CRITERIA FOR STRUCTURAL SYNTHESIS AND CLASSIFICATION OF MECHANISMS

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Abstract. *A current topic of research in kinematics is the structural synthesis and classification of kinematic chains. The structural synthesis consists of the generation of a complete list of kinematic chains based on methods that enumerate all kinematic chains with a determined mobility. However, these methods normally generate a large number of isomorphisms. A significant and unsolved problem in structural synthesis is the precise elimination of all isomorphisms. In the early stage of design, it is preferable the generation of duplicate chains to the omission of a potentially useful chain. In the synthesis process, thousands of chains enumerated must be classified to find out the most promising ones that satisfy the functional requirements required by the task. For the classification of these kinematic chains we can use the concepts of connectivity, redundancy and variety. This paper reviews methods of structural synthesis aiming at identifying the most promising method for the generation of all kinematic chains without isomorphisms. Other goal of this paper is to present selection criteria of kinematic chains based on the concepts of connectivity, redundancy and variety. The synthesis of kinematic chains for robot hands is used as an illustrative example.*

Keywords: *kinematic chains, structural synthesis, isomorphism, variety, connectivity, redundancy.*

1. INTRODUCTION

The most important phase in the design of a mechanism is to find the most adequate topology for the accomplishment of a determined task. This phase is called structural synthesis or enumeration of kinematic chains with determined mobility and number of links. The topology characteristics of mechanisms are entirely determined by the pattern of interconnections among links and are unaffected by metric properties. The mobility of a kinematic chain is the number of independent parameters required to completely specify the configuration of the kinematic chain in the space, with respect to one link chosen as the reference. The mobility of a kinematic chain, with n links and j single-degree of freedom joints, may be calculated by the general mobility criterion

$$M = \lambda(n - j - 1) + j \quad (1)$$

where λ is the order of the screw system to which all the joint screws belong.

A kinematic chain can be uniquely represented by the graph whose vertices correspond to the links of the chain and whose edges correspond to the joints of the chain. In graph theory terms, the structural synthesis of kinematic chains corresponds to the enumeration of graphs satisfying the general mobility criterion and having given a number of vertices and edges. However, the problem of graph enumeration is NP-Hard. All method of graphs enumeration generate a great amount of isomorphisms which must be eliminated, whithout eliminating any chain with useful potential for the accomplishment of the task. In practice, since the number of kinematic chains generated is often too large, it is dificult to manually consider the individual merits of each chain. For this reason, the concepts of connectivity, variety and redundancy can be used as criteria to classify kinematic chains according to the constraints required for the task.

This paper first reviews the methods of structural synthesis and the concepts of connectivity, variety and redundancy. After that, based in kinematic restrictions for robot hand in Mason and Salisbury (1985) summarized in Tischler et al. (1995b), we investigate the functional requirements of a robot hand and transform these functional requirements into purely kinematics characteristic. Then we enumerate all the kinematic chains without isomorphisms and apply the criterias (connectivity, variety and redundancy) to classify the enumerated chains to find alternative mechanisms for robot hands.

2. LINK ASSORTMENTS

The first common step of the works in enumeration of kinematic chains is the determination of the possible assortments of binary, ternary, quaternary, etc. links that can exist in the desired chains. These are given by the solutions of the following equations:

$$n = n_2 + n_3 + n_4 + \cdots \quad (2)$$

$$2j = 2n_2 + 2n_3 + 2n_4 + \cdots \quad (3)$$

where n_i is the number of links with i connections each, n is the number of links and j is the number of single-degree of freedom joints.

The subsequent step is the formation of distinct structural patterns in which polygonal (non-binary) links can be connected together, the addition of available binary links to the polygonal-link patterns in all possible ways to produce closed chains and finally discarding degenerate chains and structurally equivalent or isomorphic chains to produce the set of distinct chains.

For the purpose of classification, each link assortment is called a *partition*. Algorithms for finding all the partitions are well documented (James and Riha, 1976). Table 1 shows the partitions for constructing ten-bar kinematic chains with $\lambda = 3$ (not necessarily planar motion) and $M = 3$, where number 2 represents binary links, 3 ternary links, and so on.

Table 1. Partitions of the kinematic chains with ten links, with $\lambda = 3$ and $M = 3$.

