A NON-PARAMETRIC PROBABILISTIC APPROACH TO MODEL UNCERTAINTIES IN THE VOCAL FOLD OSCILLATION

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Abstract. *In this article, a non-parametric probabilistic approach is followed to model uncertainties in a mechanical representation of the vocal fold system at phonation. The nonparametric model is based on the use of a probabilistic model for symmetric positive-definite real random matrices using the Maximum Entropy Principle. The theory is discussed and numerical examples are presented to show that the predictability of the model may be improved when model uncertainties are taken into account. Some realizations of the output radiated pressure obtained are similar with samples of voice signals obtained from people with some pathologies in their vocal folds.*

Keywords: *Uncertainties, nonparametric probabilistic approach, voice production, modeling*

1. INTRODUCTION

To improve the predictability of models of mechanical systems, uncertainties from different sources must be considered. The first source concerns the data from which the various parameters of the model are derived. Usually these are uncertainties related to the geometry and properties of the material, and other similar parameters. One way to incorporate them is the parametric probabilistic approach, in which the parameters are expressed as random variables. The second source of uncertainties concerns the structure of the model itself. Those uncertainties arise from the fact that, to compute the response of a mechanical problem, a particular model is chosen, that does not necessarily capture accurately the real behavior of the mechanical system. Model uncertainties are difficult to quantify because they depend heavily on the type of problem, and their effect on the estimated response of a mechanical system is not obvious. Recently, a general non-parametric probabilistic approach of model uncertainties for dynamical systems has been proposed using random matrix theory (Soize, 2000).

The aim of this work is to discuss model uncertainties in the case of the biomechanics of phonation. A number of mechanical models of voice production have been proposed in past years; but, in general, they have a deterministic nature. In a previous work, Cataldo et al. (2007) incorporated data uncertainties to a two-mass model of the vocal folds (Ishizaka and Flanagan, 1972), to perform a probabilistic analysis of the fundamental frequency of the oscillation. The present work is intended as a follow-up, to explore the effect of model uncertainties.

2. MEAN MODEL

The two-mass model of the vocal folds, originally proposed by Izhizaka and Flanagan (1972), has provided a simple and effective representation of that system to study the underlying dynamics of voice production. Figure 1 shows a diagram of the model.

For a complete representation of the vocal system, a vocal tract model must be coupled to the vocal fold model. Here, we adopt a simple two-tube aproximation of the vocal tract (Titze, 1994). Here, only uncertainties in the vocal fold system are taken into account. The parameters of the vocal tract will be considered deterministic.

The dynamics of the system is given by Eqs. (1) and (2):

$$\psi_1(\mathbf{w})\dot{u}_g + \psi_2(\mathbf{w})|u_g|u_g + \psi_3(\mathbf{w})u_g + \frac{1}{\tilde{c}_1}\int_0^t (u_g(\tau) - u_1(\tau))d\tau - y = 0 \tag{1}$$

$$[M]\ddot{\mathbf{w}} + [C]\dot{\mathbf{w}} + [K]\mathbf{w} + \mathbf{h}(\mathbf{w}, \dot{\mathbf{w}}, u_g, \dot{u}_g) = 0 \tag{2}$$

where $\mathbf{w}(t) = (x_1(t), x_2(t), u_1(t), u_2(t), u_r(t))^t$, the functions x_1 and x_2 are the displacements of the masses, u_1 and u_2 describe the air volume flow through the (two) tubes that model the vocal tract and u_r is the air volume flow through the mouth. The subglottal pressure is denoted by y and u_g is the function that represent the glottal pulses signal. The function output radiated pressure p_r is given by $p_r(t) = u_r(t)r_r$, in which $r_r = \frac{128\rho v_c}{9\pi^3 y_2^2}$, ρ is the air density, v_c is the sound velocity, and y_2 is the radius of the second tube.

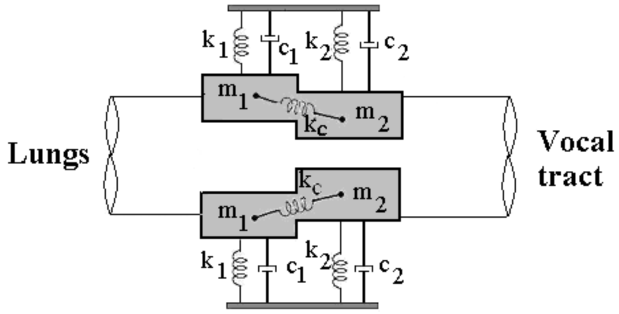

Figure 1. Two-mass model of the vocal folds.

