Direct Computations of Impulsive and Indicial Unsteady Aerodynamic Responses with Modern CFD Techniques

Alexandre Noll Marques
Instituto Tecnológico de Aeronáutica, CTA/ITA
12228-900 - São José dos Campos - SP - Brazil
ale_noll@yahoo.com

João Luiz F. Azevedo
Instituto de Aeronáutica e Espaço, CTA/IAE/ALA
12228-903 - São José dos Campos - SP - Brazil
azevedo@iae.cta.br

Abstract. *Unsteady aerodynamic impulsive or indicial responses can be extremely helpful for efficient aeroelastic analysis purposes. However, the discontinuous nature of impulsive and indicial motions has led to misinterpretations about their numerical implementation. For this reason, many unsteady CFD applications consider the use of smooth input excitations in order to resolve a portion of the frequency content of the flow dynamics. The present work presents how new interpretations of CFD solvers as discrete-time systems change the way impulsive and indicial responses can be directly obtained. The objective is to demonstrate that the rigorous relationships theoretically established among the aerodynamic responses to impulsive, indicial, harmonic and smooth inputs can be reproduced numerically with modern CFD solvers. Although the numerical results presented herein are obtained with a single CFD tool, the argument is valid for every numerical solution scheme. The CFD tool in question solves the two-dimensional Euler equations with an explicit time march, using a finite volume discretization which supports fully unstructured grids. The results are compared both in the time and the frequency domains, which yields a more complete understanding of details of the numerical solutions.*

Keywords:*Impulsive Response, Indicial Response, Unsteady Aerodynamics, CFD, Discrete-time*

1. INTRODUCTION

Indicial and impulsive responses are essential to the determination of the dynamic characteristics of any system. In the aerodynamic case, such responses are important to the fields of flight control and stability, aeroelasticity, and aeroservoelasticity, just to name a few. Traditionally, the methods developed for determining the unsteady aerodynamic behavior for subsonic and supersonic regimes are based on linearized formulations (Bisplinghoff, Ashley and Halfman, 1955) which do not present the same satisfactory results in the transonic range. According to Tijdeman (1977), this occurs due to the nonlinearity of transonic flows characterizing a significant alteration of the flow behavior, even when a profile is submitted to small perturbations. Along the last three decades, at least, there have been many attempts to numerically solve more elaborate aerodynamic models for the unsteady transonic regime. Beam and Warming (1974) presented one of the earliest papers on solving the unsteady Euler equations. They used an explicit, third-order, non centered, finite difference scheme to obtain indicial solutions for plunging flat plates and parabolic arc airfoils in the transonic regime. However, the boundary conditions were imposed in a small-disturbance fashion on a mean-surface approximation to the thick airfoil. This procedure was not considered satisfactory for more practical airfoils. This fact led to the development of numerical solvers based on the nonlinear transonic potential formulations. Traci, Albano and Farr (1975) developed the codes STRANS2 and UTRANS2 to solve the transonic steady and unsteady equations by the relaxation method. Later, Ballhaus and Goorjian (1978) developed the code LTRANS2 to time-accurately solve the low-frequency transonic small-disturbance (TSD) formulation, which is able to capture small shock motions. Yang, Guruswamy and Striz (1982) present a compilation of the results obtained with the application of these and others transonic codes to different cases of aeroelastic analyses. Ashley (1980) reported the use of semi-empirical corrections to linearized theory results as a mean of improving flutter predictions. Nevertheless, Ashley (1980) himself believed that really satisfactory aeroelastic quantitative predictions of the transonic regime should be possible only when accurate, three-dimensional, unsteady CFD codes were developed.

Computational Fluid Dynamics (CFD) is a subject that has played an extremely important role in recent aerodynamics studies. The possibility of numerically treating a broad range of phenomena which occur in flows over bodies of practically any geometry has numerous advantages over experimental determinations, such as greater flexibility together with time and financial resource savings. However, obtaining more reliable numerical results for a growing number of situations has been one of the major recent challenges in many science fields. Hirsch (1992) shows that, particularly in aerodynamics, the general phenomena are governed by the Navier-Stokes equations, which constitute a system of coupled nonlinear partial differential equations that has no general analytical solution. Space and time discretization schemes, as well as convergence acceleration techniques, boundary condition establishment and other numerical integration tools are available and largely used in order to solve such the mathematical models that describe the aerodynamic system.

Nevertheless, the use of indicial and impulse excitations with complex CFD solvers has led to misinterpretations in the beginning of the 90's. Time-marching schemes possess the ability to solve perturbations which advance through the flowfield with velocities up to some determined limit. This limit depends on the scheme itself and mesh resolution (Hirsch, 1992). Consequently, care must be taken when evaluating indicial and impulsive responses with CFD codes, once such excitations usually cause large perturbation velocities (Raveh, 2001). But, despite of the difficulties in dealing with these velocities, they are finite in magnitude, opposite to what Bakhle *et al.* (1991) have stated. This misinterpretation is easily understood in light of the usual definition of the impulse and indicial functions, in which a discontinuous change of the function value causes an indefinite derivative. This indefinite value is many times considered infinite, what, consequently, leads to infinite velocities. Actually, although the terminology "impulse function" is used throughout this paper, it should be noted that it is not rigorously a function, but rather a "generalized function" (Davies, 1978), and that the derivative of both the impulse and indicial functions becomes mathematically consistent only in certain integral applications.

