Waldron, 1981). The workspace of a manipulator robot is considered a great interest from theoretical and practical viewpoint. The accurate calculation of workspace is important because of its influence on the manipulator design, the manipulator position in the work environment and its dexterity.

The presence of voids added to the fact that the objective function presents several local maxima and is extremely nonlinear greatly increase the difficulties involved in the optimization process, justifying the use of different optimization techniques to validate the results. Voids are areas not reached by the manipulator's end-effector and which are surrounded by the workspace.

Several investigations have focused on the properties of the workspace of open chain robots with the purpose of emphasizing its geometric and kinematic characteristics, and to devise analytical algorithms and procedures for its design. Ceccarelli (1996) presented an algebraic formulation to determine the workspace of revolution manipulators. In that paper the workspace boundary is obtained from the envelope of a torus family which is traced by the parallel circles cut in the boundary of a revolving hyper-ring. The formulation is a function of the dimensional parameters in the manipulator chain and specifically of the last revolute joint angle, only. The formulation developed by Ceccarelli (1996) is used in this work to obtain the equation of the family of plane curves that represents the workspace boundary. The work developed by Ceccarelli is very important because of the workspace mathematical formulation, but do not consider the manipulators optimal design. Abdel-Malek *et al.* (2000) proposed a generic formulation to determine voids in the workspace of serial manipulators. Wenger (2000) demonstrated that it is possible to consider a manipulator's execution of non-singular changing posture motions in the design stage. Lanni *et al.* (2002) investigated and solved the design of manipulators in the form of an optimization problem that takes into account the characteristics of the workspace. They applied two different numerical techniques; the first using sequential quadratic programming (SQP) and the second involving a random search technique (simulated annealing). It is worth to notice that this methodology can not be applied to calculate the workspace volume in case there is a void.

Some researches have focused on determining the workspace boundary and on detecting the presence of voids and singularities in the workspace. Bergamaschi *et al.* (2006) proposed a form of characterizing the workspace boundary, formulating a general analytical condition to deduce the existence of cusp points at the internal and external boundaries of the workspace.

This paper proposes to approach the design of manipulators as an optimization problem that takes into account the characteristics of the workspace. In order to determine the workspace volume presented here it is necessary to know the area of its radial cross-section. This research proposes a numerical formulation to approximate the cross section area, through its discretization within a rectangular mesh. Only the points that belong to the workspace contribute to the calculation of this area. An optimization problem is formulated with the objective of determining the geometric parameters of 3R manipulators so that the maximum workspace volume is obtained. The maximization is accomplished forcing the workspace to occupy the largest set of points inside of a pre-established region, which, in this case, is a cylinder whose axis coincides with the axis of rotation of the workspace. In the optimization procedure a function that establishes the difference between the volume of the pre-established region and the volume of the workspace is minimized. The formulation of the objective function of this way is a contribution of this work. Additional constraints are included to obtain manipulator dimensions within practical values, and to impose limits at the workspace.

An optimization strategy that considers a hybrid methodology is proposed. Thus, the local search property of Sequential Quadratic Programming (SQP) can be used to obtain an optimal solution combined with metaheuristic. In the first step, the volume maximization is achieved by means of a sequential quadratic programming technique. In the second step, the numerical procedure is based on Differential Evolution (DE). This hybrid methodology is a contribution of this work too.

2. MATHEMATICAL MODELING

One of the most used methods to describe geometrically a general open chain 3R manipulator with three revolute joint is the one which uses the Hartenberg and Denavit (H-D) notation, whose scheme is exhibited in Fig. (1). The design parameters for the link size are represented as a_1, a_2, a_3, d_2, d_3, α_1, α_2, (d_1 is not meaningful since it shifts the workspace up and down).

In this paper, the homogeneous transformation matrix is written obeying the following order of the steps, for i=0,1,2:

- 1st step: a clockwise rotation of angle α_i around the axis X_i;
- 2nd step: a displacement of a_i units along the axis X_i;
- 3rd step: a displacement of d_{i+1} units along the axis Z_{i+1};
- 4th step: a counterclockwise rotation of angle θ_{i+1} around the axis Z_{i+1}.

Hence, adopting the Hartenberg and Denavit notation and in the hypothesis that both reference 1 and reference 0 have the same origin and the same axes z, the transformation matrixes of a reference on the previous are:

$$T_i^{i+1} = \begin{bmatrix} c\theta_{i+1} & -s\theta_{i+1} & 0 & a_i \\ s\theta_{i+1}c\alpha_i & c\theta_{i+1}c\alpha_i & s\alpha_i & d_{i+1}s\alpha_i \\ -s\theta_{i+1}s\alpha_i & -c\theta_{i+1}s\alpha_i & c\alpha_i & d_{i+1}c\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

in which $a_0=\alpha_0=0$, $d_1=0$, $c\alpha_i=\cos\alpha_i$, $s\alpha_i=\sin\alpha_i$, $c\theta_{i+1}=\cos\theta_{i+1}$, and $s\theta_{i+1}=\sin\theta_{i+1}$, for $i=0,1,2$.