Partition 1	3	3	3	3	2	2	2	2	2	2
Partition 2	4	3	3	2	2	2	2	2	2	2
Partition 3	4	4	2	2	2	2	2	2	2	2
Partition 4	5	3	2	2	2	2	2	2	2	2
Partition 5	6	2	2	2	2	2	2	2	2	2

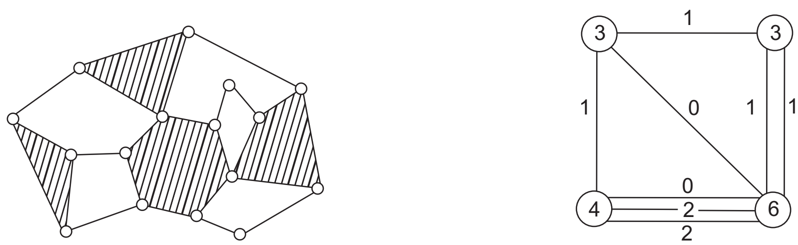
3. REVIEW OF THE METHODS OF SYNTHESIS

Applied to the field of project of mechanisms, Reuleaux (1875) defines synthesis as the process of transformation of the project specifications of a mechanism. Structural synthesis is the process of finding the arrangements of a given number of bodies and joints which result in kinematic chains of the desired mobility.

We now consider the traditional methods of synthesis. However, they generate isomorphisms whose elimination requires a great computational effort.

3.1 Method of Franke

The Franke's notation is a graphical simplification of the representation of kinematic chains (Franke, 1958). In the Franke's notation, each polygonal link is represented by one circle with a label n inside, that corresponds to number of connections of the link and binary links, are represented by lines. Figure 1(a) shows one 12-links kinematic chain and Fig. 1(b) shows the corresponding Franke's notation of 12-links kinematic chain.



(a) (b)

Figure 1. Franke's notation.

In the synthesis procedure based on Franke's notation, we first consider all the possible mappings of the polygonal (non-binary) links for each possible partition. For each partition, each circle is connected by lines in all possible ways, being the incident line number in the circle equal to label of it. Each line receives a number $k \geq 0$, $k = 0$ if no binary link exists between two polygonals (Davies and Crossley, 1966).

Care must be taken to guarantee that degenerate chains containing immobile subchains are not produced. A disadvantage of the method is that it generates a great number of isomorphisms which must be eliminated.

3.2 Method of Assur

Another approach for structural synthesis is due to Assur (1913). He introduced the concept of fundamental groups, later called Assur's groups. Assur's groups are kinematic chains in which some links contain free or unpaired elements such that when the group is connected to the frame through all its free elements it becomes a structure with zero mobility.

Assur also proposed that chains of greater complexity (i.e. with greater number of links) could be built up by the sequential addition of these Assur's groups to simpler chains (i.e. with fewer links). The basis for this idea lies in the fact that addition of an Assur's group to a link or links of an existing chain do not modify the mobility of the original chain. The method is based on visual inspection and does not require determination of partitions. Degenerate chains do not arise if the initial simpler chains are free from immobile subchains and if the free elements of an Assur's group are not all added to a single link. Figure 2 shows the addition of an Assur's group to a 4-link chain.

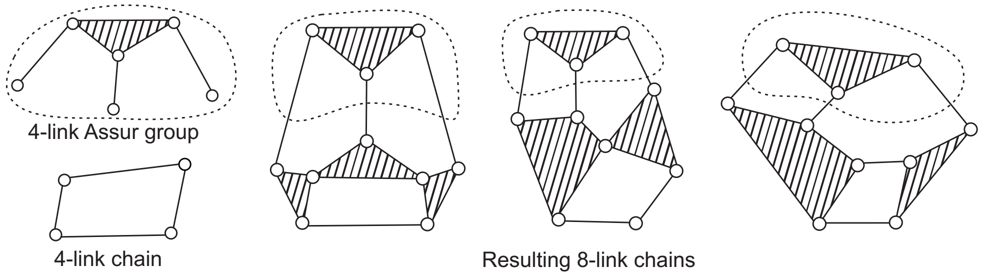


Figure 2. Aggregation of the Assur's group to 4-link chain.

However the method produces a large number of isomorphs (Mruthyunjaya, 2003) . Also, it is necessary to have available atlases of chains with mobility M and number of links less that n, as well as complete atlases of all Assur groups with $(n - M - 1)$ links.