Hence, the numerical implementation of such discontinuous functions can be quite controversial and some authors even stated that it is not feasible (Oliveira, 1993). However, recently Silva (1997) showed that, once the flow governing equations are discretized, the resultant system can no longer be viewed as a continuous-time one, but it should be treated as a discrete-time system. Furthermore, linear discrete-time systems present properties that are very similar to those of linear continuous-time systems and, more importantly, the convolution theorem is valid (Oppenheim and Schafer,1989). Therefore, there are discrete-time sequences that, when applied as input to discrete-time systems, possess the exact same properties of the impulse and indicial functions. Moreover, these sequences do not contain any singularities, and their numerical implementation is straightforward. In fact, although they are well-defined sequences for discrete-time systems, and they are by no means mere approximations of continuous functions, these sequences have been used as approximations of the impulse and step excitations in many numerical applications over the years (Beam and Warming, 1974, Ballhaus and Goorjian, 1978). Hence, the new interpretation proposed by Silva (1997) comes to validate and reaffirm as a rigorous procedure what was thought to be an approximation until recently.

The difficulties of treating the high velocities induced by the impulse and indicial excitations can be overcome by the use of smoother excitations (Davies and Salmond,1980, Bakhle *et al.*,1991, Oliveira, 1993). Although such smooth excitations do not possess all the characteristics of the impulse and indicial ones, neither are capable of uniformly exciting the entire frequency domain, they can be calibrated in order to excite certain frequency ranges of interest. The convolution sum establishes a relationship between the desired impulsive response and the one actually obtained with the smooth excitations for the frequency range of interest (Oppenheim and Schafer, 1989). Nevertheless, after appropriately defining the aerodynamic input function, Silva (1997) states that the smooth excitation approach is not valid and that it has presented reasonable results because the smooth sequences excite only low frequencies and this led to small errors.

The authors believe that they are finally able to present a unified view of the different approaches for obtaining impulsive and indicial aerodynamic responses through the application of modern CFD solvers. In fact, this unified view should demonstrate that these different approaches are rigorously equivalent and, when correctly implemented, generate identical results within the numerical accuracy expected from any numerical methodology. Actually, the present work is based on the finite volume formulation, where a CFD tool is applied with two-dimensional unstructured meshes around lifting surfaces to acquire unsteady responses to harmonic, smooth pulse, discrete step and unit sample motions. The unit sample sequence is defined latter as the discrete-time sequence equivalent to the continuous-time impulse function. Although this investigation is conducted with a particular CFD solver, it is believed that the results presented here are representative of most numerical schemes used in modern CFD solvers.

The development of the CFD tool applied in the present work is a result of the increased demand for aerodynamic parameters that followed the evolution of the work and projects performed by Instituto de Aeronáutica e Espaço (CTA/IAE). Nevertheless, the application of CFD tools in these parametric analyses has always been limited by the need of adequate code development and the lack of computational resources compatible with the work to be performed. Therefore, a progressive approach has been adopted in the development of CFD tools in CTA/IAE and in Instituto Tecnológico de Aeronáutrica (ITA), as presented by Azevedo (1990), Oliveira (1993), Azevedo, Fico and Ortega (1995), Azevedo, Strauss and Ferrari (1999), Bigarelli, Mello and Azevedo (1999), Simões and Azevedo (1999), Bigarelli and Azevedo (2002), Marques (2004), Marques and Azevedo (2006), and Marques (2007).

2. CFD SOLVER AS A DISCRETE-TIME SYSTEM

As aeronautical researchers are generally used to deal with continuous-time problems, it has been very common in the literature to look at CFD solvers as mere approximations to continuous-time systems. Therefore, it is equally common to use continuous-time system properties and thinking when performing CFD simulations. They are approximations, indeed, but discrete-time approximations. Hence, as shown by Silva (1997), once the governing equations have been discretized, the resulting numerical scheme is actually a discrete-time system, which has its own properties and peculiarities.

This misinterpretation has led many authors (Davies and Salmond, 1980, Mohr, Batina and Yang, 1989, Bakhle *et al.*, 1991, Oliveira, 1993) to justify the use of smooth pulse excitations, since the theoretical continuous-time impulse and indicial excitations are not numerically feasible. Nevertheless, Silva (1997) has suggested the use of equivalent discrete-time excitations: unit sample and discrete step (Oppenheim and Schafer, 1989). The unit sample and discrete step sequences formally hold properties very similar to the ones attributed to the impulse and step functions, respectively. Namely, if a linear time-invariant discrete-time system is subjected to a unit sample excitation, then the corresponding response will contain all the information about the system, and the response to every other input is given by the convolution sum (Oppenheim and Schafer, 1989). Moreover, the unit sample rigorously excites uniformly the complete frequency domain.