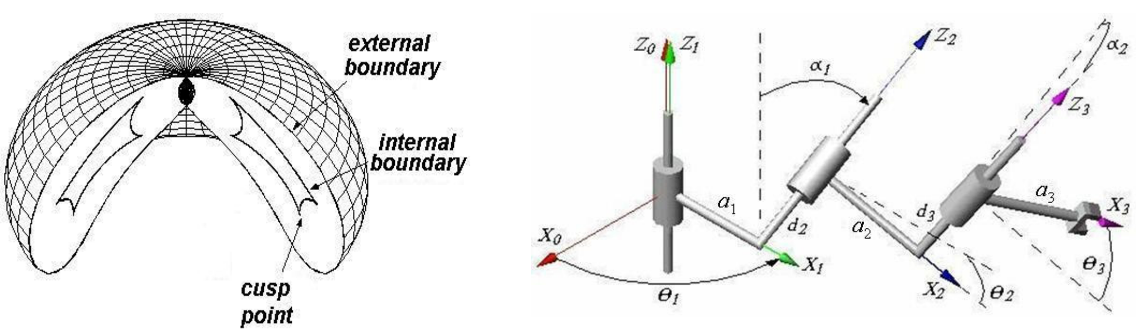


Figure 1. The workspace for 3R manipulators and design parameters.

According to Gupta and Roth (1982), the workspace $W(H)$ is the set of all attainable points for a point H of the end-effector when the joint variables sweep its definition interval entire. Point H is usually chosen as the center of the end-effector, or the tip of a finger, or even the end of the manipulator itself. The position of this point with respect to reference $X_3Y_3Z_3$ can be represented by the vector

$$H_3=[a_3\ 0\ 0\ 1]^T, \tag{2}$$

where H_3 denote the point H in the reference $X_3Y_3Z_3$, the superscripts T means transposed vector and a_3 is the distance from the end-effector to the last joint. The first procedure to investigate the workspace is to vary the angles θ_1, θ_2 and θ_3 in their interval of definition and to estimate the coordinates of point H_3 with respect to the manipulator base frame, that is, obtained from the transformation matrixes as follows:

$$H_0 = \begin{bmatrix} H_0^x & H_0^y & H_0^z & 1 \end{bmatrix}^{\mathrm{T}} = T_0^1\, T_1^2\, T_2^3\, H_3 \tag{3}$$

By expanding Eq. (3) one can obtain

$$H_2 = T_2^3\ H_3 = \begin{bmatrix} a_3\, c\theta_3 + a_2 \\ a_3\, s\theta_3\, c\alpha_2 + d_3\, s\alpha_2 \\ -a_3\, s\theta_3\, s\alpha_2 + d_3\, c\alpha_2 \\ 1 \end{bmatrix} \tag{4}$$

$$H_1 = T_1^2\ H_2 = \begin{bmatrix} H_2^x\, c\theta_2 - H_2^y\, s\theta_2 + a_1 \\ H_2^x\, s\theta_2\, c\alpha_1 + H_2^y\, c\theta_2\, c\alpha_1 + H_2^z\, s\alpha_1 + d_2\, s\alpha_1 \\ -H_2^x\, s\theta_2\, s\alpha_1 - H_2^y\, c\theta_2\, s\alpha_1 + H_2^z\, c\alpha_1 + d_2\, c\alpha_1 \\ 1 \end{bmatrix} \tag{5}$$

$$H_0 = T_0^1\ H_1 = \begin{bmatrix} H_1^x\, c\theta_1 - H_1^y\, s\theta_1 \\ H_1^x\, s\theta_1 + H_1^y\, c\theta_1 \\ H_1^z \\ 1 \end{bmatrix} \tag{6}$$

where H^x_j , H^y_j and H^z_j represent the 1st, 2nd and 3rd components of vector H_j, respectively, for j=1,2.

