SCREW-BASED RELATIVE JACOBIAN FOR MANIPULATORS COOPERATING IN A TASK

Luiz Ribeiro, ribeiro@ime.eb.br
Raul Guenther, guenther@emc.ufsc.br
Daniel Martins, daniel@emc.ufsc.br
Universidade Federal de Santa Catarina
Departamento de Engenharia Mecânica
Laboratório de Robótica
Caixa Postal 476
Campus Universitário – Trindade
88040 900 – Florianópolis, SC – Brazil

Abstract. *A current trend in Robotics is the analysis and the design of cooperative manipulation system, i.e. systems composed of multiple manipulator units interacting one another in a coordinated way. A cooperative system is characterized by higher manipulability and load capacity with respect to single-manipulator systems. The concept of the relative Jacobian is used to generate trajectories in the joint space, for two robots in cooperation executing a specified task. This paper proposes an alternative method to derive the relative Jacobian, here designated screw-based relative Jacobian, that relates the velocities of a tool (attached to one manipulator), relative to a blank (attached to another manipulator) as a function of the manipulators joint velocities, using the screw representation of differential kinematics, and the method of successive screw displacements. It is presented a systematic method to calculate the relative Jacobian in a compact, direct and simple form. The presented method is specially suitable when the geometry of manipulators becomes more general and for systems with spatial manipulators. A Cartesian-space tool path, defined relative to the blank, is resolved in the joint space.*

Keywords*: Multiple manipulators systems, relative Jacobian, kinematics, screw theory*

1. INTRODUCTION

Due to the increasing industrial needs, there are great interests in using multiple robot arms to cooperatively manipulate an object, to handle workpieces or to perform part mating operations. Specially, the cooperative motion of two manipulators is considered to be import. Compared with a single manipulator, a system with two manipulators working in a cooperative way could increase its available workspace and its load capability. A significant improvement of the system reachability, manipulability, and productivity can also be achieved (Huang and Lin, 2003).

This paper is centered on synthesizing the optimal motion trajectory for two cooperating robots in a common continuous path while satisfying task requirements, joint limits constraints, and avoiding collisions among the robots and the environment.

The kinematics of cooperation can be broadly divided into two classes. The first class is an extension of the single robot kinematics in which the joint equations for each manipulator are solved separately. The master-slave paradigm is an example of this type of approach in which the master manipulator joint positions are solved according to the specification of the task and then the slave manipulator joint positions are solved to satisfy the constraint equations resulting from the closed kinematic chain.

In the second class, the robot system is described by a single set of equations. The whole system, composed by the manipulator and the workpiece(blank), results redundant and its inverse kinematics are solved by calculating its differential kinematics and, subsequently, by integrating the differential equations to obtain the joint positions. This approach allows to solve the system differential inverse kinematics by optimizing a performance measure along the trajectory (Owen et al., 2003, 2004, 2005), and the operations may be described in tool coordinate of the task rather then in the word coordinates.

The system differential kinematics could be formulated using the concept of relative Jacobian of two manipulators introduced in Lewis (1996), as the matrix that relates the velocities of the tool (attached to one manipulator) relative to the blank (attached to another manipulator) as a function of the manipulators joint velocities.

Lewis (1996) derives the relative Jacobian differentiating the position equation of the closed kinematic chain, and following the Denavit-Hartenberg's (D-H's) convention, attaching a Cartesian coordinate to each link of each manipulator. We remark that this procedure may be cumbersome whenever the geometry of manipulators becomes more general or for systems with spatial manipulators.

In order to overcome this difficulty, we propose an alternative method to derive the matrix, here designated screw-based relative Jacobian, that relates the velocities of a tool (attached to one manipulator), relative to a blank (attached to another manipulator) as a function of the manipulators joint velocities using the screw representation of differential kinematics.

Our intention is to present a systematic method to derive the relative Jacobian in a compact, direct and simple form. This generalization and this method are the main contributions of the paper.

This paper is organized as follows. Section 2 shortly presents the fundamental kinematic tools employed. Section 3 describes the approach using the direct differentiation of the position equation. Section 4 describes our main result, the method to calculate the screw-based relative Jacobian is described. Section 5 outlines the main conclusions.

2. FUNDAMENTAL KINEMATIC TOOLS

Our approach is based on the method of successive screw displacements and on the screw representation of differential kinematics. Both techniques are shortly described in this section.

2.1. Method of Successive Screw Displacements

The method of successive screws displacements provides a representation of the location of a link in a serial kinematic chain with respect to a coordinate frame based on displacements along a series of screws in an appropriate order (successive screws). To describe the method of successive screw displacements, we first present the transformation matrix associated with a screw displacement. Next, we describe the concept of the resultant screw of two successive screw displacements.

Homogeneous transformation screw displacement representation

Chasles's theorem states that the general spatial displacement of a rigid body is a rotation about an axis and a translation along the same axis. Such a combination of translation and rotation is called a screw displacement (Bottema and Roth, 1979). In what follows, we derive a homogeneous transformation that represents a screw displacement (Tsai, 1999).