The discrete step response also characterizes a discrete-time system since one can reproduce the unit sample response from it. As demonstrated by Marques and Azevedo (2006), the backward difference of the discrete step response yields the unit sample one, *i.e.*,

$$S[n] - S[n-1] = h[n], \tag{1}$$

where $S[n]$ designates the system response to a discrete-step input. Thus, theoretically, it would be more convenient, and even computationally cheaper, because the transient solution should die out more rapidly, to acquire CFD results submitting the body to either unit sample or discrete step type perturbations than to smooth inputs. As mentioned earlier, this has been accomplished by some authors (Beam and Warming, 1974, Ballhaus and Goorjian, 1978, Silva, 1997, Silva and Raveh, 2001, Raveh, 2001), but some numerical complications may arise when simulating such cases. Raveh (2001), particularly, has performed a thorough inspection of these responses and concluded that numerical errors may occur due to the large velocities induced by sharp motions, when these exceed the propagation velocity the numerical time marching scheme can resolve. Naturally, it all depends on the numerical formulation used in the CFD solver, input amplitude, and time step. Additionally, Raveh (2001) states that simulations carried out with the discrete step excitation tend to be less influenced by the simulation parameters than those using the unit sample.

All this reasoning does not invalidate the use of smooth excitations, although it changes the way they are understood. This is because they should be seen as discrete input sequences derived from sampling the corresponding continuous functions. However, it is clear that, in face of these new proposals, the authors are interested in evaluating their usage with the present CFD solver, and also in comparing results and efficiency. Such discussions are presented in the results part of the paper. The smooth input used here is the one suggested by Bakhle *et al.* (1991), which is defined as

$$f_p(t) = \begin{cases} 4\left(\frac{t}{t_{max}}\right)^2 exp\left(2 - \frac{1}{1-\frac{t}{t_{max}}}\right), & 0 \le t < t_{max}, \\ 0, & t \ge t_{max}, \end{cases} \tag{2}$$

where t_{max} is the excitation duration. The function defined in Eq. (2) guarantees a smooth motion, and both the function and its first derivative go to zero when $t = 0$ and $t = t_{max}$.

As the exponentially-shaped input is not a unit sample excitation, the real unit sample response is evaluated using a well-known property of the convolution theorem (Oppenheim and Schafer, 1989),

$$g[n] = f_p[n] * h[n] \Longrightarrow G[n] = F_p[n]H[n], \qquad H[n] = \frac{G[n]}{F_p[n]}, \tag{3}$$

where $h[n]$ represents the time response to a unit sample excitation, and $g[n]$ is the response to the sampled exponentially-shaped excitation, $f_p[n]$. The sequences in capital letters are the discrete Fourier transforms of the corresponding sequences in lower case letters. Therefore, the frequency domain unit sample responses can be obtained by dividing the discrete Fourier transform (DFT) of the responses to the sampled exponentially-shaped pulse by the DFT of the input sequence. Although the input is not the exact unit sample excitation, it is capable of exciting the reduced frequencies of interest in aeroelastic studies. The corresponding frequency domain points resulting from this procedure are presented by Oliveira (1993) and Marques (2007). Finally, the time domain response results from the inverse discrete Fourier transform (Oppenheim and Schafer, 1989) of the frequency domain response obtained with the previous procedure. However, this time domain response is not exactly identical to the unit sample response, because only part of the frequency domain content is properly solved.

3. FLOW SOLVER

The CFD tool applied in this work is based on the 2-D Euler equations, which represent two-dimensional, compressible, rotational, inviscid and nonlinear flows. Therefore, it is completely capable of capturing the shock waves present in transonic flows. The spatial discretization is based on a cell centered, finite volume scheme, following the ideas described by Jameson and Mavriplis (1986). This scheme requires the addition of artificial dissipation terms in order to damp high frequency errors and also oscillations that occur near shock waves. The artificial dissipation model used in the present work is similar to the one presented by Mavriplis (1990). Furthermore, the numerical solution is advanced in time using a second-order accurate, 5-stage, explicit, hybrid scheme which evolved from the consideration of Runge-Kutta time stepping schemes (Jameson and Mavriplis, 1986, Mavriplis 1990). Further details on the flow solver used in the present work are described by Marques (2007).

4. RESULTS AND DISCUSSION

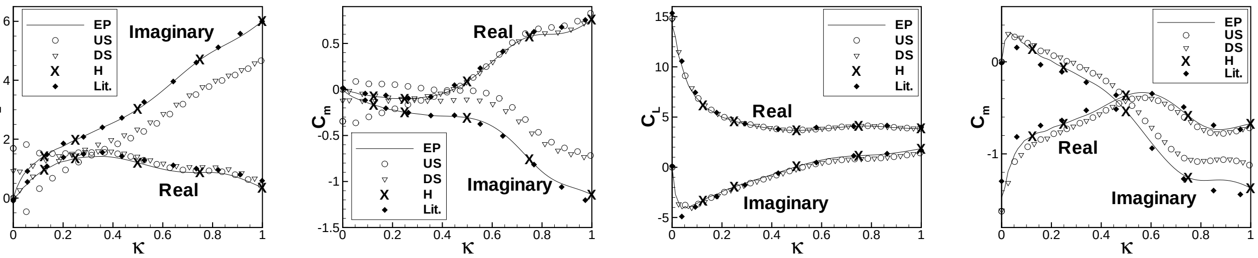
4.1 General considerations

Before attempting applications of the proposed analyses, some validation simulations are performed with the CFD tool. This has been done throughout the entire development of this code, as presented by Azevedo (1992), Oliveira (1993), Simões and Azevedo (1999), Marques (2004), and Marques and Azevedo (2006). Once the CFD tool was tested and proved to be reliable, the next step was to proceed in obtaining the unsteady responses of interest. The test-case considered throughout this paper involves a NACA 0012 airfoil at $M_\infty = 0.8$, *i.e.*, in transonic regime, and $\alpha_0 = 0$. The steady-state solution obtained with the present CFD solver is shown by Marques (2007).