The workspace of a three revolute open chain manipulator can be given in the form of the radial reach r and axial reach z with respect to the base frame (Ceccarelli, 1996). For this representation, r is the radial distance of a generic workspace point from the z-axis, and z is the distance of this same point at X_1Y_1-plane. Thus, using the Eq. (6), the parametric equations (of parameters θ_2 and θ_3) of the geometrical locus described by point H on a radial plane are

$$r^2 = \left(H_0^x\right)^2 + \left(H_0^y\right)^2 = \left(H_1^x c\theta_1 - H_1^y s\theta_1\right)^2 + \left(H_1^x\, s\theta_1 + H_1^y c\theta_1\right)^2\ ; \qquad z = H_1^z \tag{7}$$

In addition, using Eq. (5),

$$r^2 + z^2 = \left(H_2^x\right)^2 + \left(H_2^y\right)^2 + \left(H_2^z + d_2\right)^2 + 2a_1\left(H_2^x c\theta_2 - H_2^y\, s\theta_2\right) + a_1^2 \tag{8}$$

and, by multiplying the second equation of Eqs. (7) by ($2\, a_1 / \sin\alpha_1$) with the hypotheses $a_1 \neq 0$ and $\sin\alpha_1 \neq 0$, and using Eqs. (5) and (6), one obtains

$$\frac{2a_1}{s\alpha_1} z - \frac{2a_1\left(H_2^z + d_2\right)c\alpha_1}{s\alpha_1} = -2a_1\left(H_2^x s\theta_2 + H_2^y c\theta_2\right) \tag{9}$$

Squaring both sides of Eqs. (8) and (9) and adding the resulting equations as in Ceccarelli (1996), one can obtain

$$f(\,r,\,z,\,\theta_3\,) = \left[r^2 + z^2 - A\right]^2 + \left[C\,z + D\right]^2 + B = 0 \tag{10}$$

where A, B, C, D coefficients are called the architecture coefficients. These are functions of the Denavit and Hartenberg parameters a_1, a_2, a_3, d_2, d_3, α_1, α_2 and θ_3 in the form

$$\begin{aligned} &A = a_1^2 + r_2^2 + \left(z_2 + d_2\right)^2 \quad B = -4a_1^2\left[\left(H_2^x\right)^2 + \left(H_2^y\right)^2\right] = -4a_1 r_2^2 \quad z_2 = d_3 c\alpha_2 - a_3 s\theta_3 s\alpha_2 \\ &C = \frac{2a_1}{s\alpha_1} \quad D = -2a_1\left(H_2^z + d_2\right)\frac{c\alpha_1}{s\alpha_1} = -2a_1\left(z_2 + d_2\right)\frac{c\alpha_1}{s\alpha_1} \quad r_2 = \left[\left(a_3 c\theta_3 + a_2\right)^2 + \left(a_3 s\theta_3 c\alpha_2 + d_3 s\alpha_2\right)^2\right]^{1/2} \end{aligned} \tag{11}$$

Eq. (10) is the function of the three-revolute manipulator workspace described by reference point H. This equation represents a family of plane curves and it is a function of the parameters θ_2 and θ_3.

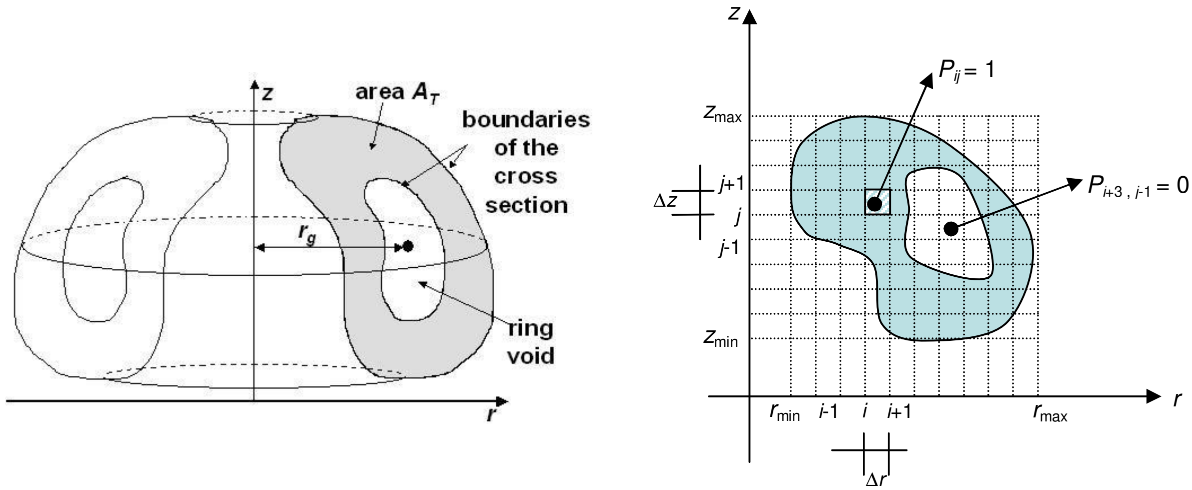


Figure 2. a) A scheme for evaluating the workspace volume of 3R manipulators; b) Discretization of cross section area by using a rectangular mesh

The workspace volume V can be evaluated by the Pappus-Guldin Theorem, presented by Beer and Johnston (2004), using the following equation (see Fig. 2a):

$$V = 2\pi r_g A_T, \tag{12}$$

where A_T is the cross section area, which is formed by the family of curves given by Eq. (10).

