TAYLOR COUETTE FLOW IN AN ECCENTRIC ANNULAR CHANNEL

Elie Luis M. Padilla
Federal University of Uberlândia
epadilla@mecanica.ufu.br

André Leibsohn Martins
CENPES - PETROBRAS
aleibsohn@petrobras.com.br

Aristeu Silveira Neto
Federal University of Uberlândia
38400-902, Uberlândia, MG, Brazil
aristeus@mecanica.ufu.br

Abstract. *Normally, the Taylor-Couette flow occurs in the gap of two concentric cylinders, inner rotating cylinder and outer stationary cylinder, and it is generated by the rotation of inner cylinder. When the Taylor number (governing parameter) is increased above the critical value, appear a set of counter-rotating torroidal vortices in the annular channel. These vortices, called Taylor vortices, appear in eccentric configurations too. The present work studies the behavior of these flows and the influence of fixed and variable eccentricity in three-dimensional domain. The immersed boundary methodology with virtual physics model is used to define the geometrical configuration with rotational and eccentric movement.*
Keywords: *Taylor vortex, eccentric annulus, immersed boundary method.*

1. INTRODUCTION

Second Taylor (1923), who studied experimentally and analytically flows between rotating concentric cylinders, for small gaps between the cylinders (compared with the radii of the internal cylinder) the problem simplifies and becomes dependent on the Taylor number. When this parameter increases above of the critical value appears the named Taylor-Couette instabilities, which consist of counter-rotating axissimetric vortices of torroidal shape. Later, many other researches had been carried (Davey, 1962; Eagles, 1971; Wereley and Lueptow, 1994) due to the great number of applications in several areas of engineering. Taylor-Couette flow with superposed axial flow, also has been object of many investigations, for same reasons previously mentioned. In particular, the Taylor-Couette flow with superposed Poiseuille flow (Kaye and Elgar, 1957; DiPrima, 1960; Lueptow et al., 1992) and Taylor-Couette flow with superposed Couette flow (Ludweig, 1964; Weisberg *et al*., 1992; Hwang and Yang, 2003), are of interest in the well drilling engineering for oil and gas production.

In real problem, the types of flows found in well drilling processes are much more complex than the flows presented, because there are other additional problems, for instance: eccentric movement determined by the interaction of internal and external flows (related to internal channel) and fluids with changeable viscosity due the stress rate (non-Newtonian fluids). Considering, of simplified form of additional problems before mentioned, some works are found in literature, between them: Lockett *et al*. (1992) and Escudier and Gouldson (1995) for concentric configurations and non-Newtonian fluid; Escudier *et al*. (2002) and Escudier *et al*. (2002-b) for fixed eccentric configurations and non-Newtonian fluid.

With the objective of analyze the influence of the variable eccentricity of inner cylinder on the Taylor-Couette flow, in the present work is used the immersed boundary method (Peskin, 1977) with virtual physical model (Lima and Silva *et al*., 2003) to represent the eccentric movement of the inner cylinder. The preliminary results are presented, which confirm the qualities of Lagrangian-Eulerian methodology.

2. PROBLEM FORMULATION

The geometry of problem is depicted in Fig. 1, which is formed for two eccentric horizontal channels, where the inner channel is rotating and posse prescript eccentric movement around of center line of outer channel. Both prescript movements of inner channel are anticlockwise. The radii of inner and outer channels and radii of eccentric movement are R_i, R_o and R_{ex}, respectively, and the length of channels is L. The angular velocity is ω and the eccentric velocity is defined as ω_{ex}. Additionally, are defined the radii relation $R = R_o/R_i$ and the aspect ratio $C = L/R_o$.

The fluid inside eccentric annular space is isothermal and incompressible, with constant proprieties. The computational modeling requires solving three-dimensional flows equations, i.e., solving the mass conservation and the

Navier-Stokes equations. These equations in dimensional form, considering the immersed boundary method, are presented as follows:

$$\nabla.\vec{u} = 0\,, \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + \nabla.(\vec{u}\vec{u}) = -\frac{1}{\rho}\nabla p + \nabla.\left[\nu(\nabla\vec{u} + \nabla\vec{u}^{T})\right] + \vec{f}_e\,, \tag{2}$$

where the velocity vector $\vec{u}$ has components u, v, w in x, y, z directions, respectively, p is the pressure field, $\vec{f}_e = \vec{f}/\rho$ is the Eulerian force that represent the static or moving interface. The fluid considered has density ρ and kinematic viscosity ν.