3.3 Method of Heap

Heap's method produces all the graphs with j vertices (or bodies) and i edges (or joints) by extending all the distinct graphs with $j - 1$ vertices and $i - i_j$ edges, where i_j is the degree of the jth vertex (Heap, 1972). For the purpose of generating kinematic chains, all the ways of joining the jth vertex using i_j edges are found and the process is repeated until the number of vertices in the graph is n and the number of edges is j.

The advantage of this method is that no initial graphs are needed since more complicated graphs are gradually built up as each vertex is added. The disadvantages of this method is that the intermediate graphs do not necessarily represent kinematic chains, Heap's method generates some graphs that correspond to improper kinematic chains and Heap's method also generates isomorphs (Tischler et al., 1995a).

3.4 Method of Farrell

We implement a modified version of the Farrell's method for enumeration of kinematic chains avoiding to enumerate the fractionated kinematic chains; therefore, it will be described here with more detail and our method will be described in the section 3.7. The Farrell's method imposes a tree structure in the kinematic chains generation process and is summarized in the following steps (Farrell, 1977) (Tischler et al., 1995a):

Step 1: Each body in the partition is assigned by a numerical label according to its degree. One of the bodies with the highest degree is given the number "1", while the body with the lowest degree is given the highest number. Two bodies cannot be assigned by the same number. For example, the partition 1 in Tab. 1 has four ternary bodies, which we now label 1, 2, 3, and 4, and six binary bodies we label 5, 6, 7, 8, 9, and 10. At this stage all bodies are unconnected. Sees the Fig.3.

Step 2: The body with the lowest number (i.e. 1) is selected and the remaining bodies, {2, 3, ... , 10} are grouped so that connecting body 1 to any member of the group would result in an identical, partially connected, form. Here, two distinct groups materialise, namely a group of ternary bodies, {2, 3, 4}, and a group of binary bodies, {5, 6, 7, 8, 9, 10}. Connecting body 1 to any member in the group {2, 3, 4} would result in two connected ternary bodies, and connecting body 1 to any member of {5, 6, 7, 8, 9, 10} would result a ternary body connected to a binary body.

Step 3: The number of connections c needed to make the body with the lowest number fully connected is determined. In this case $c = 3$, because body 1 is ternary and no connections have yet been made. All the different ways of selecting $c = 3$ bodies to connect to body 1 from the groups of Step 2 are found. These are; three ternary bodies {2, 3, 4}, two ternary bodies and one binary body {2, 3, 5}, one ternary body and two binary bodies {2, 5, 6}, and three binary bodies {5, 6, 7}. The partial forms which result from each of these selections are shown in Fig.3. In each case the lowest numbered members of each group are selected first. Each of the four partial forms represents a branch in the tree.

Step 4: Each of the branches in Step 3 are selected in turn and any bodies which are fully connected are ignored; Steps 2, 3 and 4 are repeated for the next lowest numbered body which is not fully connected. In this case the lowest numbered body will be body 2. Steps 2, 3 and 4 are repeated until all other bodies are fully connected or it is impossible to connect the remaining bodies. When either of these two situations arises the algorithm back-tracks and continues with the next unexplored branch.

Step 5: When no unexplored branch remains the next partition is selected, and all of the above steps are repeated until no further partitions remain.

Step 6: Elimination of improper kinematic chains and isomorphisms and finality enumeration of the found kinematic chains.

One of the disadvantages of the method is that it generates many isomorphisms which must be eliminated and the elimination requires a great computational effort.

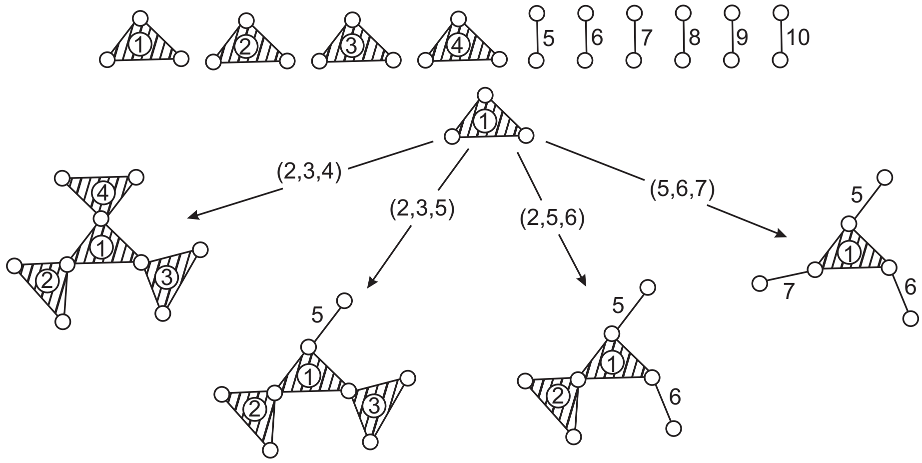


Figure 3. Example Farrell's method with possible connections for boby 1.