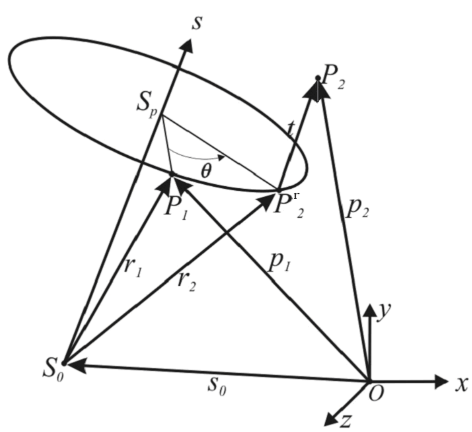

Figure 1. Vector diagram of a spatial displacement.

Figure 1 shows a point P of a rigid body that is displaced from a first position P_1 to a second position P_2 by a rotation θ about a screw axis followed by a translation of t along the same axis. The rotation brings P from P_1 to P_2^r, and the translation brings P from P_2^r to P_2. In Fig. 1, $\mathbf{s} = [s_x \;\; s_y \;\; s_z]^T$ denotes a unit vector along the direction of the screw axis, and $\mathbf{s_0} = [s_{0x} \;\; s_{0y} \;\; s_{0z}]^T$ denotes the position vector of a point lying on the screw axis. The rotation angle θ and the translational distance $\mathbf{t}$ are called the screw parameters. These screw parameters together with the screw axis completely define the general displacement of a point attached to a rigid body. So, they completely define the general displacement of a rigid body.

Representing the first position P_1 by the vector $\mathbf{p_1} = [p_{1x} \;\; p_{1y} \;\; p_{1z}]^T$ and the second position P_2 by the vector $\mathbf{p_2} = [p_{2x} \;\; p_{2y} \;\; p_{2z}]^T$, the general screw displacement for a rigid body can be given by the Rodrigues's formula as follows:

$$\mathbf{p_2} = \mathbf{R}(\theta)\, \mathbf{p_1} + \mathbf{d}(t) \tag{1}$$

where $\mathbf{R}(\theta)$ is the rotation matrix corresponding to the rotation θ about the screw axis and $\mathbf{d}(t)$ is displacement vector corresponding to the translation of t along the screw axis.

Considering the augmented vectors $\hat{\mathbf{p}}_\mathbf{1} = \left[\mathbf{p_1}^T \ \ 1\right]^T$ and $\hat{\mathbf{p}}_\mathbf{2} = \left[\mathbf{p_2}^T \ \ 1\right]^T$ the general displacement of a rigid body (Eq. (1)) can be represented by a homogeneous transformation given by:

$$\hat{\mathbf{p}}_\mathbf{2} = \mathbf{A}(\theta, t) \ \hat{\mathbf{p}}_\mathbf{1} \tag{2}$$

where

$$\mathbf{A}(\theta, t) = \begin{bmatrix} \mathbf{R}(\theta) & \mathbf{d}(t) \\ \mathbf{0} & 1 \end{bmatrix} \tag{3}$$

and the elements of $\mathbf{R}(\theta)$ and of $\mathbf{d}(t)$(see Tsai (1999) for details):

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta + s_x^2\,(1-\cos\theta) & s_y s_x\,(1-\cos\theta) - s_z \sin\theta & s_z s_x\,(1-\cos\theta) + s_y \sin\theta \\ s_x s_y\,(1-\cos\theta) + s_z \sin\theta & \cos\theta + s_y^2\,(1-\cos\theta) & s_z s_y\,(1-\cos\theta) - s_x \sin\theta \\ s_x s_z\,(1-\cos\theta) - s_y \sin\theta & s_y s_z\,(1-\cos\theta) + s_x \sin\theta & \cos\theta + s_z^2\,(1-\cos\theta) \end{bmatrix} \tag{4}$$

$$\mathbf{d}(t) = t\ \mathbf{s} + [\mathbf{I} - \mathbf{R}(\theta)]\ \mathbf{s_0} \tag{5}$$

Successive screw displacements

We now use the homogeneous transformation screw representation to express the composition of two or more screw displacements applied successively to a rigid body.

Figure 2 shows a rigid body σ which corresponds to a second moving link and is moved by two successive screw displacements: a first one, called the fixed joint axis $\$_\mathbf{1}(q_1)$, applied to the joint axis situated between the ground (fixed base) and the first link (first link screw axis), and a second one, called the moving joint axis $\$_\mathbf{2}(q_2)$, applied to the joint axis between the first and the second link (second link axis).

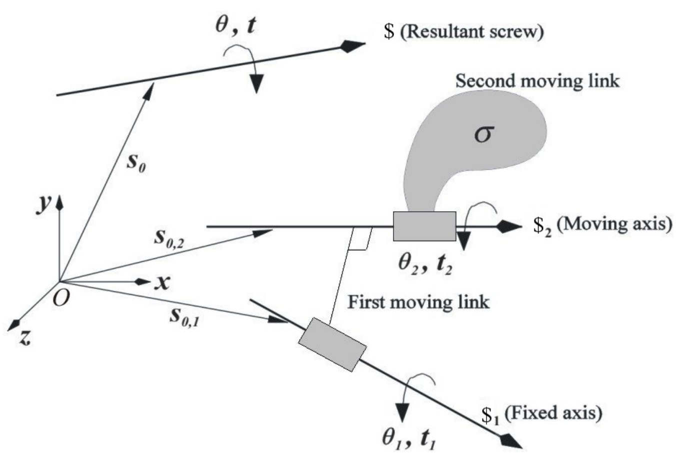

Figure 2. Two-link chain and its associated screw displacements.

