DYNAMICAL MODELING, CONTROL AND SIMULATION OF SINGLE SHAFT GAS TURBINE

Rafael Rocha Rebouças, rafaelrr@ita.br
Elder Moreira Hemerly, hemerly@ita.br
Marco Aurélio da Cunha Alves, maurelio@ita.br
José Alfredo Ruiz Vargas, vargas@ieee.org
Technological Institute of Aeronautics, Praça Mal. Eduardo Gomes, 50, Vila das Acácias, 12228-900
São José dos Campos, São Paulo, Brazil

Jesus Franklin Andrade Romero, jesus.romero@ufabc.edu.br
ABC Federal University, Catequese St., 242 - Jardim 09210-170, Santo André, São Paulo, Brazil

Abstract. *This paper deals with gas turbine dynamic modeling and the control system designed for the transient analysis of single shaft gas turbines. The main subsystem comprising a gas turbine, have been modeled: compressor, combustor chamber and turbine. The mathematical models of each component are described with the aid of unsteady one-dimensional governing equations and steady-state component characteristics. The adopted controller stucture should maintain constant rotational speed and target temperature of the turbine inlet. The simulation code, developed in Matlab-Simulink using an object-oriented approach, is flexible and can be easily adapted to any kind of plant configuration. Simulations of the transient dynamics have been carried out for a single-shaft gas turbine engine. Time plots of the main variables associated with the gas turbine dynamic behavior are shown.*

Keywords*: gas turbines, modeling, control.*

1. INTRODUCTION

Gas turbines suffer transient operation during startup, load change and shutdown as well as under abnormal conditions such as emergency. During the transient operation, system response time should be as short as possible and temporal peaks of main states such as turbine inlet temperature and rotational speed should not exceed certain reference values required for safe and reliable operation. Therefore, a precise prediction of the transient behavior of gas turbine engines is very important for stable operation, fault diagnosis, controller design, and so on.

Analytical and experimental investigations of the transient behavior of gas turbines began around the early 1950s. Several models and methods for predicting the transient behavior of gas turbines have been proposed and applied in analyzing system dynamic characteristics. In the early stages of those works, the gas turbine was approximated as first-order system with the response of rotor speed to a step change on fuel flow being a simple lag. These models have been used and modified for designing a speed controller with a frequency domain analysis (Rowen, 1983). In another works more recent (Camporeale *et al.*,1997, Kim *et a.l*, 2001, Schobeiri *et al.*,1994) dynamic simulations have been executed in the time domain introducing conservation equation sets.

In this work, models are developed to describe the transient behavior of gas turbines, and applied to the dynamic simulation of a real engine. The mathematical model describing system dynamics is derived on the basis of unsteady one-dimensional conservation equations and steady state characteristics of components (Camporeale *et al.*,1997, Schobeiri *et al.*,1994). The model is simulated by using Simulink/MATLAB© with an object-oriented approach, this code is flexible and can be easily adapted to any kind of plant configuration. The adopted control system, that to used in previous works of gas turbine control (Hajagos and Berube,2001, Hannet and Khan,1993, Rowen 1983) show a structure that consists in two control-loops: speed and temperature, both based on PID control. An approach used in previous work (Carrera,2006a, Carrera,2006b) based on optimization technique, is proposed for the tuning of the control system parameters. This controller should maintain constant rotational speed and target temperature of the turbine inlet or outlet. Simulation tests of the transients are carried out for a single-shaft gas turbine engine. Time plots of the main variables that describe the gas turbine dynamic behavior are shown and discussed.

2. GAS TURBINE MODEL

The model adopted in this paper consider compressors and turbines as volume less elements; a capacity plenum is introduced between these elements in order to take into account the unsteady mass balance; this procedure was adopted in works (Alves, 2003, Camporeale *et al.*, 1997, Silva, 2006, Horobin, 1999, Saravanamuttoo *et al.*, 1990, Schobeiri *et al.*, 1994, Schobeiri, 2004).

The burner is modeled as a pure energy accumulator. For all the components, the algebraic equations are arranged in such a way that the output values can be obtained from the input variables without iterative calculations. In most cases, the input variables are referred to the entering flow. Similarly, the output variables are referred to the exiting flow. However, in order to determine the matching between compressor and first stage turbine, it is necessary to take into account the pressure in the downstream blocks. For this reason, in some cases, a variable referred to the inlet flow is evaluated as a function of the properties of the exiting flow.