As the objective is to discuss uncertainties in the model of the vocal folds, the matrices $[M]$, $[C]$ and $[K]$ will be written as block matrices:

$$[M] = \begin{bmatrix} [M_{vf}] & 0 \\ 0 & [M_{vt}] \end{bmatrix} \quad , \quad [C] = \begin{bmatrix} [C_{vf}] & 0 \\ 0 & [C_{vt}] \end{bmatrix} \quad , \quad [K] = \begin{bmatrix} [K_{vf}] & 0 \\ 0 & [K_{vt}] \end{bmatrix} . \tag{3}$$

The functions ψ_1, ψ_2, ψ_3, $\mathbf{h}$, and also the matrices $[M_{vf}]$, $[M_{vt}]$, $[C_{vf}]$, $[C_{vt}]$, $[K_{vf}]$ and $[K_{vt}]$ are described in the appendix. Figure 2 shows the output radiated pressure, considering the following values of the parameters:
$\widehat{m}_1 = 0.125\,\mathrm{g}$, $\widehat{m}_2 = 0.125\,\mathrm{g}$, $\widehat{k}_c = 25\,\mathrm{N/m}$, $\widehat{k}_1 = 80\,\mathrm{N/m}$, $\widehat{k}_2 = 8\,\mathrm{N/m}$, $\xi_1 = 0.1$, $\xi_2 = 0.6$, $\ell_g = 1.4\mathrm{cm}$, $d_1 = 0.25\,\mathrm{cm}$, $d_2 = 0.05\,\mathrm{cm}$ and for the vocal tract model (Ishizaka and Flanagan, 1972) (Goldstein, 1980): $S_1 = 1\,\mathrm{cm}^2$, $S_2 = 7\,\mathrm{cm}^2$, $L_1 = 8.9\,\mathrm{cm}$, $L_2 = 8.1\,\mathrm{cm}$.

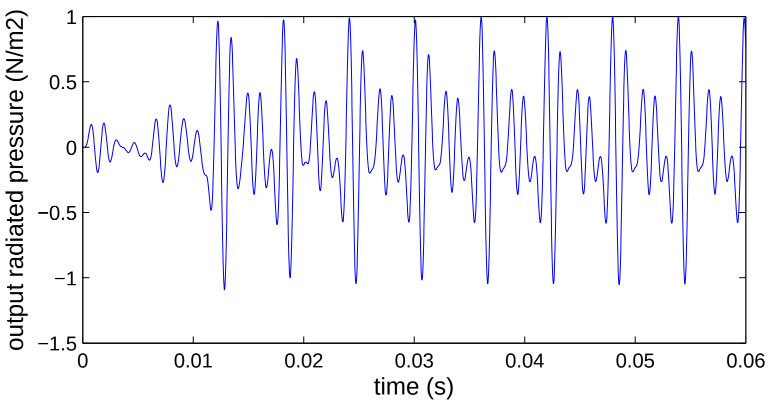

Figure 2. Output radiated pressure (normalized) computed from the mean model.

3. STOCHASTIC MODEL

To construct the corresponding stochastic model, a nonparametric approach is used. Probability density functions are constructed for the mass, damping and stiffness matrices, in order to incorporate uncertainties present on the linear part of the system.

Other quantities might be also considered as uncertain, but we restrict our analysis to the above parameters for simplicity. The general goal is to investigate the limits and the application of the non-parametric probabilistic approach to model voice production.

The matrices $[M_{vf}]$, $[C_{vf}]$ and $[K_{vf}]$ are substituted by random matrices $[\mathbf{M}_{vf}]$, $[\mathbf{C}_{vf}]$ and $[\mathbf{K}_{vf}]$, respectively. Probability density functions are constructed for the matrices, based on a non-parametric approach developed by Soize (2000). Consequently, the matrices $[M]$, $[C]$ and $[K]$ in Eq.(2) are substituted by the random matrices $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$, respectively. Vectors $\mathbf{h}$ and $\mathbf{w}$ are substituted by the random vectors $\mathbf{H}$ and $\mathbf{W}$, respectively. Function u_g is also substituted by the random process U_g. Therefore, Eq. (6) becomes:

$$[\mathbf{M}]\ddot{\mathbf{W}} + [\mathbf{C}]\dot{\mathbf{W}} + [\mathbf{K}]\mathbf{W} + \mathbf{H}(\mathbf{W}, \dot{\mathbf{W}}, U_g, \dot{U}_g) = 0 \tag{4}$$

where matrices $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$ are given by

$$[\mathbf{M}] = \begin{bmatrix} [\mathbf{M}_{vf}] & 0 \\ 0 & [M_{vt}] \end{bmatrix} \ , \ [\mathbf{C}] = \begin{bmatrix} [\mathbf{C}_{vf}] & 0 \\ 0 & [C_{vt}] \end{bmatrix} \ , \ [\mathbf{K}] = \begin{bmatrix} [\mathbf{K}_{vf}] & 0 \\ 0 & [K_{vt}] \end{bmatrix} . \tag{5}$$

There are some parameters of stiffness and damping that are present in the nonlinear part of the system, that is, in the definition of function $\mathbf{h}$, but for simplicity of the analysis, they will not be considered as uncertain.