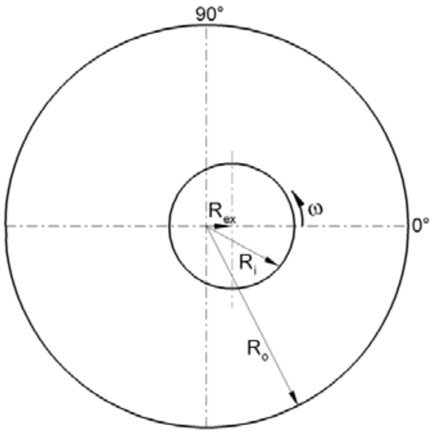
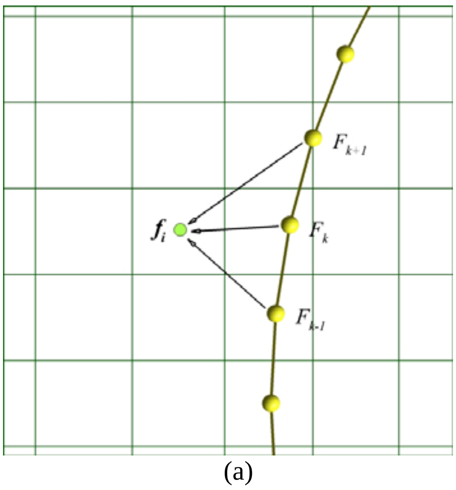


Figure 1.Two-dimensional geometric configuration of physical problem.

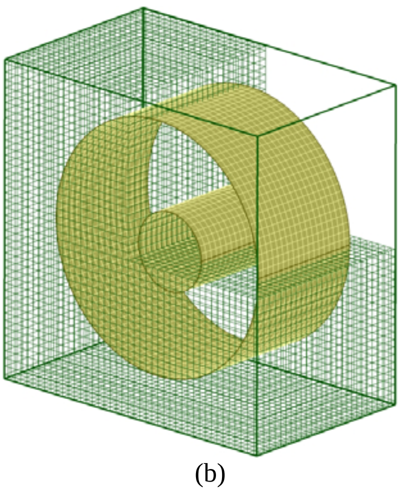
The Eulerian force result from the distribution of Lagrangean force F_k. This distribution process, as shown in Fig. 2(a) is realized using a Gaussian- like function D (Juric, 1996), according the expression:

$$f_e = \sum DF_k \Delta A_k \Delta S_k\,, \tag{3}$$

where ΔA_k and ΔS_k are the area and characteristic length of Lagrangean grid element. The characteristic length is represented for the distances between the centroid of Lagrangean grid element.



(a)



(b)

Figure 2. (a) Two-dimensional sketch of Lagrangian force distribution, (b) Eulerian and Lagrangian domain.

The Lagrangean force is obtained using the physic virtual model, proposed by Lima e Silva *et al.* (2003). This model consist in calculate the force that the solid interface excerce over the fluid through of a momentum balance (with Eq. 2) for one particle positioned in interfacial surface. So, the Lagrangean force is defined as:

$$F_k = \frac{\partial \vec{u}}{\partial t} + \nabla.(\vec{u}\vec{u}) + \frac{1}{\rho}\nabla p - \nabla.\left[\nu(\nabla\vec{u} + \nabla\vec{u}^T)\right]. \quad (4)$$

Each force term, i. e., acceleration, inertial, viscous and pressure term, are calculate using interpolating schemes over the Eulerian velocity and pressure field (Campregher, 2005; Oliveira, 2006). After, F_k is distributing using Eq. (3).