2.1. Compressor Model

During the transient the compressor module is considered as a volume less component. Thus, it is assumed that the behavior is considered as a quasi-steady component, so that a steady state compressor map is used. The map provides the corrected mass air flow rate (m_c) and the isentropic efficiency of compressor (η_c), as a function of pressure ratio $P_R = p_{out}/p_{in}$ and corrected rotational speed ($n/\sqrt{\theta}$).

$$m_c = f_1(P_R, n/\sqrt{\theta}) \quad (1)$$

$$\eta_c = f_2(P_R, n/\sqrt{\theta}) \quad (2)$$

The functions represented by Eq. (1) and Eq. (2) are numerically evaluated through interpolation on digitized maps as illustrate in the Figure 1. The function block "Look-up-tables" present in a library of the Simulink/MATLAB© is used to interpolate the maps.

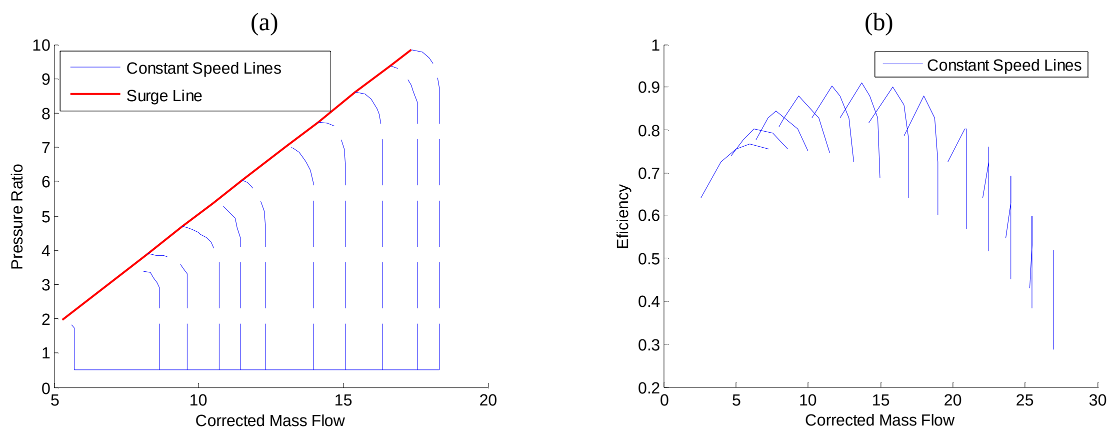

Figure 1. Compressor maps.

The corrected mass flow is defined by

$$m_c = m\left(\frac{\sqrt{\theta}}{\delta}\right) \quad (3)$$

where m is defined as mass flow rate and,

$$\theta = \frac{T_D}{T_s} \quad \text{and} \quad \delta = \frac{P_D}{P_s} \quad (4)$$

T_D and P_D are the temperature and pressure at the design point, and the P_s and T_s are the standard day pressure and temperature. The air temperature at the compressor exit is given by

$$T_{out} = T_{in}\left[1 + \frac{1}{\eta_c}({P_R}^{\gamma_a} - 1)\right] \quad (5)$$

where γ_a is defined as,

$$\gamma_a = \frac{R}{C_{p,air}M} \quad (6)$$

$C_{p,air}$, R and M are the air specific heat, universal gas constant and molar weight, respectively. The specific heat ratio $C_{p,air}$ is evaluated at the temperature T_m that is the arithmetic average between the inlet T_{in} and the outlet T_{out} temperatures. The compressor mechanical power can be evaluated by Eq. (7).

$$Pw_c = \dot{m}(T_{out}C_{p,air}(T_{out}) - T_{in}C_{p,air}(T_{in})) \quad (7)$$

2.2. Plenum

In the model adopted in this paper, the compressor and the turbine units are considered as volume less elements, and the unsteady mass balance is modeled through an adiabatic capacity plenum. A plenum is placed at the compressor outlet, considering the unsteady mass balance in: compressor ducts, compressor discharge and combustion chamber, and others plena are placed between the turbine stages (Alves, 2003, Camporeale *et al.*,1997, Saravanamuttoo *et al.*, 1990, Schobeiri *et al.*, 1994, Schobeiri, 2004). The flow speed is assumed to be negligible inside the plenum, while temperature and pressure are supposed to vary with a polytrophic law with exponent α. Applying the mass conservation law, it follows that.

$$\frac{dP_{out}}{dt} = \propto \frac{RT_{out}}{V_p}(m_{in} - m_{out}) \quad (8)$$

where V_p is the volume of plenum and $\propto$ is defined by

$$\propto = \frac{C_p}{C_v} \quad (9)$$

where $C_p$is the specific heat at the constant pressure, and C_v is the specific heat at the constant volume.