As the rigid body rotates about and/or translates along these two joint axes, the best way to obtain its resultant displacement is to displace the rigid body σ about/along the fixed axis and, in what follows, displace the body about/along the moving axis. In this way, the initial location of the moving joint axis can be used for derivation of transformation matrix $\mathbf{A_2}(q_2)$, which represents the $\$_\mathbf{2}(q_2)$ screw displacement while the fixed joint axis is used for derivation of matrix $\mathbf{A_1}(q_1)$, which represents the $\$_\mathbf{1}(q_1)$ screw displacement (see details in Tsai (1999)).

Consequently, the resulting transformation matrix is given by a premultiplication of the two successive screw displacements,

$$\mathbf{A_r}(q_1, q_2) = \mathbf{A_1}(q_1)\mathbf{A_2}(q_2) \tag{6}$$

By generalizing this procedure, the resulting homogeneous matrix $\mathbf{A_r}(q_1, \ldots, q_{i-1})$ can be calculated by

$$\mathbf{A_r}(q_1,\ldots,q_{i-1}) = \prod_{j=1}^{i-1} \mathbf{A_j} \tag{7}$$

2.2. Screw representation of differential kinematics

The Mozzi theorem states that the general spatial differential motion of a rigid body consists of a differential rotation about, and a differential rotation along an axis named instantaneous screw axis (see Cecarelli (2000)). In this way, the velocities of the points of a rigid body with respect to an inertial reference frame *O-xyz* may be represented by a differential rotation $\mathbf{w}$ about the instantaneous screw axis and a simultaneously differential translation $\boldsymbol{\tau}$ about this axis. The complete movement of the rigid body, combining rotation and translation, is called screw movement or twist and is here denoted by $\$$. Figure 3 shows a body "twisting" around the instantaneous screw axis. The ratio of the linear velocity and the angular velocity is called pitch of the screw $h= \|\boldsymbol{\tau}\|/\|\mathbf{w}\|$.

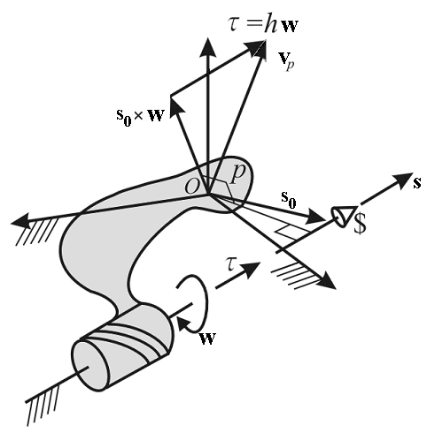

Figure 3. Screw movement or twist.

The twist may be expressed by a pair of vectors, i.e. $\$=[\mathbf{w}^T\ \mathbf{v}_p^T]^T$, where $\boldsymbol{\omega}$ represents the angular velocity of the body with respect to the inertial frame, and $\mathbf{v}_p$ represents the linear velocity of a point P attached to the body which is instantaneously coincident with the origin O of the reference frame. A twist may be decomposed into its magnitude (the terms amplitude and intensity are also found in the literature) and its corresponding normalized screw. The twist magnitude, denoted as $\dot{q}$ in this work, is either the magnitude of the angular velocity of the body, $\|\mathbf{w}\|$, if the kinematic pair is rotative or helical, or the magnitude of the linear velocity, $\|\mathbf{v}_p\|$, if the kinematic pair is prismatic. The normalized screw, $\hat{\$}$, is a twist in which the magnitude is factored out, i.e.

$$\$ = \hat{\$}\ \dot{q} \tag{8}$$

The normalized screw coordinates (Davidson and Hunt, 2004) may be given by,

$$\hat{\$} = \begin{bmatrix} \mathbf{s} \\ \mathbf{s_0} \times \mathbf{s} + h\mathbf{s} \end{bmatrix} \tag{9}$$

where, as above, the vector $\mathbf{s} = [s_x\ \ s_y\ \ s_z]^T$ denotes a unit vector along the direction of the screw axis, and the vector $\mathbf{s_0} = [s_{0x}\ \ s_{0y}\ \ s_{0z}]^T$ denotes the position vector of a point lying on the screw axis (see Fig. 4).

So, the twist given in Eq. (8) expresses the general spatial differential movement (velocity) of a rigid body with respect to an inertial reference frame *O-xyz*. The twist could also represent the movement between two adjacent links of a kinematic chain. In this case, the twist $\$_\mathbf{i}$ represents the movement of link i with respect to link $(i-1)$.

In Robotics, generally, the movement between a pair of bodies is determined by either a rotative or a prismatic joint. For a rotative joint, the pitch of the twist is null $(h=0)$. In this case, the normalized screw of the i^{th} joint is expressed by:

$$\hat{\$}_i = \begin{bmatrix} \mathbf{s}_i \\ \mathbf{s_0}_i \times \mathbf{s}_i \end{bmatrix} \tag{10}$$

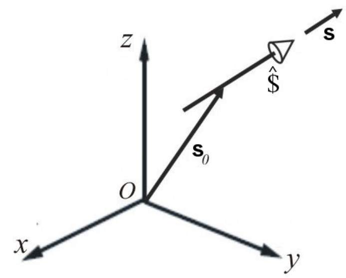

Figure 4. Normalized screw.