3.5 Method of Melbourne

Tischler et al. (1995a) proposed a method of enumeration of kinematic chains, called Melbourne's method. The Melbourne method's is a modification of Farrell's method with the objective to reduce the number of isomorphism in the output list. The modification consists of a set of four rules. To apply these rules they introduced four concepts; symmetrical body, equivalent body, proper connections and canonical connections, for more details consult (Tischler et al., 1995a). However, the method also generates isomorphic chains which must be eliminated.

The Melbourne's method was applied to synthesise kinematic chains suitable for application as robot hands (Tischler et al., 1995b).

3.6 Method of Sunkari and Schmidt

Recently, Sunkari and Schmidt (2006) presented a method of synthesis of kinematic chains based on the group theory techniques. He uses the McKay's method for generation of a isomorphism class representative in combination with an efficient degeneracy testing algorithms. According to the authors of the method, the algorithm is computationally efficient and it generates 318,162 planar kinematic chains whit 14 link and $M = 1$ in 37.28s on Pentium III 1.7GHz with 512MB RAM. The authors claims that the computational speed at which the kinematic chains are generated depend on McKay-type algorithms that greatly minimize the explicit isomorphism detection by using group theoretic techniques.

3.7 Proposed Method

The proposed method in this paper is a modification of the Farrell's method in order to avoid generation of fractionated kinematic chains. We notice that, in the majority of the applications, the fractionated chains are generated without necessity. We present, as illustrative example, the project of a robotic hand where fractionated chains do not attend the project specifications. We also notice that some methods do not enumerate fractionated chains and the authors of these methods do not justify why they do not enumerate them. We are working in a fractionated chains generation method for aggregation similar to the Assur's method , thus we enumerate fractionated chains only when is necessary (i.e. fractionated chains satisfy the project specifications). One of the disadvantages of the proposed method is that it generates many isomorphisms which must be later eliminated.

The method was implemented in C++ using graphs as data structure. The method imposes a tree structure in the generation process similar to the Farrell's method, see Fig. 4. The input data of the algorithm is the number of vertices and the degree of each vertex. The vertices are orderly decreasing of degree and labeled with gradual number. The graph of the root of the tree is formed by a set of vertices labelled. Combinations of the degrees of the vertices are made and edges are connected in accordance with the label of each vertex. The process of adding edges is repeated to complete the degree of all the vertices. In the generation process, if a graph has a connected subgraph with the degrees of the vertices complete except one of them such graph do not generate more children because in this case the children will originate fractionated kinematic chains, see Fig. 5. Some fractionated chains are generated in leaves of the tree, in this case we use

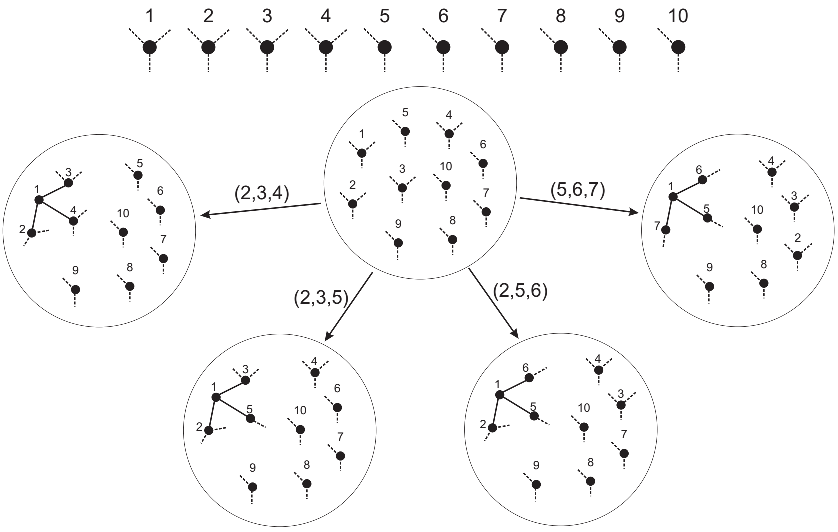


Figure 4. Structure of proposed method.

the test of biconnectivity (time complexity is polynomial) of the Boost Graph Library (BGL, 2002) to exclude them. Thus we avoid the generation of graphs that originate fractionated kinematic chains. In the graphs of leaves of the tree we run the test of isomorphisms of the Boost Graph Library (BGL, 2002) whose worst-case time complexity is $O(|V|!)$, where V is the number of vertices.