2.3. Combustion Chamber

The combustion chamber has been represented as a pure energy accumulator, neglecting the mass balance that is instead attributed to the upstream plenum block. The inside temperature and pressure is assumed homogeneous and equal to the combustor outlet values (Camporeale *et al.*,1997, Schobeiri *et al.*, 1994). The temperature dynamics is obtained from the unsteady energy conservation describe by.

$$\frac{dT_{out}}{dt} = \beta(m_{air}h_{air} + m_f(h_f + \eta_f LHV) - m_{gas}h_{gas}) \quad (10)$$

where h is defined as enthalpy, η_f as fuel's efficiency and LHV as low heating value. The subscribes $f\ and\ gas$ represent respectively, the fuel and the combustion gases. The m_{gas} and β are:

$$m_{gas} = m_{air} + m_f \quad (11)$$

$$\beta = \frac{RT_{out}}{C_v P} \quad (12)$$

The enthalpy of the air and the gases are calculated based on the specific heats, $c_{p,air}$ and $c_{p,gas}$. The coefficients are the functions of temperature evaluated by the polynomial,

$$c_p = 4186\big((((C_5T + C_4)T + C_3)T + C_2)T + C_1\big) \quad (13)$$

whose coefficients was obtained in (Barbosa, 2005), and are shown in Table 1.

Table 1. Coefficients of polynomial c_p.

Coefficients	Air (200 – 800 K)	Gases of combustion (800 – 2000 K)
C_1	0.24336328	0.19075549
C_2	$-0.329211648 * 10^{-4}$	$0.12752498 * 10^{-3}$
C_3	$0.439514 * 10^{-7}$	$-0.54651988 * 10^{-7}$
C_4	$0.10126885 * 10^{-9}$	$0.89378182 * 10^{-11}$
C_5	$-0.89883655 * 10^{-13}$	0

The pressure drop across the industrial combustion chamber is around 2%, and it's evaluated by

$$P_{in} = \kappa P_{out} \tag{14}$$

where k is the constant based on each type of the combustion chamber (Barbosa, 2005, Saravanamuttoo *et al.*, 2005).

2.4. Turbine

The model adopted to represent the expansion turbine is similar to the compressor model, under the hypothesis that the turbine module behavior can be considered a quasi-steady component. The turbine characteristic curves as illustrate by Figure 1.b and 2, can be used to estimate the corrected mass flow m_c and isentropic efficiency η_t (Camporeale *et al.*, 1997, Saravanamuttoo, 1990, Schobeiri 1994) . These variables are functions of the expansion ratio P_R and the turbine corrected speed $(n/\sqrt{\theta})$ (Eq. (1) and Eq. (2)). Knowing the value of the parameter it is possible to evaluate the mass flow rate across turbine as a function of temperature and pressure at the turbine inlet using the Eq. (15).

$$m_{in} = m_c \frac{\delta}{\sqrt{\theta}} \tag{15}$$

The temperature at the end of expansion in the turbine is given by the relationship expressed by

$$T_{out} = T_{in}\left[1 - \eta_t\left(1 - P_R^{\gamma_g}\right)\right] \tag{16}$$

where γ_g is defined as,

$$\gamma_g = \frac{R}{C_{p,gas}M} \tag{17}$$

The power Pw_t produced by the expanding gas is given by

$$Pw_t = m_{in}(h_{in} - h_{out}) \tag{18}$$

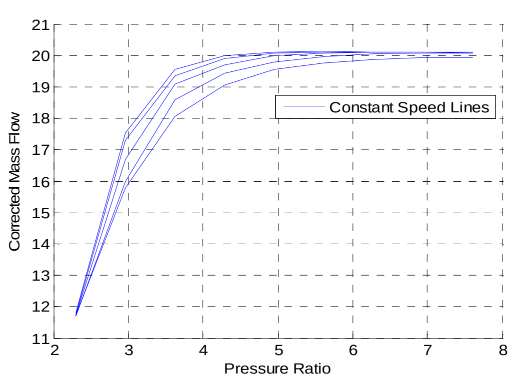

Figure 2. Turbine map.