For a prismatic joint, the pitch of the twist is infinite $(h = \infty)$ and the normalized screw of the i^{th} joint reduces to:

$$\hat{\$}_i = \begin{bmatrix} \mathbf{0} \\ \mathbf{s}_i \end{bmatrix} \tag{11}$$

The twist, defined in Eq. (8), is given by:

$$\$_i = \hat{\$}_i \dot{q}_i \tag{12}$$

where $\dot{q}_i$ is the magnitude of the i^{th} joint twist, given by $\dot{q}_i = \dot{\theta}_i$ for a revolution joint and $\dot{q}_i = \dot{d}_i$ for a prismatic joint.

2.3. Screw-based Jacobian

The motion of a rigid body may be considered as a result of instantaneous twists about several screw axes. So, for a serial manipulator, the movement of the end-effector results from the instantaneous twists about the joint axes of an open-loop chain.

Let $\mathbf{w}$ be the end-effector angular velocity, and $\mathbf{v_0}$ be the linear velocity of a point in the end-effector of a serial manipulator that is instantaneously coincident with the origin of the fixed reference frame. According the twist definition given before, in which it is expressed as a pair of vectors, and considering that the resulting movement of the end-effector is obtained adding linearly the joint twists (Tsai, 1999), the end-effector twist (velocity pair vector) is given by:

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{w} \\ \mathbf{v_0} \end{bmatrix} = \sum_{i=1}^{n} \hat{\$}_i \dot{q}_i \tag{13}$$

where $\hat{\$}_i$ is the i^{th} joint unit screw defined in Eqs. (10) or (11).

Equation (13) could be rewritten as

$$\dot{\mathbf{x}} = \mathbf{J}\ \dot{\mathbf{q}} \tag{14}$$

where $\mathbf{J}$ is the Jacobian matrix given by:

$$\mathbf{J} = \left[\hat{\$}_1\ \ldots\ \hat{\$}_n\right] \tag{15}$$

By Eqs. (10) and (11) it is clear that to compute the Jacobian matrix, the directions $(\mathbf{s})$ and the locations of the joints axes $(\mathbf{s_0})$ relative to a reference frame should be determined first. This could be done using the successive screw displacements method as presented in the sequel.

Consider the augmented vectors $\bar{\mathbf{s}}_i = [s_{xi}\ s_{yi}\ s_{zi}\ 0\,]^T$ and $\bar{\mathbf{s}}_{\mathbf{0}_i} = [s_{0_{xi}}\ s_{0_{yi}}\ s_{0_{zi}}\ 1]^T$, corresponding to the direction and location of the i^{th} joint.

Let $\bar{\mathbf{s}}_{i_{ref}}$ and $\bar{\mathbf{s}}_{\mathbf{0}_{i_{ref}}}$, with ref index, be the vectors $\bar{\mathbf{s}}_i$ and $\bar{\mathbf{s}}_{\mathbf{0}_i}$ at the reference position, i.e., in case θ_i and t_i are null.

As the vectors defining the i^{th} joint axis direction and location depend on the movement of the $(i-1)$ preceding joints, the augmented vectors are calculated by:

$$\bar{\mathbf{s}}_i = \mathbf{A_r}(q_1,\ldots,q_{i-1})\bar{\mathbf{s}}_{i_{ref}} \tag{16}$$

$$\bar{\mathbf{s}}_{\mathbf{0_i}} = \mathbf{A_r}(q_1,\ldots,q_{i-1})\bar{\mathbf{s}}_{\mathbf{0}_{i_{ref}}} \tag{17}$$

where $\mathbf{A_r}(q_1,\ldots,q_{i-1})$ is the resulting matrix given in Eq. (7)

3. RELATIVE JACOBIAN

The relative Jacobian was first defined by Lewis (1996) for two manipulators operating in kinematic cooperation, with the movement of the tool (attached to the end-effector of one of the manipulators) relative to the blank (attached to another manipulator). More specifically, the relative Jacobian gives the velocities of the tool relative to the blank as a function of the manipulators joint velocities. The same definition is adopted by Owen et al. (2003) (2004) (2005).

Lewis (1996) calculates the relative Jacobian by deriving the position vector. For completeness, we reproduce this procedure shortly in this section.

Let the velocity vector of the tool relative to the blank in the blank's frame ($\dot{\mathbf{x}}_{\mathbf{p,t}}$) given by:

$$\dot{\mathbf{x}}_{\mathbf{p,t}} = \mathbf{J_R}\ \dot{\mathbf{q}} \tag{18}$$

where, $\mathbf{J_R}$ is the relative Jacobian, and $\dot{\mathbf{q}}$ is the vector of the manipulators joint velocities, obtained by combining the tool manipulator joint velocities ${}^{\mathbf{1}}\dot{\mathbf{q}}$ and the blank manipulator joint velocities ${}^{\mathbf{2}}\dot{\mathbf{q}}$:

$$\dot{\mathbf{q}} = \begin{bmatrix} {}^{\mathbf{1}}\dot{\mathbf{q}}^T & {}^{\mathbf{2}}\dot{\mathbf{q}}^T \end{bmatrix}^T \tag{19}$$

The velocity vector of the tool relative to the blank is usually given by:

$$\dot{\mathbf{x}}_{\mathbf{p,t}} = \begin{bmatrix} \dot{\mathbf{r}}^{\mathbf{p}\,T}_{\mathbf{p,t}} & \mathbf{w}^{\mathbf{p}\,T}_{\mathbf{p,t}} \end{bmatrix}^T \tag{20}$$

where, $\dot{\mathbf{r}}^{\mathbf{p}}_{\mathbf{p,t}}$ and $\mathbf{w}^{\mathbf{p}}_{\mathbf{p,t}}$ are the vectors of the linear and angular velocities of the tool (point $\mathbf{t}$), in the blank's frame ($\mathbf{p}$).