3. NUMERICAL PROCEDURE

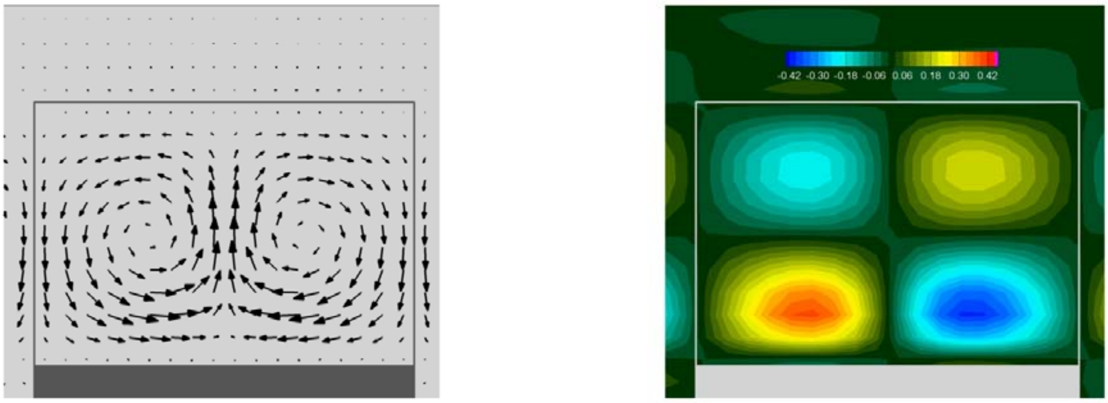
In order to perform the discretization of the equations, the finite volume method was employed on staggered grid, having second order schemes in space and time: central differencing an Adams-Brashforth schemes, respectively. The pressure velocity coupling method was done using the fractional step (Kim and Moin, 1985), where the steps named predictor and corrector are used. The pressure correction is evaluated by solving the Poisson equation using a strongly implicit procedure method, as proposed by Stone (1968).

The time step is evaluated following the CFL stability criteria. Moreover, and non-uniform (concentrated near the walls) grid is employed for the Eulerian domain and uniform grid is employed for the Lagrangian domain, as observed in Fig. 2(b).

The Eulerian part of present numerical code was rigorously validated for laminar-turbulent internal flows by Padilla *et al.* (2005), Padilla and Silveira Neto (2005) and Padilla *et al.* (2006).

4. RESULTS

The dimensionless parameters that govern this problem like is the Taylor number, as commented in section 1. The Taylor number is defined, as suggested by Hwang and Yang (2004), as $Ta = \omega R_i (R_o - R_i)/\nu$. The geometrical parameters R and C are selected as 3.2 and 1. The simulations were performed with a 42x42x24 Eulerian non-uniform grid in x, y, z directions, respectively, and a 106x19 (outer channel) and 34x19 (inner channel) Lagrangian uniform grid in θ, z directions. Periodic boundary condition is considered in axial direction.



(a) (b)

Figure 3. Taylor-Couette Flow at Ta = 100; (a) Taylor vortices, (b) axial velocity component.

Lueptow and Docter (1992) reported that the critical value of Taylor number for R = 3.2 is between 65-70, so, over this value the Taylor vortices are formed. Considering this information, was simulated de Taylor-Couette flow for Ta = 100. Initially, the Taylor-Couette flow was considered to validate the Eulerian-Lagrangean numerical code. The results obtained are shown in Fig. 3. In this figure, one can bee two counter-rotating vortices (Fig. 3a), Taylor vortices, with wave-length equal $C/2$, wave-length forced by the imposed aspect ratio. On the other hand, the axial velocity field (Fig. 3b) present typical cellular distribution. These qualitative results agree very well with experimental results obtained by Cole (1965), Andereck *et al.* (1985) and Wereley and Lueptow (1999).

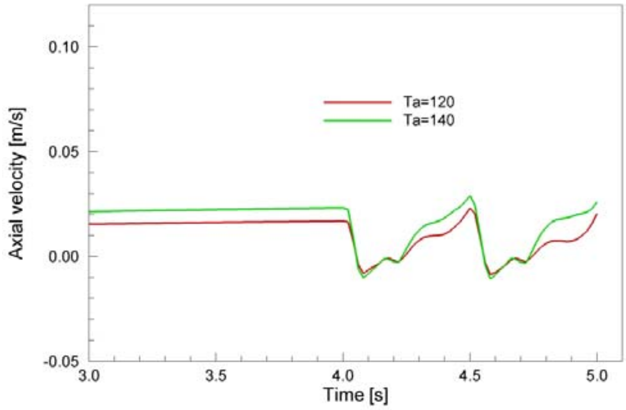


Figure 4. Temporal axial velocity distribution at the center point of the gap.