Probability density functions will be constructed for the matrices $[\mathbf{M}_{vf}]$, $[\mathbf{C}_{vf}]$ and $[\mathbf{K}_{vf}]$ (see next section) and the Monte Carlo Method will be applied. Thus, for each realization θ, Eq. (6) will be written as

$$[\mathbf{M}(\theta)]\ddot{\mathbf{W}}(\mathbf{t},\theta) + [\mathbf{C}(\theta)]\dot{\mathbf{W}}(\mathbf{t},\theta) + [\mathbf{K}(\theta)]\mathbf{W}(\mathbf{t},\theta) + \mathbf{H}(\mathbf{W}(\mathbf{t},\theta), \dot{\mathbf{W}}(\mathbf{t},\theta), \mathbf{U_g}(\mathbf{t},\theta), \dot{\mathbf{U}}_\mathbf{g}(\mathbf{t},\theta)) = \mathbf{0}\,. \tag{6}$$

4. NONPARAMETRIC APPROACH

The non-parametric approach, rather than assessing the uncertainties on given parameters, tries to provide a quantification of the uncertainties on a higher level and specifically, at the level of the matrices of mass, damping and stiffness of the system. The method therefore relies on random matrices. The corresponding probability density functions are constructed by the Maximum Entropy Theorem (Shannon, 1948, Jaynes, 1957a, Jaynes, 1957b), using algebraic properties on the matrices and their mean values as constraints. An unique parameter for each matrix is introduced to control the dispersion level.

Two sets of matrices better suited to characterize the stiffness, mass and damping matrices of a dynamical system are introduced here: the ensembles S_A^+ and S_N^+, defined in the following.

The ensemble S_A^+ is defined as the set of all the random real symmetric positive-definite matrices whose probability density function is constructed by using the Maximum Entropy Principle and the ensemble S_N^+ is defined as the set of the real random positive-definite matrices, but with **identity mean value**, whose probability density function is also constructed by using the Maximum Entropy principle. It should be noted that $S_N^+ \subset S_A^+$.

The construction of the nonparametric probabilistic model is based on replacing the matrices $[M_{vf}]$, $[C_{vf}]$, $[K_{vf}]$ by the random matrices $[\mathbf{M}_{vf}]$, $[\mathbf{C}_{vf}]$, $[\mathbf{K}_{vf}]$. The probabilistic model of each one of these matrices is constructed in S_A^+, as described in the following.

Let $[\mathbf{A}]$ be the matrix that represent each one of the matrices $[\mathbf{M}_{vf}]$, $[\mathbf{C}_{vf}]$ and $[\mathbf{K}_{vf}]$. Let $\mathbb{M}_2^+(\mathbb{R})$ be the set of all 2×2 real symmetric positive-definite matrices. The random matrix $[\mathbf{A}]$, in S_A^+, verifies the following properties:

(i) $[\mathbf{A}] \in \mathbb{M}_2^+(\mathbb{R})$, almost surely;

(ii) Matrix $[\mathbf{A}]$ is a second-order random variable: $E\{||[\mathbf{A}]||_F^2\} < +\infty$

(iii) The mean value $[\underline{A}]$ of $[\mathbf{A}]$ is a given matrix in $\mathbb{M}_2^+(\mathbb{R})$: $E\{[\mathbf{A}]\} = [\underline{A}]$, where $E\{.\}$ is the expected value.