2.5. Shaft Dynamic

The angular acceleration of the shaft joining compressor, turbine and, eventually, applied load is given by the angular momentum equilibrium. This equilibrium depend on moment of inertia I which includes the inertia effects of the shaft and of the other connected devices (Camporeale,1997, Carrera,2006a, Carrera,2006b, Schobeiri 1994) and is given by

$$\frac{dw}{dt} = \frac{1}{Iw}(Pw_t - Pw_c - Pw_L) \tag{19}$$

where w represents the angular velocity, Pw_t is the internal mechanical power of the turbine, Pw_c is the internal mechanical power required by the compressor and, Pw_L is the mechanical power required by the electrical load.

2.6. Single Shaft Gas Turbine Flow Diagram

The flow diagram of the single shaft model is illustrated by Figure 3. This diagram shows the interaction between the models of the compressor, plenum, combustion chamber and shaft, and their respective equations are indicated.

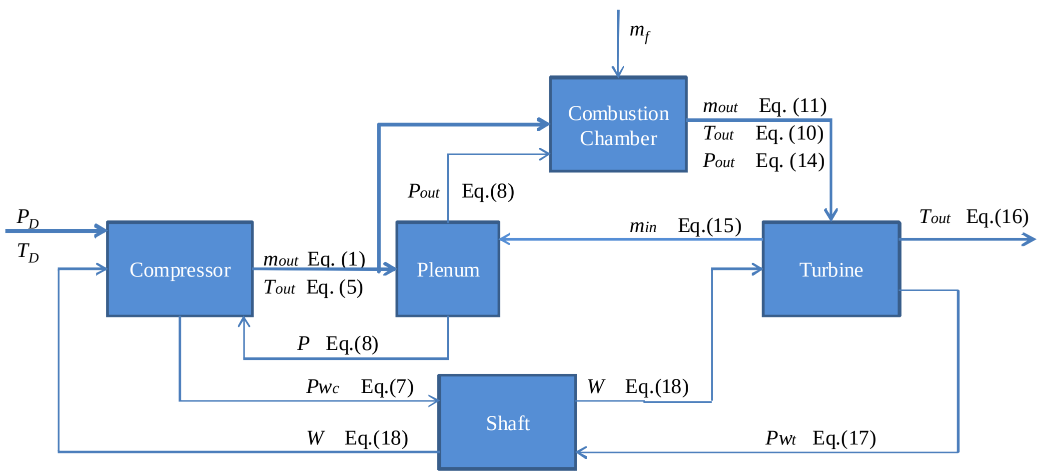

Figure 3. Flow diagram of single shaft gas turbine, with the respective equations.

2.7. Simulation Results

This section shows the simulation results of single shaft gas turbine model proposed in this paper. A generic engine with 3MW of power was utilized for this purpose. Figure 4 illustrate the response of the engine submitted to steps input of fuel flow.