This research proposes numerical formulation to approximate the cross section area, through its discretization within a rectangular mesh. Initially, the extreme values of vectors r and z should be obtained as

$$\begin{aligned} r_{\min} &= \min\{r\} \quad \text{and} \quad r_{\max} = \max\{r\} \\ z_{\min} &= \min\{z\} \quad \text{and} \quad z_{\max} = \max\{z\} \end{aligned} \tag{13}$$

Adopting n_r and n_z as the number of intervals chosen for the discretization along the r and z axis, the sizes of the elementary areas of the mesh can be calculated:

$$\Delta r = \frac{r_{\max} - r_{\min}}{n_r} \quad \text{and} \quad \Delta z = \frac{z_{\max} - z_{\min}}{n_z} \tag{14}$$

The n_r and n_z values must be adopted so that the sizes of the elementary areas (Δr or Δz) are at least 1% of the total distances considered in the discretization (r_{max} - r_{min} or z_{max} - z_{min}). Every point of the family of curves form the cross section of the workspace is calculated by Eq. (10). Using this equation, varying the values of θ_2 and θ_3 in the interval $[-\pi , \pi]$, it is possible to obtain the family of curves of the workspace. Given a certain point (r, z), its position inside the discretization mesh is determined through the following index control:

$$i = \text{int}\left(\frac{r - r_{\min}}{\Delta r}\right) + 1 \quad \text{and} \quad j = \text{int}\left(\frac{z - z_{\min}}{\Delta z}\right) + 1 \tag{15}$$

where i and j are computed as integer numbers. As shown in Fig. 2b, the point of the mesh that belongs to the workspace is identified by $P_{ij} = 1$, otherwise $P_{ij} = 0$, which means:

$$P_{ij} = \begin{cases} 0, \, if \quad P_{ij} \notin W(H) \\ 1, \, if \quad P_{ij} \in W(H) \end{cases} \text{; where } W(H) \text{ indicates workspace region.} \tag{16}$$

In this way, the total area is obtained by the sum of every elementary areas of the mesh that are totally or partially contained in the cross section. In Eq. (17), it is observed that only the points that belong to the workspace contribute to the calculation of the area A_T. The coordinate r_g of the center of the mass is calculated considering the sum of the center of the mass of each elementary area, divided by the total area, using the following equation:

$$A_T = \sum_{i=1}^{i_{max}} \sum_{j=1}^{j_{max}} \left(P_{ij}\ \Delta r\ \Delta z \right) \quad \text{and} \quad r_g = \frac{\sum_{i=1}^{i_{max}} \sum_{j=1}^{j_{max}} \left(P_{ij}\ \Delta r\ \Delta z \right)\left((i-1)\ \Delta r + \frac{\Delta r}{2} + r_{min} \right)}{A_T} \tag{17}$$

Finally, after the calculation of the cross section area and the coordinate of the center of the mass, given by Eqs. (16) and (17), the workspace volume of the manipulator can be evaluated by using Eq. (12).

3. FORMULATION OF AN OPTIMAL DESIGN PROCEDURE

The purpose of the proposed manipulator design procedure is to come up with a dimensional synthesis of 3R manipulators. An optimization problem is formulated with the objective of determining the geometric parameters of 3R manipulators so that the maximum workspace volume is obtained. The maximization is accomplished forcing the workspace to occupy the largest set of points inside of a pre-established region, which, in this case, is a cylinder whose axis coincides with the axis of rotation of the workspace. In the optimization procedure a function that establishes the difference between the volume of the pre-established region and the volume of the workspace is minimized. The optimization problem is therefore defined as:

$$\min f(x) = V_{cil} - V(x); \qquad 0.01 < x_i < 1.0, \text{ for } i = 1, \ldots, 5; \qquad 0.5° < x_k < 90°, \text{ for } k = 6, 7, \tag{18}$$

where $x = [a_1\ \ a_2\ \ a_3\ \ d_2\ \ d_3\ \ \alpha_1\ \ \alpha_2]$ is the design vector, the workspace volume V(x) is given by Eq. (12) and the cylinder volume is given by:

$$V_{cil} = 2\,\pi\, b\, A, \qquad b = \frac{L_{max}}{2}, \qquad A = \left(L_{max}\right)^2, \tag{19}$$

where

$$L_{max} = \sqrt{\left(a_1^u\right)^2 + \left(d_2^u\right)^2} + \sqrt{\left(a_2^u\right)^2 + \left(d_3^u\right)^2} + a_3^u\,. \tag{20}$$

The values a_i^u and d_j^u are the maximum values that the respective parameters a_i and d_j can assume during the optimization process, for i = 1, 2, 3 and j = 2, 3.