(iv) $[\mathbf{A}]$ is such that $E\{ln(det[\mathbf{A}])\} = \nu_A$, $|\nu_A| < +\infty$

Since $[\underline{A}]$ is positive definite, there is an upper triangular matrix $[\underline{L}]$ in $\mathbb{M}_2(\mathbb{R})$ such that $[\underline{A}] = [\underline{L}]^T[\underline{L}]$ and the ensemble S_A^+ can be defined as the set of random matrices $[\mathbf{A}]$ which are written as $[\mathbf{A}] = [\underline{L}]^T[\mathbf{N}][\underline{L}]$ in which $[\mathbf{N}]$ is in S_N^+. Each matrix $[\mathbf{N}]$ satisfies the following properties:

(i) $[\mathbf{N}] \in \mathbb{M}_2^+(\mathbb{R})$, almost surely;

(ii) Matrix $[\mathbf{N}]$ is a second-order random variable: $E\{||[\mathbf{N}]||_F^2\} < +\infty$;

(iii) The mean value $[\underline{N}]$ of $[\mathbf{N}]$ is the identity matrix $[I_2]$ in $\mathbb{M}_2^+\mathbb{R} : E\{[\mathbf{N}]\} = [\underline{N}] = [I_2]$;

(iv) $[\mathbf{N}]$ is such that $E\{ln(det[\mathbf{N}]\} = \nu_N \ , \ \mid \nu_N \mid < +\infty$

The last constraint yields the fundamental property that the inverse of $[\mathbf{N}]$ is also a second-order random variable: $E\{||[\mathbf{N}^{-1}||_F^2\} < +\infty$. For the computation of the probability density function of $[\mathbf{N}]$, it was also shown that rather than considering $\nu_{[N]}$, the value of which does not bear a simple physical meaning, it was interesting to replace it by a parameter δ_N, which measures the dispersion of the probability model of the random matrix $[\mathbf{N}]$ around the mean value $[\underline{N}] = [I_2]$.

Realizations of the matrices of S_A^+ can be computed from those of matrices of S_N^+ so that only the probability density function of the latter ensemble will be described, computed using the Maximum Entropy Theorem. It is defined with respect to the measure $\tilde{d}A$ on the set $\mathbb{M}_2^S(\mathbb{R})$ of 2×2 real symmetric matrices, where $\tilde{d}N$ is such that

$$\tilde{d}N = 2^{1/2} \prod_{1 \leq i \leq j \leq 2} d[N]_{ij} \tag{7}$$

where $dN = \prod_{i \leq i, j \leq n} d[N]_{ij}$ is the Lebesgue measure on $\mathbb{R}^2$. With the usual normalization condition on the probability density function, it can be shown to be (Soize, 2000, 2001):

$$p_{[\mathbf{N}]}([N]) = \mathbf{1}_{\mathbb{M}_2^+(\mathbb{R})}([N]) \times C_{[\mathbf{N}]} \times (det[N])^{3(1-\delta^2)/(2\delta^2)} \times exp\left\{-\frac{3}{2\delta^2} tr[N]\right\} \tag{8}$$

in which $[N] \mapsto \mathbf{1}_{\mathbb{M}_2^+(\mathbb{R})}([N])$ is a function from $\mathbb{M}_2(\mathbb{R})$ (set of all real matrices 2×2) into $\{0, 1\}$ that is equal to 1 when $[N]$ is in $\mathbb{M}_2^+(\mathbb{R})$ and 0 otherwise, and where constant $C_{[\mathbf{N}]}$ is equal to

$$C_{[\mathbf{N}]} = \frac{(2\pi)^{-1/2} \left(\frac{3}{2\sigma^2}\right)^{3/\sigma^2}}{\prod_{j=1}^{n} \Gamma\left(\frac{3}{2\sigma^2} + \frac{1-j}{2}\right)} \tag{9}$$

with $z \mapsto \Gamma(z)$ the gamma function defined for $z > 0$ by $\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt$.

The dispersion parameter δ_N is a real parameter defined, for any random matrix $[\mathbf{N}]$, with mean value $[\underline{N}]$, by

$$\delta_N = \left\{ \frac{E\{||[\mathbf{N} - I_2]||_F^2\}}{||[\underline{N}]||_F^2} \right\}^{1/2} . \tag{10}$$

where $||.||_F$ is the Frobenius norm. For matrices of S_N, this definition reduces to

$$\delta_N = \frac{1}{2} E\{||[\mathbf{N} - [I_2]||_F^2\}^{1/2} . \tag{11}$$

This dispersion parameter should be chosen independent of n and such that $0 < \delta < \sqrt{\frac{3}{7}}$, to ensure that the condition on the integrability of the inverse of the random matrices is verified.

The probability density functions of the three matrices of mass, damping and stiffness have been described as if they were independent. But, they correspond to the same physical problem and it could be argued that they should be dependent. In fact, they are, through their mean values, which are computed using the same mechanical parameters. But concerning their probabilistic model, since we did not specify any particular condition of dependence, the Maximum Entropy Theorem describes the matrices as independent.