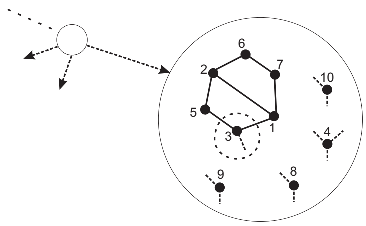


Figure 5. Eliminated graph, avoiding generate fractionated kinematic chains.

Table 2 shows some of the results known in the enumeration of planar kinematic chains. A reason for the discrepancies in the results of Tab. 2 is the generation of kinematic chains without fractionation for Sunkari and Schmidt (2006) and Tuttle (1996) and with fractionation for Hwang and Hwang (1992) and Tischler et al. (1995a). Another reason can be related the imperfections in the tests of isomorphisms and detection of degenerated kinematic chais. In the tested cases, the results of our method are in accordance with of Sunkari and Schmidt (2006).

After this review we evidence that the problem of generation of kinematic chains is still an unsolved problem.

Table 2. Summarises of some known cases of planar kinematic chains enumeration.

Loops	Mobility			
	1	2	3	4
2	2	3 [1][2], 4 [3]	5 [1][2], 7 [3]	6 [2], 10 [3]
3	16	35 [1][2], 40 [3]	74 [1][2], 98 [3][4]	126 [2], 189 [3]
4	230	753 [1][2], 839 [3]	1962 [1][2], 2442 [3]	4356 [2], 5951 [3]
5	6856 [1][2], 6862 [3]	27496 [1][2],29704 [3]	83547 [1][2]	216291 [2]
6	318126 [1], 318162 [2]	1432608 [1],1432730 [2]	4805382 [1], 4805764 [2]	13743920 [2]

Legend of references:
[1] - (Tuttle, 1996); [2] - (Sunkari and Schmidt, 2006);
[3] - (Hwang and Hwang, 1992); [4] - (Tischler et al., 1995a);

4. DETECTION OF ISOMORPHISMS

A major problem in the study of kinematic structures is that of detecting a possible isomorphism (structural equivalence) between two given chains. Two kinematic chains or mechanisms are said to be isomorphic if they share the same topological structure. In terms of graphs, there exists a one-to-one correspondence between their vertices and edges that preserve the incidence. We now consider the traditional methods of detecting isomorphism and their weaknesses, admitting that, in the general case, no efficient solution of the graph isomorphism problem has been found yet.

Uicker and Raicu (1975) suggested that the characteristic polynomial could be used to test for isomorphism. However, if two kinematic chains are isomorphic, it is necessary, but not sufficient, that their characteristic polynomials are identical as there are counter-examples where this method fails (Tischler et al., 1995a)(Mruthyunjaya, 2003).

Ambekar and Agrawal (1987) suggested a method of identification called the optimum code. The method involves a technique for labeling the links of a kinematic chain such that a binary string obtained by concatenating the upper triangular elements of the adjacency matrix row by row, excluding the diagonal elements, is maximized. This is called the MAX code. We can also search for a labeling of the chain that minimizes the binary string of the upper triangular elements, called the MIN code. There is a need to develop a more eficient heuristic algorithm for determination of the optimum code (Tsai, 2001)(Mruthyunjaya, 2003).

Rao and Raju (1991) present a method for detecting isomorphs based on Hamming numbers of the adjacency matrix. Although no counter-examples are known, when the algorithm was applied to the detection of isomorphs among the number of inversions of the planar, $M = 1$, ten links, some non-isomorphic inversions were omitted (Tischler et al., 1995a).

The algorithm of generation of kinematic chains implemented in this paper uses the test of isomorphisms detection of the Boost Graph Library (BGL, 2002) whose worst-case time complexity is $O(|V|!)$, where V is the number of vertices.