The optimization problem can be subject to the constraint:

$$g_1(x) = z > 0; \tag{21}$$

With the imposition of this constraint, it is possible to obtain the optimal design within an area of practical interest. The condition $z > 0$ means the manipulator's end-effector only reaches areas above its base.

4. OPTIMIZATION STRATEGY

In this research, the local search property of sequential quadratic programming (SQP) has been used to obtain an optimal solution combined with the metaheuristic called differential evolution (DE). In the first step, the volume maximization is achieved by means of a sequential quadratic programming technique, using the code DOT (Design Optimization Tools) developed by Vanderplaats (1995). In this code a pseudo-objective function is written using the augmented Lagrange multiplier method. The optimal results of SQP are used in the second step, where the numerical procedure is based on Differential Evolution. The computational code of the DE was developed in MATLAB® by the authors. A brief review of the methods used in the optimization process is presented below.

4.1. A Overview on Differential Evolution (DE)

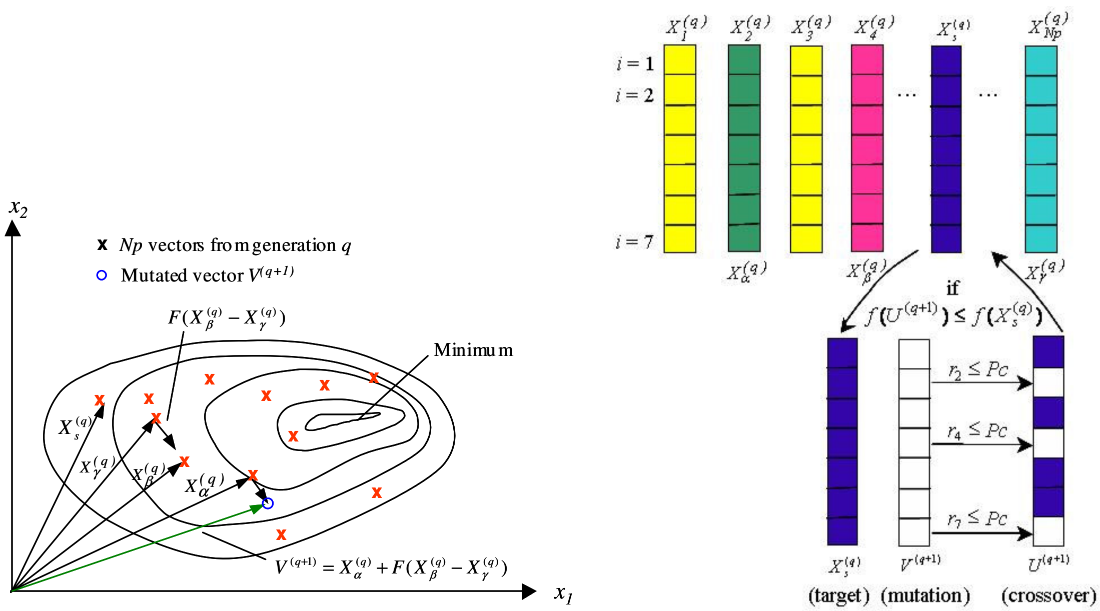
Let the initial population chosen randomly consisting by Np individuals called vectors. This population should cover the entire search space. For a problem with n design variables each vector has n parameters. Generally, this population is created by uniform probability distribution. In this way the population follows a natural evolution, but Np does not change during the minimization process. According to Storn and Price (1997), the main idea of differential evolution is to generate new individuals, called mutated vector or donor vector, by adding the weighted difference between two population random individuals to a third individual. This operation is called *mutation*. The new donor individual's parameters are then mixed with the parameters of another individual randomly chosen, denoted target vector or vector to be replaced, to yield the called trial vector. This process is often referred to as *crossover* in the evolutionary strategy community. If the trial vector cost yields a lower value than the target vector cost, then the trial vector replaces the target vector in the following generation. This last operation is called *selection*. The process is ended when the limit of the maximum number of generations is attaint or through the stagnation concept, i.e., when after several serial iterations any improvement in the population is observed.

The differential evolution operators are based on a natural evolution principle which the aim is to keep the population diversity.