This can be written using a more mathematical formalism. Let us consider a set of m random matrices $[\mathbf{A}_1], \ldots, [\mathbf{A}_m]$ in S_A^+, for which mean values are given, but no correlation tensor between any two of the random matrices is provided. The Maximum Entropy Theorem can be used to show that the probability density function $([A_1], \ldots, [A_m]) \mapsto p_{[\mathbf{A_1}],\ldots,[\mathbf{A_m}]}([A_1], \ldots, [A_m])$ from $(\mathbb{M}_2^+)^m$ into $\mathbb{R}_+$ with respect to the measure $\tilde{d}A_1 \times \ldots \times \tilde{d}A_m$ on $(\mathbb{M}_2^S)^m$ is written as

$$p_{[\mathbf{A_1}]([A_1],\ldots,[A_m])} = p_{[\mathbf{A_1}]}([A_1]) \times \ldots \times p_{[\mathbf{A_m}]}([A_m]) \tag{12}$$

which means that the $[A_1], \ldots, [A_m]$ are independent random matrices.

Having values in $\mathbb{M}_2^+(\mathbb{R})$, $[\mathbf{N}]$ can be written $[\mathbf{N}] = [\mathbf{L}^T][\mathbf{L}]$, where $[\mathbf{L}]$ is an upper triangular random matrix with values in $\mathbb{M}_2(\mathbb{R})$. Let us introduce $\sigma_2 = \frac{\delta}{\sqrt{3}}$. It can be shown that

(i) random variables $([\mathbf{L}])_{1 \leq i \leq j \leq n}$ are independent;

(ii) for $i < j$, $[\mathbf{L}]$ can be written $[\mathbf{L}]_{ij} = \sigma_2 U_{ij}$, where U_{ij} is a Gaussian random variable with real values, zero mean and unit variance.

(iii) for $i = j$, $[\mathbf{L}]_{ij}$ can be written $[\mathbf{L}]_{ii} = \sigma_2\sqrt{2V_i}$, where V_i is a gamma random variable with positive real values and a probability density function $p_{V_i}(v)$ (with respect to dv) in the form

$$p_{V_i}(v) = \mathbf{1}_{\mathbb{R}^+}(v)\frac{1}{\Gamma\left(\frac{3}{2\sigma^2}+\frac{1-i}{2}\right)}v^{\frac{3}{2\sigma^2}-\frac{1+i}{2}}e^{-v}\,. \tag{13}$$

This algebraic structure of $[\mathbf{N}]$ allows an efficient procedure to be defined for the Monte Carlo numerical simulation of random matrix $[\mathbf{N}]$.

The nonparametric approach to stochastic modeling consists in setting the probability density functions of these matrices, ensuring that certain algebraic properties are verified. For the construction of these probability distributions, the mean value and a dispersion parameter have to be supplied for each of the matrices. We discarded the problem of the identification of the dispersion parameters, but we still have to address that of the mean value of the matrices $[\underline{M}_{vf}]$, $[\underline{C}_{vf}]$ and $[\underline{K}_{vf}]$.

5. RESULTS

The Monte Carlo method is a very general resolution technique, that can deal with complicated systems, with many random variables or processes. Its basic steps are: (1) generate samples of the input random parameter vector following its prescribed set of marginal laws, (2) compute the response of the system for each realization independently, (3) compute statistics of the response using these response samples. Then, the Monte Carlo method will be used. The main problem - and limitation- of this method derives from the computational time required to assess the statistics of the response, as it is the time necessary for one deterministic computation multiplied by the number of trials. The corresponding stochastic solver is based on a Monte Carlo numerical simulation. Realizations $[\mathbf{M}_{vf}](\theta)$, $[\mathbf{C}_{vf}](\theta)$, $[\mathbf{K}_{vf}](\theta)$, $A_{g0}(\theta)$ and $Y_s(\theta)$ of the random matrices $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$ are obtained from the probability density functions defined before.

One of the goals here is to construct probability density functions of the fundamental frequency when uncertainties in the model are taken into account using a nonparametric probabilistic approach. Then, for each realization of the stochastic problem associated, the fundamental frequency of the voice signal produced has to be calculated.

Let $F_0(\theta)$ be the fundamental frequency of the radiated pressure $P_r(t,\theta)$ evaluated for each realization. Let $T(\theta)$ be the period of the signal $U_g(t,\theta)$ for each realization θ. Then, $F_0(\theta) = 1/T(\theta)$.