Köbler et al. (1993) have examined the structural complexity of the graph isomorphism problem and state that there is strong evidence to suggest that no efficient algorithms exist for this problem (i.e. the problem of isomorphisms is NP-Hard).

5. DEGENERATED KINEMATIC CHAINS

5.1 Fractionation

Sets of kinematic chains with mobility $M > 1$ contain some chains that are fractionated; these members can present body-fractionation and joint-fractionation.

A body-fractionated chain contains a body which divides the chain into two closed, independent, kinematic chains. A closed kinematic chain is one in which every body is connected to at least two other bodies. A body-fractionated chain must have at least two independent loops and a mobility $M \geq 2$ (Tischler et al., 1995a). Figure 6(a) shows one body-fractionated planar kinematic chain with three loops and $M = 3$.

A joint-fractionated chain is one in which the remotion of a joint divides the chain in two closed kinematic subchains. A joint-fractionated kinematic chain must have a mobility $M \geq 3$ and at least two independent loops. When one fractionating joint is removed the combined mobility of the two resulting chains is $M - 1$ (Tischler et al., 1995a) . Figure 6(b) shows one joint-fractionated planar kinematic chains with with three loops and $M = 3$.

Our method not enumerate fractionated chains eliminating computational efforts for generation and identification of the fractionated chains.

(a) (b)

Figure 6. Fractionated planar kinematic chain.

5.2 Improper Kinematic Chains

An improper kinematic chain is a kinematic chain where at least one biconnected subchain has mobility $M' \leq 0$. The subchain with mobility $M = 0$ are called Baranov chains (Manolescu, 1979). Improper chains are of no interest in pure kinematic analysis. Some methods of synthesis generate improper chains which must be identified and eliminated.

6. CRITERIA FOR CLASSIFICATION OF KINEMATIC CHAINS

In general, the number of generated kinematic chains in the synthesis process is great and it is difficult to evaluate each chain individually. Therefore, it is necessary to develop a set of criteria to evaluate the merit of each chain without eliminating a chain with possibilities to develop the desired task. For selecting the enumerated chains the criteria of variety, conectivity and redundancy are presented.

6.1 Variety

Variety is a useful property for determining the relative connectivities within a chain and also for selecting actuated pairs. Variety may also be used to classify kinematic chains according to the constraints required (Tischler et al., 1995b).

A kinematic chain is variety V if it does not contain any loop, or subset of loops, with a mobility of less than $M - V$, but does contain at least one loop, or subset of loops, which has a mobility of $M - V$ (Tischler et al., 1995b).

Classification of kinematic chains by variety V allows generalizations to be made about the relative connectivity of bodies within the kinematic chain therefore if a variety V kinematic chain has a mobility M greater than the order of the screw system that generally prevails λ , i.e. if $M > \lambda$, then any two links, separated by at least $\lambda - V$ joints, have relative connectivity $C \geq \lambda - V$. The variety of the kinematic chains also affects the choice of the joint to be actuated. If the Variety of a kinematic chain with j joints is $V = 0$, the actuated pairs may be selected at random. The Fig. 7 shows a ten links planar kinematic chain with variety $V = 0$.

6.2 Connectivity

In a kinematic chain represented by a graph G, the connectivity between two links i and j is defined in Carboni and Martins (2006) as

$$C_{ij} = \min : \{D_{\min}[i, j], M, M'_{\min}, \lambda\} \quad (4)$$

where $D_{\min}[i, j]$ is the minimum distance between vertices i and j of G, M is the mobility of the kinematic chain considered, $M'_{\min}$ is the minimum mobility closed-loop biconnected subchain of G containing vertices i and j, and λ is the order of the screw system (Carboni and Martins, 2006).

The connectivity is an important criterion for selecting kinematic chains. For example, the Fig.8 represents a closed planar kinematic chain with mobility $M = 3$, but the connectivity between any two links i and j cannot be greater than 2. From this simple example, it is evident that connectivity, not mobility, determines the ability of an output link to perform a task relative to a frame.

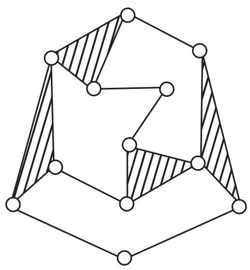


Figure 7. Planar kinematic chain with V=0.