In this work we use the frame, vector and rotation matrix notation as in Sciavicco & Siciliano (2000), in which $\mathbf{O_i}$ is the i^{th} frame and $\mathbf{r}^{\mathbf{k}}_{\mathbf{i,j}}$ is the vector from the i^{th} frame to the j^{th} frame as seen from the k^{th} frame, and $\mathbf{R}^{\mathbf{i}}_{\mathbf{j}}$ denotes the rotation matrix of frame j with respect to frame i.

In the following, the relative Jacobian is derived for two 3-dof planar manipulators as shown in Fig.5.

The procedure starts by defining four frames: $\mathbf{b_1}$ in base of tool robot; $\mathbf{b_2}$ in base of blank robot; $\mathbf{p}$ in a workpiece (blank), and $\mathbf{0}$, the inertial frame (Fig.5).

It is considered that:

i. the desired task is defined by the linear and angular velocities in the p frame (blank frame), that is, $\dot{\mathbf{r}}^{\mathbf{p}}_{\mathbf{p,t}}$ and $\mathbf{w}^{\mathbf{p}}_{\mathbf{p,t}}$;
ii. there is no relative movement between the workpiece (blank) and the gripper of the blank robot; and,
iii. the position and orientation of the bases of the manipulators with respect to the 0 frame are constant with respect to the time.

From Fig.5, the tool position vector relative to the blank in the inertial frame is:

$$\mathbf{r}^{\mathbf{0}}_{\mathbf{p,t}} = \mathbf{r}^{\mathbf{0}}_{\mathbf{0,b1}} + \mathbf{r}^{\mathbf{0}}_{\mathbf{b1,e1}} + \mathbf{r}^{\mathbf{0}}_{\mathbf{e1,t}} - \mathbf{r}^{\mathbf{0}}_{\mathbf{0,b2}} - \mathbf{r}^{\mathbf{0}}_{\mathbf{b2,e2}} - \mathbf{r}^{\mathbf{0}}_{\mathbf{e2,p}} \tag{21}$$

After expressing each term with respect to its local frame and solving for $\mathbf{r}^{\mathbf{p}}_{\mathbf{p,t}}$, the resultant expression is derived with respect to time, resulting

$$\dot{\mathrm{r}}^{\mathrm{p}}_{\mathrm{p,t}} = \mathrm{R}^{\mathrm{p}}_{\mathrm{b1}}\ {}^{1}\mathrm{J_v}\ {}^{1}\dot{\mathrm{q}} - \mathrm{R}^{\mathrm{p}}_{\mathrm{e1}}\ \Omega(\mathrm{r}^{\mathrm{e1}}_{\mathrm{e1,t}})\ \mathrm{R}^{\mathrm{e1}}_{\mathrm{b1}}\ {}^{1}\mathrm{J_w}\ {}^{1}\dot{\mathrm{q}} - \mathrm{R}^{\mathrm{p}}_{\mathrm{b2}}\ {}^{2}\mathrm{J_v}\ {}^{2}\dot{\mathrm{q}} + \mathrm{R}^{\mathrm{p}}_{\mathrm{p}}\ \Omega(\mathrm{r}^{\mathrm{p}}_{\mathrm{p,t}})\ \mathrm{R}^{\mathrm{p}}_{\mathrm{b2}}\ {}^{2}\mathrm{J_w}\ {}^{2}\dot{\mathrm{q}} + \mathrm{R}^{\mathrm{p}}_{\mathrm{e2}}\ \Omega(\mathrm{r}^{\mathrm{e2}}_{\mathrm{e2,p}})\ \mathrm{R}^{\mathrm{e2}}_{\mathrm{b2}}\ {}^{2}\mathrm{J_w}\ {}^{2}\dot{\mathrm{q}} \tag{22}$$

where

$$^{\mathbf{i}}\mathbf{J} = \begin{bmatrix} ^{\mathbf{i}}\mathbf{J_v} & ^{\mathbf{i}}\mathbf{J_w} \end{bmatrix}^T \tag{23}$$

$$\mathbf{\dot{r}^{bi}_{bi,ei}} = \mathbf{^{i}J_{v}}\ \mathbf{^{i}\dot{q}} \tag{24}$$

$$\mathbf{w^{bi}_{bi,ei}} = \mathbf{^{i}J_{w}}\ \mathbf{^{i}\dot{q}} \tag{25}$$

and $\mathbf{^{i}J_{v}}$ is the contribution to the linear velocity, $\mathbf{^{i}J_{w}}$ is the contribution to the angular velocity, and $\mathbf{^{i}J}$ is the manipulator conventional Jacobian for the i^{th} manipulator joint, using the Denavit-Hartenberg's transformation matrices (see details in Tsai (1999)), and $\mathbf{\Omega}$ is a 3 x 3 skew-symmetric matrix representing the vector $\mathbf{p^{i}_{j}}$ (expressed in the i^{th} frame).