The convergence analysis with respect to n is carried out in studying the convergence of the estimated second-order moment of F_0 defined by

$$\text{Conv}(n) = \frac{1}{n}\sum_{j=1}^{n}F_0(\theta_j)^2. \tag{14}$$

This convergence analysis is performed for $\delta_{[\mathbf{M_{vf}}]} = \delta_{[\mathbf{C_{vf}}]} = \delta_{[\mathbf{K_{vf}}]} = 0.05$ and Fig. 3 shows the graph of the function $n \mapsto \text{conv}(n)$. It can be noted that a reasonable convergence is reached for $n \geq 300$.

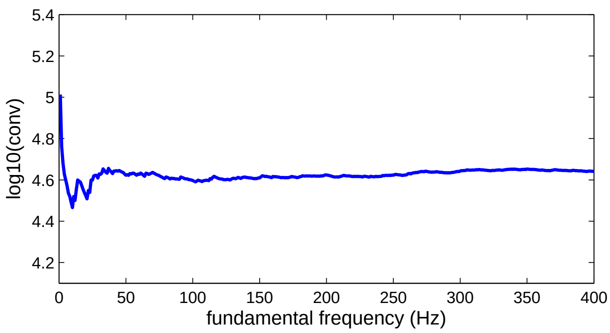

Figure 3. Convergence: graphs of functions $n \mapsto log_{10}\text{Conv}$.

The estimation of the probability density function p_{F_0} of random variable F_0 is constructed as follows. Let M be the number of intervals. Let $I_j = [\nu_j, \nu_j + \Delta\nu[$ for $j = 1,\ldots,M$ with $\nu_1 = \widetilde{f}_1$ and $\Delta\nu = (\widetilde{f}_n - \widetilde{f}_1)/M$. An estimation $\widehat{p}_{F_0}$ of the probability density function of F_0 is given by

$$\widehat{p}_{F_0}(f_0) = \sum_{j=1}^{M}\mathbf{1}_{I_j}(f_0)\frac{N_j}{n\Delta\nu}\,. \tag{15}$$

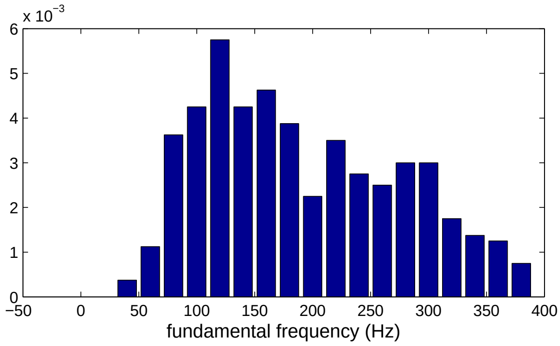

Figure 4. Probability density function.

Figure 4 shows the probability density function taking into account 300 realizations. When sounds are not produced, the fundamental frequency is set to 0, and these values are not showed.

It can be noted that the mean value is near $140\,\mathrm{Hz}$, which is inside the frequency band for fundamental frequencies for men (according to the data used). But, it is necessary to compared the distribution obtained with experimental data.

Another goal of this work is to discuss the capability of the nonparametric probabilistic approach of predicting the responses of a system, because it consider the model uncertainties.

Then, some realizations corresponding to the output radiated pressure were chosen. These results are shown in Figs. 5, 6, 7 and 8. These plots should be compared with those ones showed by Herzel (1993), obtained experimentally. Figure 5 shows a periodic pressure, which corresponds to a normal phonation, which should be compared with the output radiated pressure obtained with the mean model (Fig. 2). Figures 6, 7, and 8, resemble different cases of phonation disorders, similar to experimental measures in pathological cases (papillomas and nodulus) (Herzel, 1993).

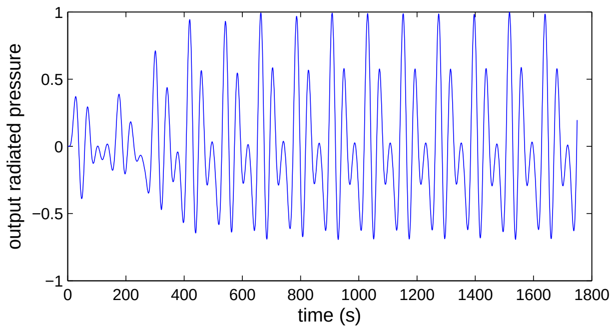

Figure 5. Output radiated pressure, periodic, similar to the output radiated pressure obtained with the mean model.

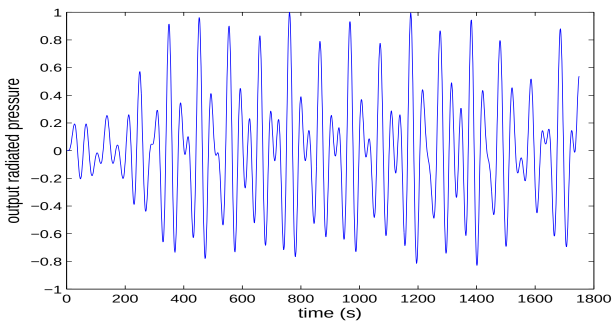

Figure 6. Output radiated pressure, non-periodic, similar to a case with papillomas of the vocal folds.