Mutation: With the purpose to obtain the mutated vector $V^{(q+1)}$, let the vectors X_α, X_β and X_γ mutually different and randomly chosen from the population with Np individuals, so that $Np \geq 4$. The random indexes $\alpha, \beta, \gamma \in \{1,\ldots, Np\}$ are integer mutually different. In generation q one pair of vectors (X_β, X_γ) defines a difference vector ($X_\beta - X_\gamma$). F multiplies this difference named weighted difference and it is used to perturb the third vector X_α or the best vector X_{best}. F is a real and constant factor belonging to interval [0,2], which controls the amplification of the difference vector. This process that yields the mutated vector $V^{(q+1)}$ can be mathematically written as:

$$V^{(q+1)} = X_\alpha^{(q)} + F(X_\beta^{(q)} - X_\gamma^{(q)}) \tag{22}$$

Figure 3a shows a two-dimensional function that illustrates the different vectors which to take part in the generation of mutated vector.



(a) (b)

Figure 3. a) The process for generating $V^{(q+1)}$ for two dimensional function; b) Illustration of the binomial crossover.

Crossover: Consider that for each target vector $X_s^{(q)}$, $s \in \{1,\ldots, Np\}$, different from indexes α, β, γ, was generated a mutated vector $V^{(q+1)}$. The crossover is introduced in order to increase the diversity of the perturbed individuals. Thus, the trial vector $U^{(q+1)}$ is formed by:

$$u(i)^{(q+1)} = \begin{cases} v(i)^{(q+1)}, & \text{if} \quad r_i \leq Pc \\ v_s(i)^{(q)}, & \text{if} \quad r_i > Pc\,, \quad i = 1,\ldots, n \end{cases}, \tag{23}$$

where r_i is i*th*-evaluation of a uniform random number generator with outcome belonging to [0, 1], $Pc \in [0, 1]$ is the crossover probability and it must be supplied by user. Pc represents the probability of the new trial vector to inherit the variable values from mutated vector. When $Pc = 1$, for example, all trial vector variables will come from mutated vector $V^{(q+1)}$. On the other hand, $Pc = 0$, all trial vector variables will come from the target vector $X_s^{(q)}$.

This crossover, developed by Storn and Price (1995), is called binomial crossover operator, due to independent binomial experiments, which is executed whenever a randomly picked number $r \in [0, 1]$ is lower than the Pc crossover probability. Figures 3b shows the binomial crossover process with seven design variables.

After the crossover, if one or more trial vector variables are out of search space then it can be brought in the bound range as following:

$$\begin{aligned}&\text{If } u(i) < x(i)^{\min},\ \text{then } u(i) = x(i)^{\min}\\&\text{If } u(i) > x(i)^{\max},\ \text{then } u(i) = x(i)^{\max}, i = 1,\ldots,n\end{aligned} \tag{24}$$

where $x(i)^{\min}$ and $x(i)^{\max}$ are the lower and upper limits, i.e., the side constraints, respectively.

Selection: The selection is the process of producing better offspring. Unlike many other evolutionary algorithms, the DE does not use ranking and proportional selection. Instead, the cost of each trial vector $U^{(q+1)}$ is worked out and compared with the cost of target vector $X_s^{(q)}$. If the cost of target vector is lower than that of trial vector, the target is allowed to advance for the next generation $q+1$. Otherwise, the trial vector replaces the target vector in the following generation. In other words this process can be written as:

$$\begin{aligned}&\text{If } f(U^{(q+1)}) \le f(X_s^{(q)}),\ \text{then } X_s^{(q+1)} = U^{(q+1)}\\&\text{If } f(U^{(q+1)}) > f(X_s^{(q)}),\ \text{then } X_s^{(q+1)} = X_s^{(q)}\end{aligned} \tag{25}$$

Usually, the DE algorithm performance depends mainly of the Np population size, search space and, crossover probability.

4.2. A Overview on Sequential Quadratic Programming (SQP)

Sequential quadratic programming (SQP) represents a nonlinear programming method. The main idea is the formulation of a subproblem based on a quadratic approximation of the Lagrangian function (Bazarra *et al*., 1993). Let x_k be the vector containing the design parameters at step k, the subproblem is obtained by linearizing the nonlinear constrains (Nocedal and Wright, 1999) in the form

$$\min\left\{\frac{1}{2}s^T H_k + \Delta f(x_k)^T s\right\} \tag{26}$$

subject to

$$\begin{aligned}&\nabla g_i(x)^T s + g_i(x) = 0 \qquad i = 1, \ldots, m_e\\&\nabla g_i(x)^T s + g_i(x) \le 0 \qquad i = m_e+1, \ldots, m\end{aligned} \tag{27}$$

in which T denotes the transpose operation and H_k is the Hessian matrix of the Lagrangian function Λ, given by

$$\Lambda(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i g_i(x) \tag{28}$$

In this equation, λ is Lagrangian multipliers; s is the search direction; m_e is the number of the equality constraints; m is the total number of constraints; $\nabla g_i(x)^T$ is the Jacobian matrix of the constraints; x is the vector containing the design parameters a_1, a_2, a_3, d_2, d_3, α_1, α_2. The numerical procedure starts with an initial guess of the manipulator chain solution and, during each iteration k, a quadratic programming problem is solved to provide a search direction s_k, so that the solution is updated as

$$x_{k+1} = x_k + \psi_k s_k \tag{29}$$

in which ψ_k is a step size obtained minimizing a Lagrangian function.