$$\mathbf{\Omega}(\mathbf{p^{i}_{j}}) = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix} \tag{26}$$

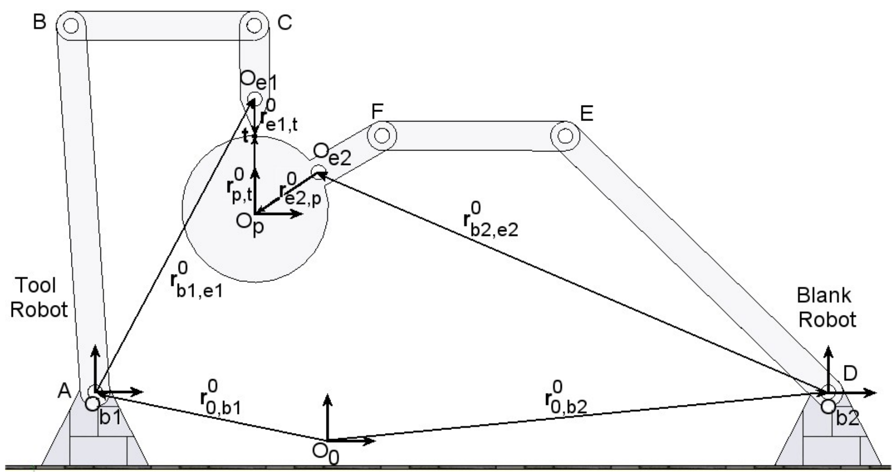

Figure 5. Two 3-dof robotic planar system.

Similarly, for the angular velocities:

$$\mathbf{w^{0}_{p,t}} = \mathbf{w^{0}_{0,b1}} + \mathbf{w^{0}_{b1,e1}} + \mathbf{w^{0}_{e1,t}} - \mathbf{w^{0}_{0,b2}} - \mathbf{w^{0}_{b2,e2}} - \mathbf{w^{0}_{e2,p}} \tag{27}$$

Expressing each term with respect to its local frame, considering Eq. (25), knowing the orientation of the bases of the manipulators with respect to the 0 frame are constant with respect to the time, then $\mathbf{w^{0}_{0,b1}} = \mathbf{w^{0}_{0,b2}} = \mathbf{0}$, and there are no relative movement between the workpiece (blank) and the gripper of the blank robot, then $\mathbf{w^{0}_{e1,t}} = \mathbf{w^{0}_{e2,p}} = \mathbf{0}$, $\mathbf{w^{p}_{p,t}}$ results:

$$\mathbf{w^{p}_{p,t}} = \mathbf{R^{p}_{b1}}\ \mathbf{^{1}J_{w}}\ \mathbf{^{1}\dot{q}} - \mathbf{R^{p}_{b2}}\ \mathbf{^{2}J_{w}}\ \mathbf{^{2}\dot{q}} \tag{28}$$

Substituting Eqs. (22) and (28) in Eq. (20) gives:

$$\mathbf{\dot{x}_{p,t}} = \begin{bmatrix} \mathbf{R^{p}_{b1}}\ \mathbf{^{1}J_{v}} - \mathbf{R^{p}_{e1}}\ \mathbf{\Omega}(\mathbf{r^{e1}_{e1,t}})\mathbf{R^{e1}_{b1}}\ \mathbf{^{1}J_{w}} & -\mathbf{R^{p}_{b2}}\ \mathbf{^{2}J_{v}} + \mathbf{\Omega}(\mathbf{r^{p}_{p,t}})\mathbf{R^{p}_{b2}}\ \mathbf{^{2}J_{w}} + \mathbf{R^{p}_{e2}}\ \mathbf{\Omega}(\mathbf{r^{e2}_{e2,p}})\mathbf{R^{e2}_{b2}}\ \mathbf{^{2}J_{w}} \\ \mathbf{R^{p}_{b1}}\ \mathbf{^{1}J_{w}} & -\mathbf{R^{p}_{b2}}\ \mathbf{^{2}J_{w}} \end{bmatrix} \mathbf{\dot{q}} \tag{29}$$

and the relative Jacobian is:

$$\mathbf{J_{R}} = \begin{bmatrix} \mathbf{R^{p}_{b1}}\ \mathbf{^{1}J_{v}} - \mathbf{R^{p}_{e1}}\ \mathbf{\Omega}(\mathbf{r^{e1}_{e1,t}})\mathbf{R^{e1}_{b1}}\ \mathbf{^{1}J_{w}} & -\mathbf{R^{p}_{b2}}\ \mathbf{^{2}J_{v}} + \mathbf{\Omega}(\mathbf{r^{p}_{p,t}})\mathbf{R^{p}_{b2}}\ \mathbf{^{2}J_{w}} + \mathbf{R^{p}_{e2}}\ \mathbf{\Omega}(\mathbf{r^{e2}_{e2,p}})\mathbf{R^{e2}_{b2}}\ \mathbf{^{2}J_{w}} \\ \mathbf{R^{p}_{b1}}\ \mathbf{^{1}J_{w}} & -\mathbf{R^{p}_{b2}}\ \mathbf{^{2}J_{w}} \end{bmatrix} \tag{30}$$

From these equations, the input data for calculating the relative Jacobian are:

i. the initial configuration of the manipulators, following Denavit-Hartenberg's (D-H's) convention;
ii. the $\mathbf{{}^{i}J}$ is the manipulator conventional Jacobian for the i^{th} manipulator joint, using the Denavit-Hartenberg (D-H) transformation matrices;
iii. the tool configuration relative to the tool manipulator;
iv. the blank configuration relative to the blank manipulator;
v. the manipulators joint movements; and,
vi. the tool point *t* position with respect to the *p* frame.