These results show the possibility of understanding phenomena of voice production related to pathological cases. A possibility is to identify what were the realizations of the mass, stiffness and damping random matrices that generated the results obtained and then to reconstruct the corresponding mechanical model. This could be a good help for preventing or diagnosing pathologies related to voice production.

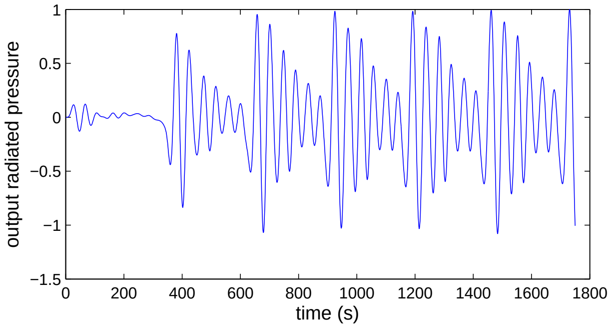

Figure 7. Output radiated pressure, non-periodic, similar to a case with papillomas of the vocal folds.

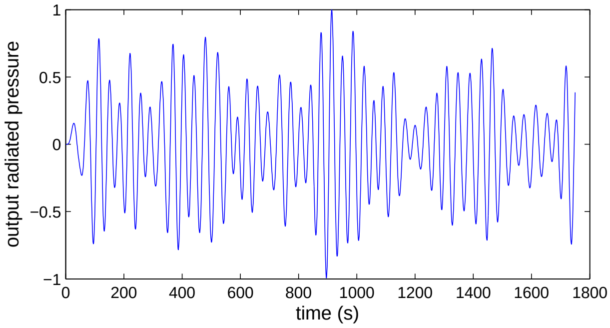

Figure 8. Output radiated pressure, non-periodic, similar to a case with nodulus of the vocal folds.

6. CONCLUSIONS

A non-parametric probabilistic approach has been proposed to model uncertainties in a model of vocal folds for voice production. As it has been presented in the literature, model uncertainties cannot be modeled by using parametric probabilistic approach, that is, if probability density functions are constructed directly for chosen uncertain parameters. The results obtained showed that the non-parametric probabilistic approach, applied to the model used, is capable of predicting realizations of the output radiated pressure which matches experimental data, both in cases of normal phonation and in voice disorders. By using the experience that has been acquired with this work and other, it can be said that some results obtained here could not be obtained by using the parametric probabilistic approach. But, this should be better investigated.

7. ACKNOWLEDGEMENTS

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9. APPENDIX

$$\psi_1(\mathbf{w}) = (\frac{\rho d_1}{a_{g0}+2\ell_g x_1} + \frac{\rho d_2}{a_{g0}+2\ell_g x_2} + \tilde{\ell}_1)$$

$$\psi_2(\mathbf{w}) = (\frac{0.19\rho}{a_{g0}+2\ell_g x_1} + 2\ell_g x_1) + \frac{\rho}{(a_{g0}+2\ell_g x_2)^2}\left[0.5 - \frac{a_{g0}+2\ell_g x_2}{a_1}\left(1 - \frac{a_{g0}+2\ell_g x_2}{a_1}\right)\right]$$

$$\psi_3(\mathbf{w}) = (12\mu\ell_g \frac{d_1}{(a_{g0}+2\ell_g x_1)^3} + 12\ell_g^2 \frac{d_2}{(a_{g0}+2\ell_g x_2)^3} + r_1)$$

$$[M_{vf}] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \ , \ [M_{vt}] = \begin{bmatrix} \tilde{\ell}_1 + \tilde{\ell}_2 & 0 & 0 \\ 0 & \tilde{\ell}_2 + \tilde{\ell}_r & -\tilde{\ell}_r \\ 0 & -\tilde{\ell}_r & \tilde{\ell}_r \end{bmatrix}$$

$$[C_{vf}] = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \ , \ [C_{vt}] = \begin{bmatrix} r_1 + r_2 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_r \end{bmatrix} \ ,$$