5. NUMERICAL EXAMPLE

Maximization of workspace volume is achieved by means of the optimization strategy described in section 4. To demonstrate the efficiency of the proposed optimization design procedure, two numerical examples are studied: the first case considers the optimization problem given by Eq. (18) (unconstrained problem); the second case considers in addition the constraint given by Eq. (21).

A total of 1500 evaluations (15 individuals; 100 generations) of cost function, given by Eq. (12), was done by the metaheuristic in each run. Other parameters used in the DE were adopted as: representation of individuals by real vectors using CR = 0.8 and f_m = 0.4. The best values are obtained after 20 trials. It is worthwhile to mention that the different values for these parameters were tested, but they do not get better results for the optimal workspace volume, considering the average values.

Tables 1 and 2 show the results of the final parameters for the case 1 and 2, respectively. The final volume and the computational time of execution for each of the applied methods are also shown. In the case 1, the optimal volume is improved by using the second step in 8.02%. In the case 2, the improvement by using the second step is 11.57%. In both cases, it was observed that the optimum configuration of the workspace depends mainly on the angles α_1 and α_2, because the parameters a_1, a_2, a_3, d_2 and d_3 assumed the largest allowed value, which in this case is 1.0.

For the case 1, the optimal results obtained through the optimization procedure are: a_1=1.0, a_2=1.0, a_3=1.0, d_2=1.0, d_3=1.0, α_1=84.18°, α_2=77.14° and the final volume is 131.98 u.v.. One can observe that the project parameters result in a manipulator with a bigger volume. The optimal cross section area of the workspace volume is presented in Fig. 4.

Figure 5 shows the optimal result obtained through the optimization procedure for the case 2, observe that he condition z > 0 was obeyed, thus the manipulator's end-effector only reaches areas above its base. The optimal parameters designs are: a_1=1.0, a_2=1.0, a_3=1.0, d_2=1.0, d_3=1.0, α_1=36.13°, α_2=29.77° and the final volume is 70.76 u.v.

Table 1. Optimal results in each optimization step – Case 1: unconstrained problem

	Initial value	First step (SQP)	Second step (DE)
Workspace volume [u.v.]	92.35	122.18	131.98
Dimensional parameters [um um um um um degree degree]	[0.5 0.5 0.5 0.5 0.5 45.0 45.0]	[1 1 1 1 1 83.47 75.18]	[1 1 1 1 1 84.18 77.14]
Computational time [min]	-	0.11	24.71
Performance	-	32.30 %	8.02 %

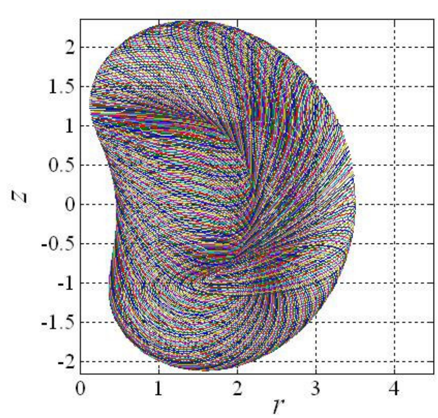


Figure 4. The optimum design of a 3R manipulator - Case 1: unconstrained problem

Table 2. Optimal results in each optimization step - Case 2: with constraint z >0.

	Initial value	First step (SQP)	Second step (DE)
Workspace volume [u.v.]	92.35	63.42	70.76
Dimensional parameters [u.m. u.m u.m. u.m. u.m. degree degree]	[0.5 0.5 0.5 0.5 0.5 45.0 45.0]	[1 1 1 1 1 19.43 60.55]	[1 1 1 1 1 36.13 29.77]
Computational time [min]	-	0.16	26.42
Min (z)	-0.64 < 0 Not obeyed	$0.252 \times 10^{-3} > 0$ obeyed	$0.102 \times 10^{-1} > 0$ obeyed
Performance		-	11.57%

It is relevant to notice that for the case 1 (unconstrained problem) the volume obtained in the first step is increased in 32,30% (see Table 1). In the other hand, the optimal volume in the case 2 decreased for 1st and 2nd steps (see Table 2), because the initial project does not obey the imposed constraint (z>0).