This procedure based on differentiating the relative position vector. Following D-H's convention, a Cartesian coordinate system is attached to each link of each manipulator. The first limitation is that modeling process is cumbersome. Second, for complex geometry of each manipulator, locating a number of local coordinate frames and extracting the corresponding parameters are tedious and prone mistakes. Third, the complexity of the relative Jacobian increases rapidly as the geometry of manipulators becomes more general.

To overcome these difficulties we propose an alternative method to calculate the relative Jacobian, based on the screw theory.

4. SCREW-BASED RELATIVE JACOBIAN

In this section, the relative Jacobian is derived in an alternative way using the screw representation of differential kinematics and the method of successive screw displacements. As before, we intend to calculate the velocity of the *t* point attached to the tool with respect to the *p* frame attached to the blank (see Fig. 6).

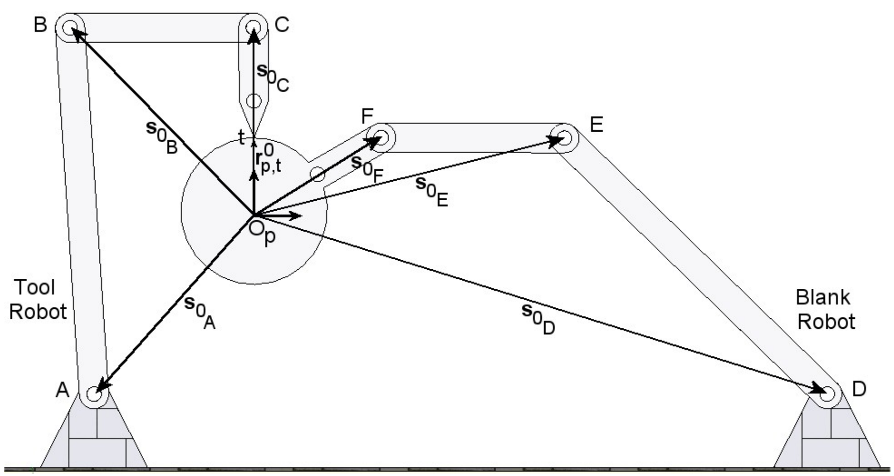

Figure 6. Two 3-dof robotic planar system – vectors $\mathbf{s_{0_i}}$.

The velocity of the tool relative to the blank in the *p* frame can be expressed by a twist as defined in section 2:

$$\dot{\mathbf{x}}^{\mathbf{P}} = \left[\begin{array}{cc} \mathbf{w}^{\mathbf{P}T} & \dot{\mathbf{r}}_{\mathbf{0}}^{\mathbf{P}T} \end{array} \right]^T \tag{31}$$

where $\mathbf{w}^{\mathbf{P}}$ is the angular velocity of the tool in the p frame and $\dot{\mathbf{r}}_{\mathbf{0}}^{\mathbf{P}}$ is the linear velocity of the tool point that instantaneously coincides with the origin of the *p* frame (O_p).

Let $\dot{q}_i$, $i = 1, ..., n_1 + n_2$, be the magnitude of the twist *i*, where n_1 and n_2 are number of joints of robot 1 (tool robot) and robot 2 (blank robot), respectively. As described in section 2, the first-order instantaneous kinematics can be written as:

$$\left[\begin{array}{cc} \mathbf{w}^{\mathbf{P}T} & \dot{\mathbf{r}}_{\mathbf{0}}^{\mathbf{P}T} \end{array} \right]^T = \sum_{i=1}^{n_1+n_2} \hat{\$}_{\mathbf{i}}^{\mathbf{P}} \dot{q}_i \tag{32}$$

where $\hat{\$}_{\mathbf{i}}^{\mathbf{P}}$ is the i^{th} joint normalized screw in *p* frame, and $\dot{q}_i$ is the corresponding twist magnitude.

Equation (32) can be rewritten in the following form:

$$\mathbf{\dot{x}^{p}} = \mathbf{J^{p}}\ \mathbf{\dot{q}} \tag{33}$$

where $\mathbf{\dot{x}^{p}}$ is given by Eq. (31) and represents the velocity of the tool in *p* frame, the vector $\mathbf{\dot{q}}$ is the vector which combines all the manipulators joint velocities, given by Eq. (19) in case we have two manipulators, and the matrix $\mathbf{J^{p}}$ is defined as:

$$\mathbf{J^{p}} = \left[\hat{\$}^{\mathbf{p}}_{\mathbf{1}}\ \ldots\ \hat{\$}^{\mathbf{p}}_{\mathbf{n_1+n_2}}\right] \tag{34}$$

where the i^{th} normalized screw ($\hat{\$}^{\mathbf{p}}_{\mathbf{i}}$) is calculated using Eqs. (10) or (11).