$$[K_{vf}] = \begin{bmatrix} k_1 + k_c & -k_c \\ -k_c & k_2 + k_c \end{bmatrix} \ , \ [K_{vt}] = \begin{bmatrix} \frac{1}{\tilde{c}_1} + \frac{1}{\tilde{c}_2} & -\frac{1}{\tilde{c}_2} & 0 \\ -\frac{1}{\tilde{c}_2} & \frac{1}{\tilde{c}_2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \ ,$$

$$\mathbf{h}(\mathbf{w}, \dot{\mathbf{w}}, u_g, \dot{u}_g) = \begin{bmatrix} s_1(x_1) + t_1(x_1)\dot{x}_1 - f_1(x_1, u_g, \dot{u}_g) \\ s_2(x_2) + t_2(x_2)\dot{x}_2 - f_2(x_1, x_2, u_g, \dot{u}_g) \\ -\frac{1}{\tilde{c}_1} u_g \\ 0 \\ 0 \end{bmatrix} ,$$

where
$\tilde{\ell}_n = \frac{\rho \ell_n}{2\pi y_n^2}$, $\tilde{\ell}_r = \frac{8\rho}{3\pi^2 y_n}$, $r_n = \frac{2}{y_n}\sqrt{\rho\mu\frac{\omega}{2}}$, $\omega = \sqrt{\frac{k_1}{m_1}}$, $a_n = \pi y_n^2$, $\tilde{c}_n = \frac{\ell_n \pi y_n^2}{\rho v_c^2}$, ℓ_n is the length of the n th tube, y_n is the radius of the n th tube, and μ is the shear viscosity coefficient.

$$s_\alpha(w_\alpha) = \begin{cases} k_\alpha \eta_{k_\alpha} x_\alpha^3 \ , \ x_\alpha > -\frac{a_{g0}}{2\ell_g} \\ k_\alpha \eta_{k_\alpha} x_\alpha^3 + 3k_\alpha \left\{ \left(w_\alpha + \frac{a_{g0}}{2\ell_g}\right) + \eta_{h_\alpha}\left(w_\alpha + \frac{a_{g0}}{2\ell_g}\right)^3 \right\} \ , \ x_\alpha \leq -\frac{a_{g0}}{2\ell_g} \end{cases} \ , \alpha = 1, 2.$$

$$t_\alpha(x_\alpha) = \begin{cases} 0 \ , \ x_\alpha > -\frac{a_{g0}}{2\ell_g} \\ 2\xi\sqrt{m_1 k_1} \ , \ x_\alpha \leq -\frac{a_{g0}}{2\ell_g} \end{cases} \ , \alpha = 1, 2.$$

$$f_1(x_1, u_g, \dot{u}_g) = \begin{cases} \ell_g d_1 p_{m_1}(x_1, u_g, \dot{u}_g) \ , \ x_1 > -\frac{a_{g0}}{2\ell_g} \\ 0 \ , \ otherwise \end{cases}$$

$$f_2(x_1, x_2, u_g, \dot{u}_g) = \begin{cases} \ell_g d_2 p_{m_2}(w_1, w_2, u_g, \dot{u}_g) \ , \ x_1 > -\frac{a_{g0}}{2\ell_g} \text{ and } x_2 > -\frac{a_{g0}}{2\ell_g} \\ \ell_g d_2 p_s \ , \ x_1 > -\frac{a_{g0}}{2\ell_g} \text{ and } x_2 \leq -\frac{a_{g0}}{2\ell_g} \\ 0 \ , \ otherwise \end{cases}$$

$$p_{m_1}(x_1, u_g, \dot{u}_g) = p_s - 1.37\frac{\rho}{2}\left(\frac{u_g}{a_{g0}+2\ell_g x_1}\right)^2 - \frac{1}{2}\left(12\mu\ell_g \frac{d_1}{(a_{g0}+2\ell_g x_1)^3} + \frac{\rho d_1}{a_{g0}+2\ell_g x_1}\right)\dot{u}_g$$

$$p_{m_2}(x_1, x_2, u_g, \dot{u}_g) = p_{m_1} - *$$
$$* = \frac{1}{2}\left\{(12\mu\ell_g \frac{d_1}{(a_{g0}+2\ell_g x_1)^3} + 12\ell_g^2 \frac{d_2}{(a_{g0}+2\ell_g x_2)^3})u_g + (\frac{\rho d_1}{a_{g0}+2\ell_g x_1} + \frac{\rho d_2}{a_{g0}+2\ell_g x_2})\dot{u}_g\right\} - \frac{\rho}{2}u_g^2\left(\frac{1}{(a_{g0}+2\ell_g x_2)^2} - \frac{1}{(a_{g0}+2\ell_g x_1)^2}\right)$$

10. Responsibility notice

The author(s) is (are) the only responsible for the printed material included in this paper