Rausch, Batina and Yang (1990) present frequency domain responses for the transonic configuration in question when the airfoil is submitted to both plunging and pitching motions. These responses are obtained numerically with a solver very similar to the one used in the present work. Furthermore, Rausch, Batina and Yang (1990) also apply the pulse transfer function technique, *i.e.*, the solutions are obtained with a smooth input, and, therefore, they serve as good standards for comparison. The present authors attempt to reproduce these solutions using exponentially-shaped, unit sample, discrete step and harmonic inputs in order to address the capabilities of obtaining impulsive and indicial aerodynamic responses with modern CFD solvers. Moreover, this is done with both deformable and rigid mesh formulations. It is important to emphasize that the pitching motion occurs over the quarter-chord point of the airfoil. Additionally, C_ℓ and C_m denote the lift and moment coefficients, respectively. Moreover, the values presented here for these coefficients are actually the differences in relation to the steady-state values. For a NACA 0012 airfoil with $\alpha_0 = 0$, one could expect for zero lift and moment, but numerical errors lead to small residual values. Finally, henceforth, the pitching moment refers to the quarter-chord point and is positive in the nose-up direction.

4.2 Deformable mesh

Figure 1 shows the impulsive frequency domain responses in terms of the aerodynamic coefficients C_ℓ and C_m for different types of excitation. The exponentially-shaped pulse (EP) input extents until the dimensionless time value of 1 and it has maximum value of $0.0001c$ for the plunge mode, and 0.001 deg. for the pitch mode. Similarly, the amplitude of both unit sample (US) and discrete step (DS) excitations is $0.000001c$ for the plunge mode, and 0.0001 deg. for the pitch mode.



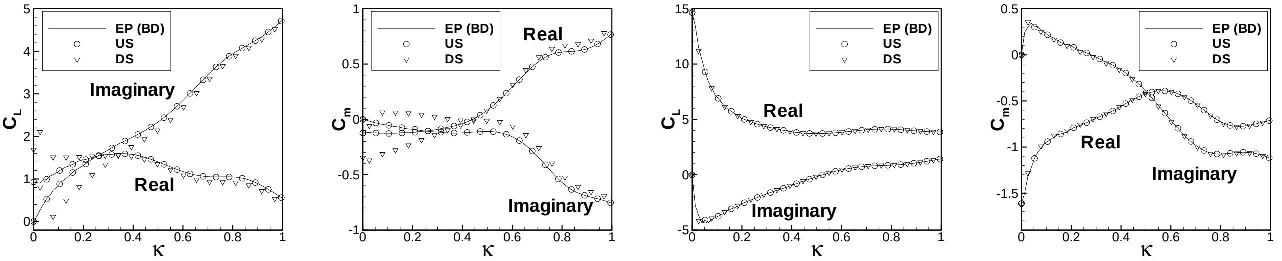
(a) C_ℓ response for the plunge mode. (b) C_m response for the plunge mode. (c) C_ℓ response for the pitch mode. (d) C_m response for the pitch mode.

Figure 1. Low reduced frequency impulsive responses of a NACA 0012 airfoil at $M_\infty = 0.8$ and $\alpha_0 = 0$.

It can be seen that the exponentially-shaped responses match the harmonic ones in both modes, which is a proof of the validity of the adopted hypothesis of linearity in relation to the motion amplitude, and that the EP input can adequately excite the frequency range of interest. Moreover, these results also agree very well with the literature data, what corroborates the quality of the present CFD solver. Nevertheless, the responses obtained directly with discrete step and unit sample inputs deviate from the expected values. Specifically, the plunge mode results for these inputs present nonzero values for a steady-state response, which makes no physical sense at all. However, the results are not completely different from the EP responses, which might indicate numerical difficulties rather than inadequacies in the solution procedure.

It is very important to recall the argument of Silva (1997), according to which it is not possible to determine impulsive aerodynamic responses in the plunge mode performing the division of the DFT of the CFD response by the DFT of a smooth excitation due to a two-channel input dependency of the aerodynamic input function. However, the perfect agreement between the impulsive frequency domain responses obtained with the EP and the values acquired with the harmonic oscillations in the plunge mode proves that the aerodynamic system input depends only on the motion, *i.e.*, the single-input premise is valid. This issue is further discussed by Marques (2007).