The vector of velocities of the *t* point relative to the blank in *p* frame, defined in Eq. (20) can be obtained from Eq. (28) by considering that the angular velocity of the *t* point with respect to the *p* frame expressed in *p* frame is

$$\mathbf{w^{p}_{p,t}} = \mathbf{w^{p}} \tag{35}$$

and that the linear velocity of the *t* point with respect to the *p* frame expressed in *p* frame is

$$\mathbf{\dot{r}^{p}_{p,t}} = \mathbf{\dot{r}^{p}_{0}} + \mathbf{w^{p}} \times \mathbf{r^{p}_{p,t}} \tag{36}$$

where $\mathbf{r^{p}_{p,t}} = \begin{bmatrix} r^{p}_{p,t_x} & r^{p}_{p,t_y} & r^{p}_{p,t_z} \end{bmatrix}^T$ is the vector from the origin of the *p* frame to the *t* point, as seen from the *p* frame.

Considering the well known relation $\mathbf{w^{p}} \times \mathbf{r^{p}_{p,t}} = -\mathbf{\Omega}(\mathbf{r^{p}_{p,t}})\ \mathbf{w^{p}}$, where $\mathbf{\Omega}(\mathbf{r^{p}_{p,t}})$ is the skew-symmetric matrix given by (see Tsai, 1999):

$$\mathbf{\Omega}(\mathbf{r^{p}_{p,t}}) = \begin{bmatrix} 0 & -r^{p}_{p,t_z} & r^{p}_{p,t_y} \\ r^{p}_{p,t_z} & 0 & -r^{p}_{p,t_x} \\ -r^{p}_{p,t_y} & r^{p}_{p,t_x} & 0 \end{bmatrix} \tag{37}$$

Eq. (36) can be expressed as:

$$\mathbf{\dot{r}^{p}_{p,t}} = \mathbf{\dot{r}^{p}_{0}} - \mathbf{\Omega}(\mathbf{r^{p}_{p,t}})\ \mathbf{w^{p}} \tag{38}$$

Using Eqs. (35) and (38) in Eq. (20) results:

$$\mathbf{\dot{x}_{p,t}} = \begin{bmatrix} \mathbf{\dot{r}^{p}_{p,t}} \\ \mathbf{w_{p,t}} \end{bmatrix} = \begin{bmatrix} -\mathbf{\Omega}(\mathbf{r^{p}_{p,t}}) & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{w^{p}} \\ \mathbf{\dot{r}^{p}_{0}} \end{bmatrix} \tag{39}$$

Substituting Eq. (33) in Eq. (39), the velocity of the *t* point attached to the tool with respect to the *p* frame attached to the blank ($\mathbf{\dot{x}_{p,t}}$) can be expressed by:

$$\mathbf{\dot{x}_{p,t}} = \begin{bmatrix} -\mathbf{\Omega}(\mathbf{r^{p}_{p,t}}) & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{J^{p}}\ \mathbf{\dot{q}} \tag{40}$$

Comparing Eq. (40) and Eq. (18), we define the screw-based relative Jacobian as:

$$\mathbf{J_{R}} = \begin{bmatrix} -\mathbf{\Omega}(\mathbf{r^{p}_{p,t}}) & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{J^{p}} \tag{41}$$

From Eqs. (41), (34), (10), (11), (16), (17) and (7) the screw-based relative Jacobian may be calculated in a systematic way, giving the following input data:

i. initial configuration of the manipulators;
ii. directions and locations of the manipulators axes with respect to the *p* frame, in the manipulators reference position;
iii. the manipulators joint movements; and,

iv. the tool point t position with respect to the p frame.

The initial configuration in this method is introduced in a direct and compact way. This makes it simpler, specially in case the system has spatial manipulators and when the geometry of manipulators becomes more general.

Moreover, in the screw-based relative Jacobian method there are only two coordinate frames, in contrast to the early method, the D-H's convention, $n+1$ local coordinate for n joints. The screw parameters should not be confused with D-H parameters. The joint variables of a screw displacement represent the actual angles of rotation and/or distances of translation needed to bring the end-effector of tool robot from a reference position to a target position.

5. CONCLUSIONS

The main objective of this paper was to synthesize the optimal motion trajectories for two cooperating robots in a common continuous path while satisfying task requirements, joint limits constraints, and avoiding collisions among the robots and the environment. The conventional method to derive relative Jacobian was described. Then a new method to derive the relative Jacobian for manipulators cooperating in a task is presented. The proposed method uses the screw representation of differential kinematics and, consequently is named screw-based relative Jacobian.

It was shown that relative Jacobian can be greatly simplified by expressing the screws in a unique reference frame. This new method is a systematic procedure to calculate the relative Jacobian in a compact, direct and simple form. There are only two coordinate frames, one attached to the blank and other to the tool; following D-H's convention, a Cartesian coordinate system is attached to each link of each manipulator. The presented method is specially suitable when the geometry of manipulators becomes more general and for systems with spatial manipulators.

Moreover, the D-H parameters do not represent the angle of rotation or the distance of translation about a joint axis. To obtain the actual displacements, it is necessary to subtract the joint variables associated with a reference position from a target position. Other advantage of using successive screw displacements is that there is a unique reference position and can be conveniently chosen.

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