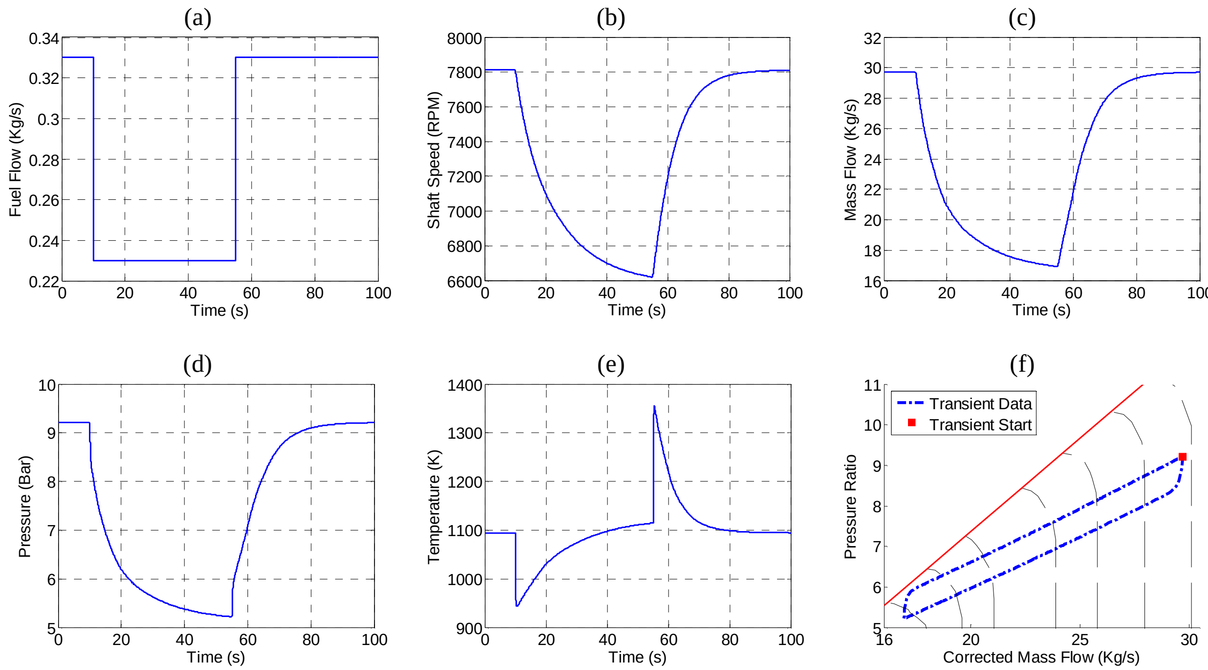

Figure 4. Simulation results. (a) fuel flow, (b) shaft speed, (c) mass flow, (d) pressure of the compressor outlet, (e) burner temperature exit, (f) transient in the compressor map.

The behavior of the speed, mass and pressure are related to each other, and shows characteristics similar as the first order systems. The temperature response, exhibit overshoots as expected, because of the fast change of the fuel flow. Every illustrations, shows a transient behavior very similar to the results generated by commercial software as GasTurb© and GSP©, and results proposed by technical literature (Alves, 2003, Camporeale *et al.*,1997, Hajagos and Berube,2001, Kim *et al.*,2001, Saravanamuttoo *et al.*, 1990).

3. CONTROL SYSTEM

The transients of the industrial gas turbines represent a very important aspect of performance in electrical power systems. The response of shaft speed to disturbances, as change load, is essential for the dynamic performance of electrical power systems. In most generator driving gas turbines, the rotational speed should be controlled at constant value after perturbations.

The adopted control system structure , with minor modifications, has been used in previous works of gas turbine control (Hajagos and Berube,2001, Hannet and Khan,1993). Its structure consists in two control-loops: speed and temperature, both based on PID control and is illustrate by Figure 5.

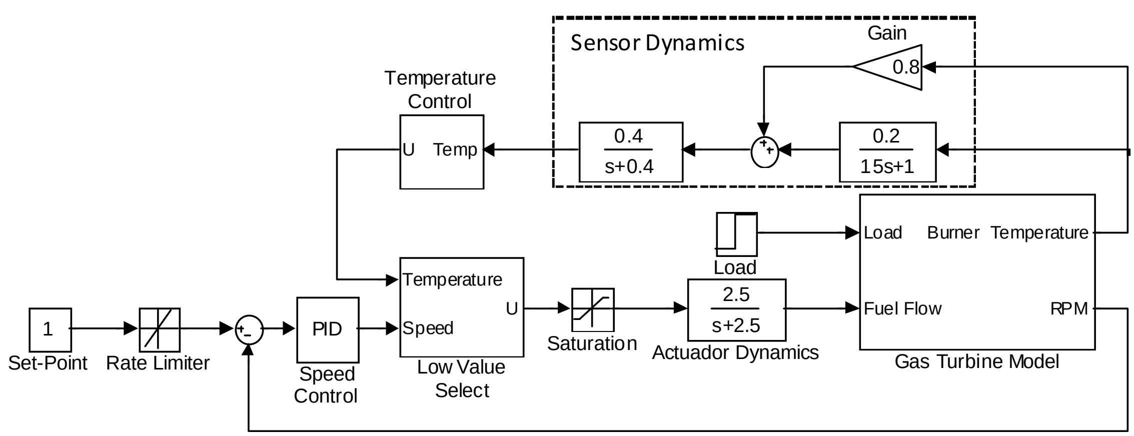

Figure 5. Block diagram for the speed and temperature control.