In order to comprehend the behavior reported in Fig. 1, it is important to understand the effect of the mesh deformation on the aerodynamic solutions. The mesh deformation algorithm implemented in the present CFD solver is constructed in a way that the elements far from the body should absorb most of the motion, while the grid around it remains practically unchanged. This is done in order to avoid, or attenuate, spurious fluctuations that might occur in the solution due to the mesh-dependency of the numerical scheme, since the mesh near the body is more important in solving the major phenomena that dominate the aerodynamic response. The fact is that this procedure works well for smooth motions, such as the exponentially-shaped pulse, but it is not effective for sharp inputs. This reasoning is reinforced by analyzing the response for the same exponentially-shaped pulse, but keeping the mesh without any deformation, except for the wall, presented in Fig. 2. Of course, this is only possible when the input amplitude is small enough so that the wall does not cross the neighboring volumes. In this case, the EP gives the same results as the DS excitation with mesh deformation in both modes. The agreement with the US solution is not excellent for the plunging motion, but it is much better than in the prior situation. On the other hand, for the pitching motion, the EP solution, considering only the wall motion, and the US solution, with mesh deformation, show an excellent matching. This indicates that, for the US and DS inputs, the dynamic mesh algorithm is not effective in avoiding the mesh deformation around the body, which causes numerical errors.



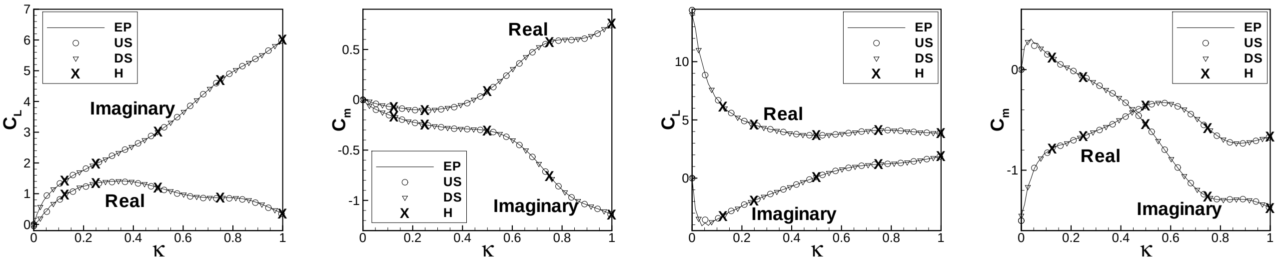
(a) C_ℓ response for the plunge mode.
(b) C_m response for the plunge mode.
(c) C_ℓ response for the pitch mode.
(d) C_m response for the pitch mode.

Figure 2. Low reduced frequency impulsive responses of a NACA 0012 airfoil at $M_\infty = 0.8$ and $\alpha_0 = 0$. BD indicates that only the body nodes move, without mesh deformation.

4.3 Rigid mesh

The mesh deformation effects observed in the results presented in the previous section drove the authors to explore the possibility of moving the mesh rigidly, following the body motion, and, consequently, without volume deformations. Naturally, this can only be done in the cases in question because the body performs rigid motions exclusively. This type of simulation is not possible for the modal analysis of deformable structures. The simulations performed with the rigid mesh algorithm are exactly the same ones analyzed before. Hence, once again, the NACA 0012 airfoil is submitted to exponentially-shaped pulse (EP), unit sample (US), discrete step (DS), and harmonic (H) inputs, in both pitch and plunge modes. The excitation amplitudes are also the same as the ones previously presented.

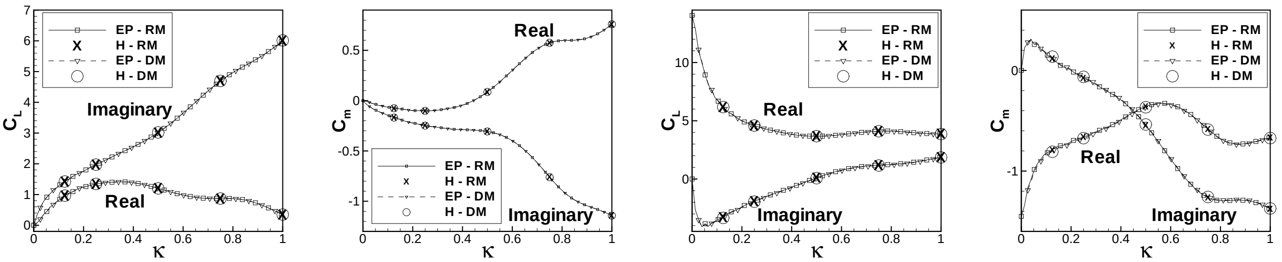
The frequency domain impulsive responses for the cases with a rigid mesh are given in Fig. 3. This figure shows that all the approaches applied to acquire the elementary aerodynamic solution generate the same results when the RM algorithm is considered. Such results confirm the reasoning that the mesh deformation is responsible for the differences presented in Fig. 1. Therefore, this is a demonstration that it is possible to numerically obtain direct aerodynamic responses to unit sample and discrete step inputs. Moreover, as presented before, these responses are exact, and not mere approximations, at least within the numerical scheme and discrete-time contexts.



(a) C_ℓ response for the plunge mode.
(b) C_m response for the plunge mode.
(c) C_ℓ response for the pitch mode.
(d) C_m response for the pitch mode.

Figure 3. Low reduced frequency impulsive responses of a NACA 0012 airfoil at $M_\infty = 0.8$ and $\alpha_0 = 0$. Simulations consider a rigid mesh.