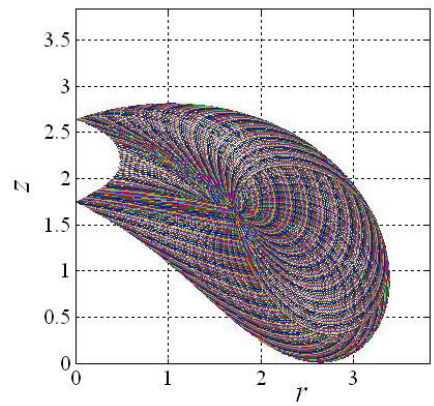


Figure 5. The optimum design of a 3R manipulator - Case 2: with constraint z >0.

6. CONCLUSIONS

A suitable formulation for the manipulator workspace was used to devise an efficient numerical procedure to solve the optimization problem. The design problem was formulated minimizing an objective function, which establishes the difference between the volume of the pre-established region and the volume of the workspace. This formulation presented for the maximization of the volume is a contribution of this work.

In this paper, the local search property of sequential quadratic programming (SQP) has been used to obtain a final solution combined with the metaheuristic called differential evolution (DE). In the first step, the volume maximization is achieved by means of a SQP, whose results are used in the second step, where the numerical procedure is based on DE. The use of this hybrid methodology presents an other contribution of this article.

This optimization strategy was applied in two examples obtained good results. Clearly the volume of the workspace increases when the values of the design parameters a_1, a_2, a_3, d_2 and d_3 also increase. This way, as it was waited, the maximization of the volume takes the parameters assumes the largest allowed value, which in this case is 1.0. Then it was observed that the optimum configuration of the workspace depends mainly on the angles α_1 and α_2. Thus, the objective function is highly sensitive to the design variables α_1 and α_2.

As expected, SQP approach is faster than the differential evolution, which is not competitive with SQP in terms of computational time. However, this technique is sensitive to the initial point and it can to stay “arrested” in a local minima. It was verified that the metaheuristics can provide a good solution even if the problem has many local optimum solutions. These methods, however, demand a large computational time. Thus, this optimization strategy, that combines two different optimization techniques, presents a good potential to work with complex problems.

7. ACKNOWLEDGEMENTS

The first author is thankful to the Fundação de Amparo a Pesquisa do Estado de Minas Gerais (Fapemig) for the partial financial support.

8. REFERENCES

Abdel-Malek K., Yeh H-J., Othman S.,2000, "Understanding Voids in the Workspace of Serial Robot Manipulators", Proc. 23rd ASME Design Engineering Technical Conf., 2000, Baltimore, Maryland.

Bazarra, M. S., Sherali, H. D., Shetty, C. M., 1993, "Nonlinear Programming. Theory and Algorithms", second edition, John Wiley & Sons, New York.

Beer, F.P. and Johnston Jr., E.R., 2004, "Vector Mechanics for Engineers: Statics and Dynamics", 7th ed. McGraw-Hill; New York.

Bergamaschi, P.R., Nogueira, A.C. and Saramago, S.F.P., 2006, "Design and optimization of 3R manipulators using the workspace features", Applied Mathematics and Computation, Elsevier, Vol. 172. No.1., pp. 439-463.

Ceccarelli M., 1996, "A Formulation for the Workspace Boundary of General N-Revolute Manipulators", IFToMM Jnl Mechanism and Machine Theory, Vol.31,pp.637-646.

Gupta K.C., Roth B.,1982, "Design Considerations for Manipulator Workspace", ASME Jnl of Mechanical Design, Vol. 104, pp.704-711.

Kumar A., Waldron K.J., 1981,"The Workspaces of a Mechanical Manipulator", ASME Jnl of Mechanical Design, Vol. 103, pp. 665-672.

Lanni, C., Saramago S.F.P., Ceccarelli, M.,2001, 2002, "Optimal Design of 3R Manipulators Using Classical Techniques and Simulated Annealing", RBCM- J. of the Brazilian Soc. Mechanical Sciences, Brazil, 24. No.4., pp. 293-301.

Nocedal, J. and Wright, S. J., 1999, "Numerical Optimization", Springer Series in Operations Research.

Storn, R. and Price, K., 1995, "Differential Evolution — a Simple and Efficient Adaptive Scheme for Global Optimization over Continuous Spaces", Technical Report TR 95-012, International Computer Science Institute, CSI, Berkeley, 1995.

Storn, R. and Price, K., 1997, "Differential Evolution -a Simple and Efficient Heuristic for Global Optimization over Continuous Spaces", Journal of Global Optimization, Vol. 11, pp. 341-359.

Vanderplaats, G., 1995, "DOT - Design Optimization Tools Program – Users Manual", Vanderplaats Research & Development, Inc, Colorado Springs.

Wenger P., "Some Guidelines for the Kinematic Design of New Manipulators", Mechanism and Machine Theory, 2000, Vol. 35, pp. 437-449.

9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.