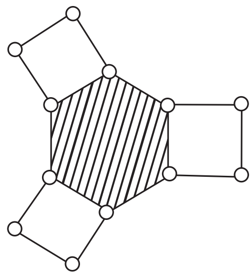


Figure 8. Planar k. c. eliminated for the connectivity.

6.3 Redundancy

Redundancy is one of the most important parameters in a kinematic chain together with connectivity and variety. To introduce the redundancy concept we need the degrees-of-control concept. In a kinematic chain represented by a graph G, the degrees-of-control between two links i and j is defined in Carboni and Martins (2006) as

$$K_{ij} = \min : \{D_{\min}[i, j], M, M'_{\min}\}. \quad (5)$$

The degrees-of-control K_{ij} between two links i and j of a kinematic chain is the minimum number of independent actuating pairs needed to determine the relative position between the two links i and j, possibly leaving some other link-relative position undetermined as when K_{ij} is less than the mobility M (Belfiore and Benedetto, 2000)(Carboni and Martins, 2006).

In a kinematic chain represented by a graph G, the redundancy between two links i and j is the diference between K_{ij} and C_{ij}

$$R_{ij} = K_{ij} - C_{ij}. \quad (6)$$

The redundancy can be used to prevent collisions in manipulators which operate in confined environment (Simas, 2005).

7. KINEMATIC CHAINS FOR ROBOT HANDS

In this section, we examine the specific application of kinematic chains as robot hands. Our starting point is the work described by Tischler et al., (1995a, 1995b). Our objective is to enumerate and to classify alternative mechanism for robot hands. Our method is applied to generate the kinematic chains and the connectivity is applied to classify kinematic chains that are suitable for application as robot hands. The results are compared with those obtained by for Tischler et al. (1995b). The contact types suitable is point contact with friction. A point contact with friction is kinematically equivalent to a spherical pair (Tischler et al., 1995b), thus spherical pairs are required to represent the contacts. Therefore, if a kinematic chain with single-freedom joints is suitable for application as a robot hand in accordance with above specifications it must contain the subchain shown in Fig.9.

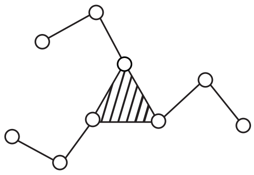


Figure 9. A three dimensional subchain representing a grasped body and three point contacts with friction which must be included in all suitable kinematic chains for potential robot hands consistent with these specifications.

To maintain static equilibrium, three point contact with friction are required. Since three point contact are required between the grasped object and the finger-tips, we must synthesise kinematic chains which contain at least one ternary body. The desired connectivity of the grasped object relative to the grounded body is $M = C$. To synthesise one entire robot hand, we need three spherical pairs to represent the contacts in the linkage. The only screw system which has full-cycle mobility and can admit three distinct spherical pairs is the general six-system, $\lambda = 6$ (Hunt et al., 1991). Since we require a ternary body in the linkage and at least two independent loops, i.e. $\nu = 2$.

Fractionated chains with $M = 6$ and $\nu = 2$ are not suitable for the specifications of robot hand. Body-fractionated chains, with only two independent loops, do not contain a ternary body which has to be present to represent the grasped body, hence they can be disregarded. Joint-fractionated chains, with $\lambda = 6$ and $\nu = 2$, are also unsuitable, because it is not possible to choose a grounded body such that the grasped body has a connectivity of $C = M$ relative to the ground.

Table 3 shows the results for the synthesis of kinematic chains with $\lambda = 6$ and $\nu = 2$. The column 1 shows the mobility, column 2 the total number of kinematic chains without fractionation for a given mobility, column 3 how many of the total number of kinematic chains in column 2 contain the subchain shown in Fig. 9, column 4 the number of useful inversions (i.e. number of choices for the grounded body, strictly only linkages can be inverted) for each of the kinematic chains represented in column 3. Of the inversions in column 4, the only suitable mechanisms for application as robot hands are those which have a relative connectivity between the grasped object and the grounded body equal to the mobility M shown in column 1.

The relative connectivity between the grasped object and the ground was calculated through the automatic method of Carboni and Martins (2006). Tischler et al. (1995b) calculate the connectivity for the method based on the variety of the kinematic chain. Of the inversions of column 4 were eliminated 181 chains in the total that has connectivity of grasped object relative the grounded $M < C$. The suitable inversions are shown in column 5. The alternative mechanisms for robot hands satisfying our specifications are derivatives of the inversions show in column 5 of Tab. 3.