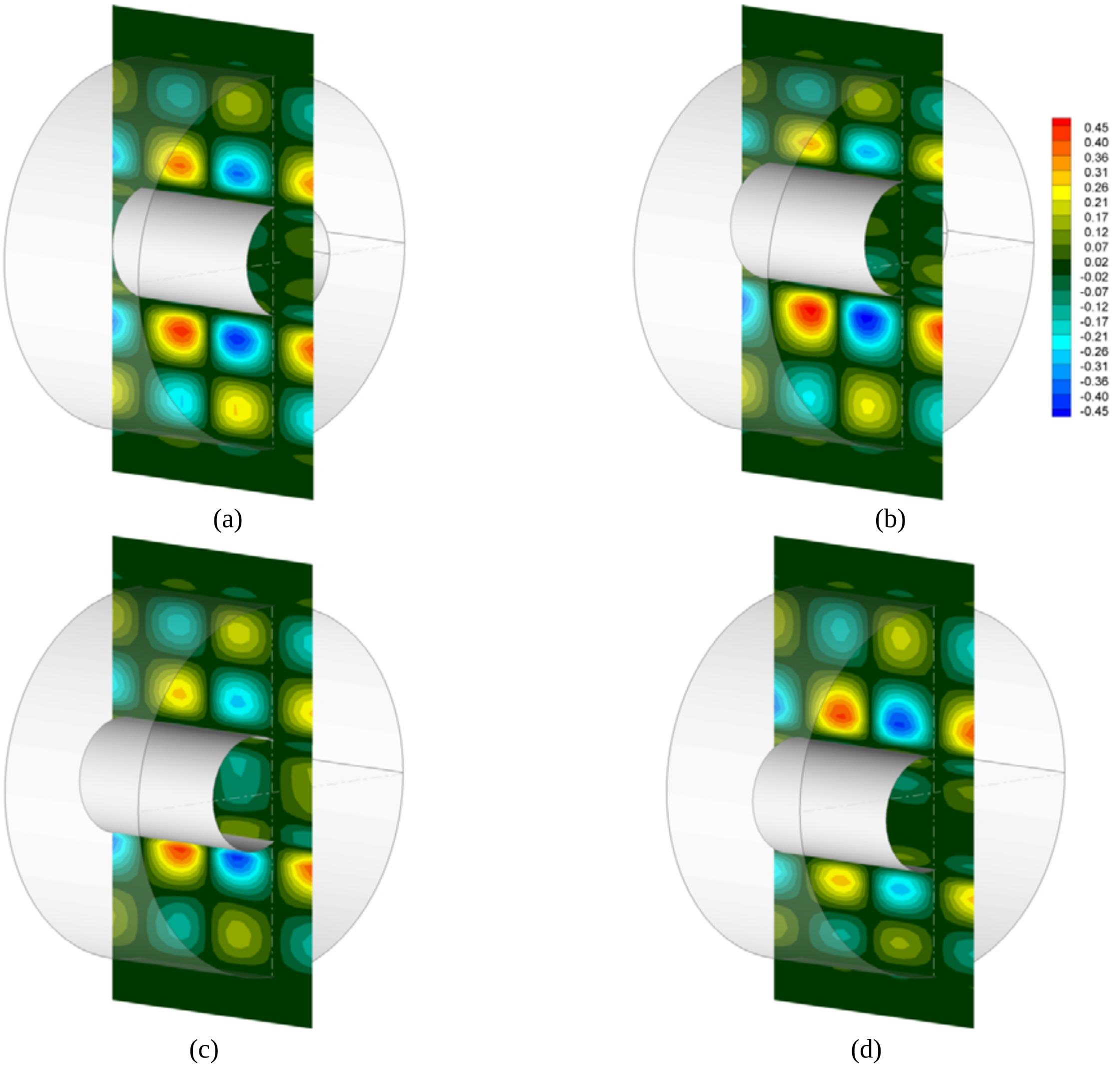


Figure 5. Instantaneous axial velocity field at Ta = 100; (a) α = 0º, (b) α = 72º, (c) α = 180º, (d) α = 252º.

The Couette flow simulations in an eccentric annular channel were performed for several values of Taylor number between $100 \leq Ta \leq 140$ with imposed eccentricity $\varepsilon = R_{ex}/(R_o - R_i) =$ 0.182. The strategy consist in begin the simulation with eccentric channel fixed in $\theta =$ 0° as initial position up to permanent regime attain (4 s), over this regime the eccentric movement begin with velocity $\omega_{ex} = 4\pi$ s^{-1}. The periodic eccentric movement of inner channel can be visualized in Fig. 4, through of temporal distribution of axial velocity at the numerical probe localized in center gap position. In this figure, one can see two cycles formed between 4 and 5 s, as well as an increment of axial velocity component as Taylor number increases.