An approach based on optimization technique is proposed for the tuning control system parameters. The PID's parameters (Table 2) were obtained by the optimization technique Nelder-Mead (Carrera,2006a, Carrera,2006b). The cost function employed is

$$J = \sum_{k}^{N-1} error(k)^2 + 10u(k)^2 \tag{20}$$

Table 2. PID parameters.

K_p	0.1808
K_I	0.1124
K_D	0.0692

3.1. Simulation Results

The proposed simulation considers that the system operates in the equilibrium point, after 1 second the load is reduced to 40% of nominal load. Figure 6 shows the transient response of the main variables of the controlled system.

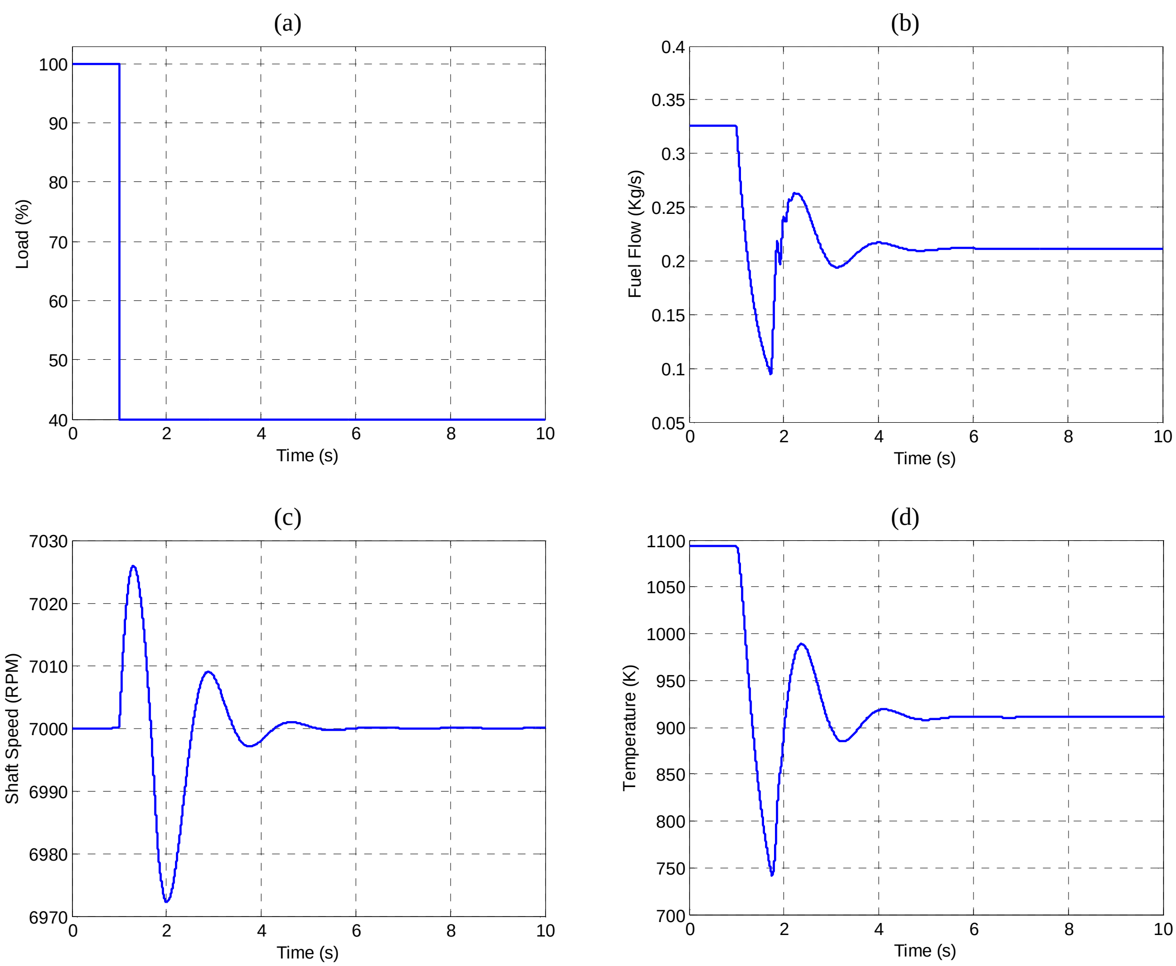

Figure 6. Simulations results of the system controlled. (a) load, (b) response from control (fuel flow), (c) shaft speed, (d) combustion chamber outlet temperature.