Table 3. Synthesis of kinematic chains with $\lambda = 6$, $\nu = 2$ and k.c's suitable as robot hands.

1	2	3	4	5
Mobility M	Total number of k.c's without fractionating	Unique k.c's containing subchain	Useful inversions of k.c's with subchain	k.c's suitable as robot hands
2	7	4	21	19
3	10	6	34	26
4	12	7	50	22
5	15	9	71	16
6	18	11	97	9

Figure 10 shows the kinematic chain and potential mechanism for robot hand that operates in the general screw system with $M = 6$ and $\nu = 2$. The mechanism of Fig. 10 is known as Stanford/JPL or Salisbury's hand (Mason and Salisbury, 1985)(Ruoff et al., 1984). The other eight structures can be found in Tischler et al. (1995b).

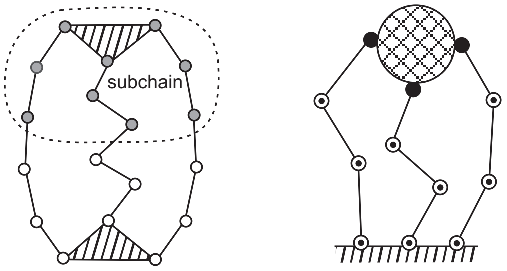


Figure 10. Potential mechanism known as Stanford/JPL or Salisbury's hand.

Figure 11(a) shows a symmetrical kinematic chain and the potential mechanism for robot hand and Fig. 11(b) show a non-symmetrical kinematic chain and the potential mechanism, both operates in the general screw system with $M = 3$ and $\nu = 2$. The others 24 mechanisms can be easily sketched.

The results of the Tab. 3 are in accordance with results obtained in Tischler et al. (1995b). The difference in the Tab. 3 is that Tischler et al. (1995b) enumerate fractionated kinematic chains which must be eliminated because they are not suitable for the specifications of robot hand and our method do not enumerate fractionated kinematic chain eliminating computational efforts for the generation and the identification these chains.

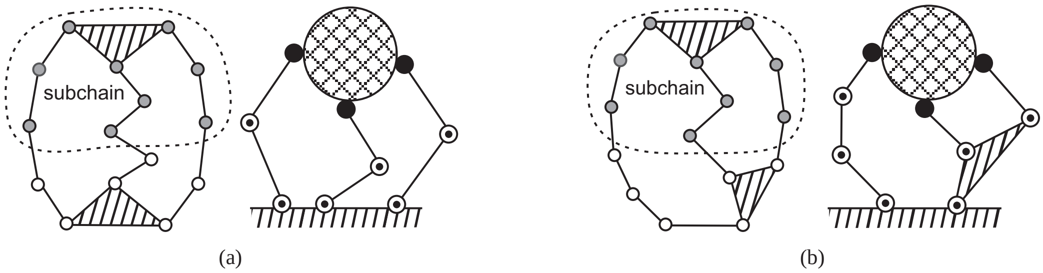


Figure 11. Potential mechanism for robot hand.

8. CONCLUSION

In this paper, we review some methods of synthesis of kinematic chains and we present our method for generation of kinematic chains. A kinematic chain can be uniquely represented by the graph whose vertices correspond to the links of the chain and whose edges correspond to the joints of the chain. Representing a kinematic chain as a graph allows consideration of its kinematic structure with minimal reference to its geometrical proportions. This is a useful simplification in the preliminary stages of design. The problem of the synthesis of kinematic chains is reduced to the problem of enumeration of graphs that satisfy the mobility criterion. One of the unsolved problem in structural synthesis is the precise elimination of all the kinematic chains structurally equivalent (i.e. isomorphism). It is necessary to study the problem of isomorphisms and search new methods to identify the isomorphisms.

We present the criteria of variety, connectivity and redundancy to classify kinematic chains. Based in kinematic constraint for robot hands in Mason and Salisbury (1985) summarized in Tischler et al. (1995b), we enumerate all the kinematic chains that satisfy the mobility criterion and number of loops for robot hands. In order to identify those kinematic chains most suitable for application as robot hands, we apply the criteria of connectivity to classify the kinematic chains generated. One table of alternative mechanisms for robot hands is presented and is in accordance with the previous work of Tischler et al. (1995b).

This application validates our method of automatic generation of kinematic chains and the criteria of classification applied.

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