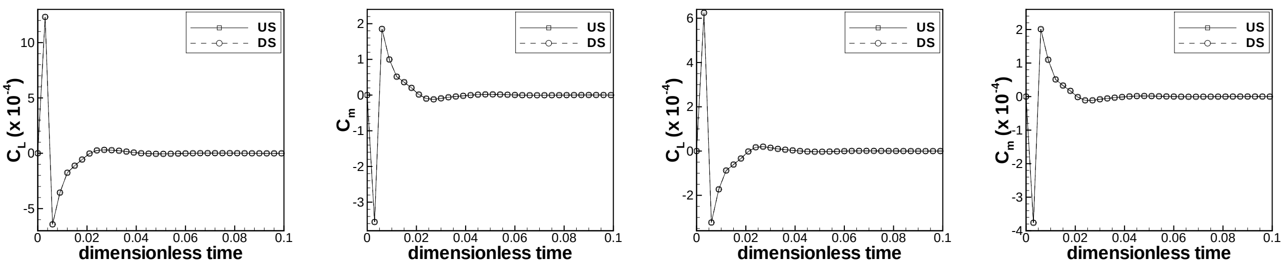
Additionally, Fig. 4 shows comparisons of the impulsive frequency domain responses obtained with deformable and rigid mesh algorithms for the EP cases. The agreement between the results shows that the use of smooth motions is not affected by the mesh deformation effects, and, hence, the smooth pulse technique is a more robust procedure than directly obtaining unit sample or discrete step responses.



(a) C_ℓ response for the plunge mode. (b) C_m response for the plunge mode. (c) C_ℓ response for the pitch mode. (d) C_m response for the pitch mode.

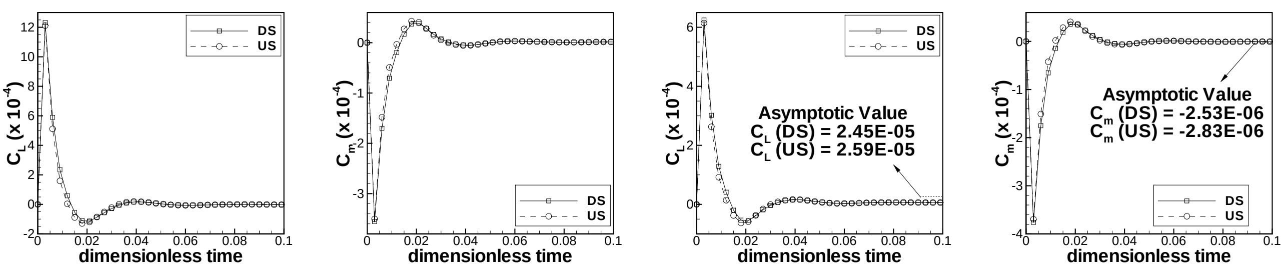
Figure 4. Low reduced frequency impulsive response of a NACA 0012 airfoil at $M_\infty = 0.8$ and $\alpha_0 = 0$ to an EP. Comparison between deformable (DM) and rigid mesh (RM) simulations.

The correspondence among the different solutions can also be verified in the time domain. For example, as presented by Marques and Azevedo (2006), the unit sample responses can be obtained with the backward difference of the discrete step ones, as indicated in Eq. (1), and shown in Fig. 5. On the other hand, the discrete step responses can be reproduced by a convolution sum between the unit sample ones and the DS sequence, as demonstrated in Fig. 6. As one can observe, the DS response evaluated by convolution presents small differences in the initial values when compared to the one obtained directly, but the asymptotic solutions are almost the same in both cases. This suggests that these time solutions differ only in the high frequency content.



(a) C_ℓ response for the plunge mode. (b) C_m response for the plunge mode. (c) C_ℓ response for the pitch mode. (d) C_m response for the pitch mode.

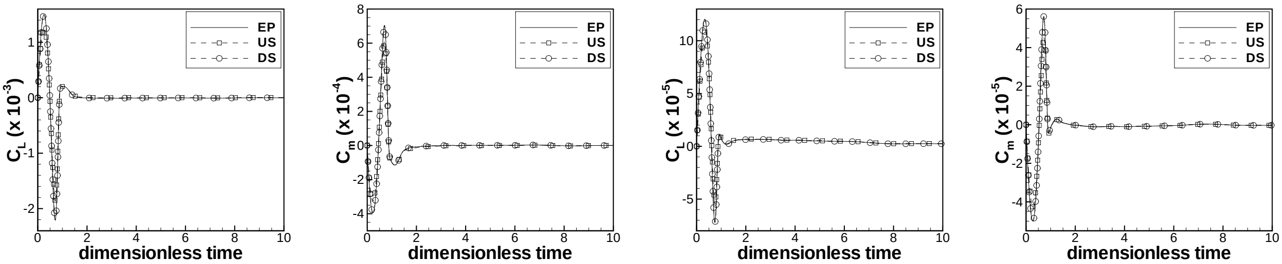
Figure 5. Initial part of the response of a NACA 0012 airfoil at $M_\infty = 0.8$ and $\alpha_0 = 0$ to a US. Comparison between the responses acquired directly (US) and through the use of discrete step (DS) responses. Simulations consider a rigid mesh.