Figure 5 shows the axial velocity field on $y-z$ plane, at Ta = 100, for several eccentric angles α (angles formed between R_o, at θ = 0°, and R_{ex}). For α = 0º (Fig. 5a), the alternated cellules in axial direction, characteristic from velocity component, approximately has the same dimensions in the upper and lower regions of the annular channel. As commented previously, the eccentric movement is initiated in this position. As the eccentric angle increases the cellular structures change its dimensions as function of the space annular reduction (considering the plane that shows this figure). Thus, the dimensions of the cells very are differentiated for θ = 72º and 252º (Figs 5b and 5d), being these greaters in the upper and lower part of the channel, respectively. For θ = 180º (Fig. 5c), the dimensions of the cellules are similar to the opposing eccentric position, θ = 0º, but the intensity of these field is bigger in lower region of channel due to the effects of inertia caused by the eccentric movement. Certainly, these images show that the dynamic of flows with eccentric movement constantly change, which reflected on the main structure, the Taylor vortices, result in deformation of these structures along a tangential direction. The local variations of axial velocity along an axial direction at r = 0.2 and θ= 90°, for the eccentric positions showed in Fig. 5, are shown in Fig. 6. In agreement with the previous figure, at position of equivalents eccentric angle, θ = 0º and 180º, the profiles are different.

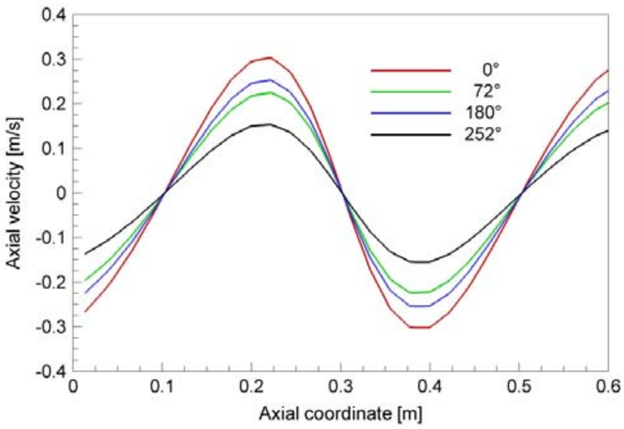


Figure 6. Axial velocity distribution associated the Fig. 5 at r = 0.2 and θ = 90°

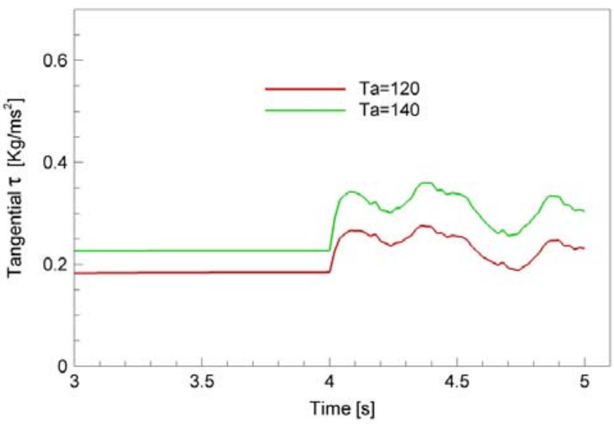


Figure 7. Temporal distribution of tangential shear stress, signal of numerical probe localized at r = 0.2, θ = 90° and z = 0.3.

The temporal distributions of tangential shear stress at numerical probe localized at r = 0.2, θ = 90° and z = 0.3, for two values of Taylor number, are presented in Fig. 7. In this figure, one can see that the inertial effects of the eccentric movement are greaters than these observed ones in axial velocity component (see Fig. 4), i. e., there is hystheresis behavior. The tangential shear stress increases as function of Taylor number, as well as the torque required by rotating the inner channel.

5. CONCLUSIONS

Preliminary results of Taylor-Couette flow simulations inside an annular channel with eccentric movement were presented. The simulations were carried using the immersed boundary methodology with virtual physical model to represent the movement of inner channel. The standard flows considering fixed and variable eccentricity is characterized by presence of deformed Taylor vortices, which deform as function of the imposed eccentricity degree. On the other hand, the quantification of shear stress is more sensible to the inertia of eccentric movement. In general form, these results confirm the potential of the methodology to analyze internal flows with presence of bodies in movement. The continuation of this work understands the accomplishment of simulations with finer grids than allow the quantitative comparison with experimental results.

6. ACKNOWLEDGEMENTS

The authors would like to tank the Support and Research of Minas Gerais State (FAPEMIG) and Research Center of PETROBRAS (CENPES) for the financial support.

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