In accord with the Figure 6.c, the controlled system demonstrated a good performance, the shaft speed change less the 0.3% of the nominal speed, and the control response (fuel flow) maintain under of the restrictions (saturation) imposed. The proposed control system with optimized parameters, regulated the shaft speed after perturbation in load as expected.

4. CONCLUSION

A dynamical nonlinear model of a gas turbine engine is proposed and applied to the realistic simulations. The main components of a gas turbine are modeled by using performance maps and differential equations are based on: conservation mass, energy and momentum. The results obtained in transients simulations of the single shaft gas turbine model, showed similar results as were proposed by technical literatures (Alves, 2003, Camporeale *et al.*, 1997, Saravanamuttoo *et al.*, 1990, Schobeiri *et al.*, 1994, Schobeiri, 2004) and generated results by commercial software as GasTurb© and GSP©.

A control system was adopted based on PID controller, and an approach for tuning the parameters is proposed. The parameters were obtained using an optimization technique, reached the purpose, and regulated the shaft speed after perturbation of load as expected.

5. ACKNOWLEDGEMENTS

The first and second authors acknowledge the support from the FINEP, and from the Reference Center of Gas Turbines of ITA, headed by Professor João Roberto Barbosa.

6. REFERENCES

Alves, M. A. C. ,2003, "Transitório não-adiabático de turbinas a gás." PhD thesis. Technological Institute of Aeronautics, Brazil.

Barbosa, J. R.,2005, "Desempenho de Turbinas à Gás". Curse notes of the Technological Institute of Aeronautics. Brazil.

Camporeale, S.M.; Fortunato, B.; Matrovito, M., 1997, "A Modular Code for Real Time Dynamic Simulation of Gas Turbine in Simulink". Transactions of the ASME: Journal of Engineering for Gas Turbine and Power, pp. 506-517.

Carrera, R.; Hemerly, E. M., 2006, "Modelagem dinâmica e controle de turbinas a gás". Congresso Brasileiro de Automática, Salvador, Brazil.

Carrera, R.; Hemerly, E. M. , 2006, "Dynamic modeling and control of gas turbines using fuel flow optimization", DINCON,Guaratinguetá, Brazil.

Hajagos, L.M., Berube, G.R., 2001, "Utility experience with gas turbine testing and modeling," IEEE Transactions for Power Systems., pp. 671–677.

Hannet, L. N., and Khan, A., 1993, "Combustion Turbine Dynamic Model Validation From Tests," IEEE Transactions for Power Systems., 8, pp. 152–158.

Horobin, M. S., 1999, "Cycle-match engine models used in functional engine design – an Overview", AGARD RTO – MP – 8.

Kim, J. H., Song, T. W., Kim, T. S., and Ro, S. T., 2000, "Model Development and Simulation of Transient Behavior of Heavy Duty Gas Turbines," Transactions of the ASME: Journal of Engineering for Gas Turbine and Power., pp. 589-594.

Saravanamuttoo, H. I. H.; Rogers, G. F. C., Cohen, H., 2001, "Gas Turbine Theory". 5th Edition. Prentice-Hall, New York, USA.

Saravanamuttoo, H. I. H., Sjolander, S. A., Stranznicky, P. V. and MacIsaac, B. D., , 1990, "Gas Turbine Performance and Desing", Carleton University, Canada.

Schobeiri, M. T., Attia, M., and Lippe, C., 1994, "GETRAN: A Generic Modularly Structured Computer Code for Simulation of Dynamic Behaviour of Aero- and Power Generation Gas Turbine Engines," Transactions of the ASME: Journal of Engineering for Gas Turbine and Power, 116, pp. 483–494.

Schobeiri, M. T., "Turbomachinery Flow Physics and Dynamic Performance". Springer-Verlag. 2004.

Silva, F. J. S., , 2006, "Simulação de Desempenho de Turbinas Gás em Regime Transitório". PhD thesis. Technological Institute of Aeronautics, Brazil.

Rowen, W.,1983, "Simplified Mathematical Representations of Heavy Duty Gas Turbines". Transactions of the ASME: Journal of Engineering for Gas Turbine and Power. pp. 865–869.

9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.