(a) C_ℓ response for the plunge mode. (b) C_m response for the plunge mode. (c) C_ℓ response for the pitch mode. (d) C_m response for the pitch mode.

Figure 6. Initial part of the response of a NACA 0012 airfoil at $M_\infty = 0.8$ and $\alpha_0 = 0$ to a DS. Comparison between the responses acquired directly (DS) and through convolution (US). Simulations consider a rigid mesh.

Although it may seem that the same unit sample responses can be obtained from the exponentially-shaped solutions through the inverse discrete Fourier transform of the frequency domain response given in Fig. 3, this is not possible. A unit sample response contains information about a large frequency range, while a smooth input is not able of properly exciting higher frequency values. Therefore, although both responses are coincident in a certain frequency range, they are not equivalent, and this is reflected in the time domain. Nevertheless, the exponentially-shaped responses can be reproduced with convolution sums, either from the unit sample or discrete step responses, as presented in Fig. 7. The direct EP response coincides with the one obtained by convolution with the DS data. On the other hand, it presents some small differences with the other convolution response in the initial portion of the time response. The small errors present when representing other solutions from the convolution of the US response demonstrates the numerical difficulty of obtaining the unit sample response through direct application of a US input.



(a) C_ℓ response for the plunge mode. (b) C_m response for the plunge mode. (c) C_ℓ response for the pitch mode. (d) C_m response for the pitch mode.

Figure 7. Initial part of the response of a NACA 0012 airfoil at $M_\infty = 0.8$ and $\alpha_0 = 0$ to an EP. Comparison between the responses acquired directly (EP) and through convolution (US and DS). Simulations consider a rigid mesh.

5. CONCLUSIONS

The authors present a thorough analysis of the application of a CFD solver for obtaining impulsive and indicial aerodynamic responses. First of all, it is demonstrated that, under the new discrete-time interpretation of CFD solvers, it is possible to obtain impulsive and indicial aerodynamic responses with the direct use of unit sample and discrete step sequences as input. Furthermore, according to such interpretation, these responses are not approximations to the expected continuous-time impulsive and indicial solutions, but they are the formal elementary solutions for the discrete-time aerodynamic system represented by the discretized equations. However, direct calculation of such responses can lead to many numerical difficulties, some of which are also presented. It is important to emphasize that the closure of such questions is very relevant in many areas, in particular in the development of reduced-order models for aeroservoelastic control laws.

Moreover, the authors also show the correctness of the use of smooth input functions in order to determine the impulsive aerodynamic responses in certain frequency ranges. This is a very important conclusion, since the application of smooth motions yields much more robust numerical applications.

Although the test-case considered throughout this paper represents a single numerical scheme applied to a particular aerodynamic configuration, the authors experience and the arguments contained herein indicate that there are no restrictions in expanding the present conclusions to more general situations. Therefore, the authors hope to have made a contribution to the understanding of the use of CFD solvers as reliable sources of aerodynamic unsteady responses.

6. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the partial support for this research provided by Conselho Nacional de Desenvolvimento Científico e Tecnológico, CNPq, under the Integrated Project Research Grants No. 501200/2003-7 and 312064/2006-3. This work is also supported by Fundação de Amparo à Pesquisa do Estado de São Paulo, FAPESP, through a MS Scholarship for the first author according to FAPESP Process No. 04/11200-1. Further support from FAPESP is provided through Process No. 04/16064-9. Finally, the authors thank Carlos F.C. Simões for valuable discussions which were extremely important for the achievement of the present successful results.

7. REFERENCES

Ashley H., 1980, "Role of Shocks in the “Sub-Transonic” Flutter Phenomenon", Journal of Aircraft, Vol. 17, No. 3, pp. 187–197.

Azevedo, J.L.F., 1990, "Euler Solutions of Transonic Nozzle Flows", Proceedings of the 3rd National Meeting on Thermal Sciences – III ENCIT, Itapema, SC, Brazil, pp. 243–248.

Azevedo, J.L.F., 1992, "On The Development of Unstructured Grid Finite Volume Solvers for High Speed Flows", Report NT–075–ASE–N/92, Instituto de Aeronáutica e Espaço, São José dos Campos, SP, Brazil.

Azevedo, J.L.F., Fico, N.G.C.R., Jr., and Ortega, M.A., 1995, "Two-Dimensional and Axisymmetric Nozzle Flow Computation Using the Euler Equations", Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol. 17, No. 2, pp. 147–170.

Azevedo, J.L.F., Strauss, D. and Ferrari, M.A.S.,1999, "Viscous Multiblock Simulations of Axisymmetric Launch Vehicle Flows", Journal of Spacecraft and Rockets, Vol. 36, No. 4, pp. 489–498.

Bakhle, M.A., Mahjan, A.J., Keith, T.G. and Stefko, G.L., 1991, "Cascade Flutter Analysis with Transient Response Aerodynamics", Computers and Structures, Vol. 41, No. 5, pp. 1073–1085.

Ballhaus, W.F. and Goorjian, P.M., 1978, "Computation of Unsteady Transonic Flows by the Indicial Method", AIAA Journal, Vol. 16, No. 2, pp. 117–124.

Beam, R.M. and Warming, R.F., 1974, "Numerical Calculations of Two-Dimensional, Unsteady Transonic Flows with Circulation", NASA TN D–7605.

Bigarelli, E.D.V., Mello, O.A.F. and Azevedo, J.L.F., 1999, "Three Dimensional Flow Simulations for Typical Launch Vehicles at Angle of Attack", Proceedings of the 15th Brazilian Congress on Mechanical Engineering – COBEM 1999, Águas de Lindóia, SP, Brazil.

Bigarelli, E.D.V. and Azevedo, J.L.F., 2002, "On Turbulence Models for 3-D Aerospace Applications", Proceedings of the 9th Brazilian Congress of Thermal Engineering and Sciences, Paper CIT02–0341, Caxambu, MG, Brazil.

Bisplinghoff, H.L., Ashley, H.L. and Halfman, R.L., 1955, "Aeroelasticity", Adison-Wesley, Cambridge, MA.

Davies, B., 1978, "Applied Mathematical Sciences – Integral Transforms and Their Applications", Vol. 25, Springer-Verlag, New York, NY.

Davies, D. and Salmond, D.J., 1980, "Indicial Approach to Harmonic Perturbations in Transonic Flow", AIAA Journal, Vol. 18, No. 8, pp. 1012–1014.

Hirsch, C., 1992, "Numerical Computation of Internal and External Flows – Fundamentals of Numerical Discretization", Vol. I, John Wiley & Sons, Chichester, England.

Jameson, A. and Mavriplis, D., 1986, "Finite Volume Solution of the Two-Dimensional Euler Equations on a Regular Triangular Mesh", AIAA Journal, Vol. 24, No. 4, pp. 611–618.

Marques, A.N., 2004, "Simulation of Unsteady Aerospace Flows Using Unstructured Meshes", Graduation Project, Instituto Tecnológico de Aeronáutica, São José dos Campos, SP, Brazil (in Portuguese).

Marques, A.N. and Azevedo, J.L.F., 2006, "Application of CFD-based Unsteady Forces for Efficient Aeroelastic Stability Analyses", AIAA Paper No. 2006–0250, 44th AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV.

Marques, A.N., 2007, "A Unified Discrete-Time Approach to the State Space Representation of Aeroelastic Systems", Master Thesis, Instituto Tecnológico de Aeronáutica, São José dos Campos, SP, Brazil.

Mavriplis, D.J., 1990, "Accurate Multigrid Solution of the Euler Equations on Unstructured and Adaptative Meshes", AIAA Journal, Vol. 28, No. 2, pp. 545–552.

Mohr, R.W., Batina, J.T. and Yang, H.T.Y., 1989, "Mach Number effects on Transonic Aeroelastic Forces and Flutter Characteristics", Journal of Aircraft, Vol. 26, No. 11, pp. 1038–1046.

Oliveira, L.C., 1993, "A State-Space Aeroelastic Analysis Methodology Using Computational Aerodynamics Techniques", Master Thesis, Instituto Tecnológico de Aeronáutica, São José dos Campos, SP, Brazil (in Portuguese).

Oppenheim, A.V. and Schafer, R.W., 1989, "Discrete-Time Signal Processing", Prentice Hall, Englewood Cliffs, NJ.

Rausch, R.D., Batina, J.T. and Yang, H.T.Y., 1990, "Euler Flutter Analysis of Airfoil Using Unstructured Dynamic Meshes", Journal of Aircraft, Vol. 27, No. 5, pp. 436–443.

Raveh, D.E., 2001, "Reduced-Order Models for Nonlinear Unsteady Aerodynamics", AIAA Journal, Vol. 39, No. 8, pp. 1414–1429.

Silva, W.A., 1997, "Identification of Linear and Nonlinear Aerodynamic Impulse Responses Using Digital Filter Techniques", NASA TM–112872, NASA Langley Research Center, Hampton, VA.

Silva, W.A. and Raveh, D.E., 2001, "Development of Unsteady Aerodynamic State-Space Models from CFD-Based Pulse Responses", AIAA Paper No. 2001–1213, 42th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Seattle, WA.

Simões, C.F.C. and Azevedo, J.L.F., 1999, "The Influence of Numerical Parameters on Unsteady Airfoil Inviscid Flow Simulations Using Unstructured Dynamic Meshes", Proceedings of the 15th Brazilian Congress of Mechanical Engineering – COBEM 1999, Águas de Lindóia, SP, Brazil.

Tijdeman, H., 1977, "Investigation of the Transonic Flow Around Oscillating Airfoils", NLR–TR–77090U, National Aerospace Laboratory, The Netherlands.

Traci, R.M., Albano, E.D. and Farr, J.L., 1975, "Small Disturbance Transonic Flows About Oscillating Airfoils and Planar Wings", AFFDL–TR–75–100, Air Force Flight Dynamics Laboratory.

Yang, T.Y., Guruswamy, P. and Striz, A.G., 1982, "Application of Transonic Codes to Flutter Analysis of Conventional and Supercritical Airfoils", Journal of Aircraft, Vol. 19, No. 3, pp. 211–220.

8. Responsibility notice

The authors are the only responsible for the printed material included in this paper.