FORCES ACTING BETWEEN ROAD AND WHEEL OF THE WHEELCHAIR

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Abstract. *This paper deals with the dynamics of the contact between the tire and the ground of the wheelchair. A mathematical tire model that calculates contact forces and torques between the road and tire was developed using the Working Model 2D® multibody software. Different analyses have been carried out varying the parameters and situations. The mathematical model needs a set of parameters to describe the tire propertie, the determination of these parameters is normally by experiments. So in this paper the lateral tire force and the longitudinal force is obtained through specific tests on the single tire. Know the lateral force produced, it is possible to determine the lateral response of the single tire, thus the relationship between lateral load, slip angle and vertical load. Through the results gotten for the analysis of each model, it will be possible to get the best design of the wheelchair and with a mathematical model that properly predicts the lateral force will be useful for future practical issues, such as controlling the motion of a vehicle for following a desired trajectory.*

Keywords*: wheelchair, tire, lateral force, dynamics, vehicle model.*

1. INTRODUCTION

This work studies the vehicular dynamics and the modeling of the contact between ground and wheel in manual wheelchair. Using the Working Model 2D for the integration of the equations of motion and visualization of the displacements, the model of the contact between ground and wheel had been implemented from the reference literature. These model had been implemented simulated and compared each other by figures and graphs. The model developed was sufficiently trustworthy and enough simple. However, trustworthy parameters do not exist for the coefficient of these models when used for the modeling of one wheelchair. So was developed an experiment to raise these parameters.

With the model of the contact between wheel and road and with the knowledge of the dynamic behavior of the wheelchair it will make possible the development of new products to help people with special needs, improving the quality of life and its integration in the society.

2. MODELING OF THE CONTACT BETWEEN ROAD AND WHEEL

For the equations of motion and the model some hypotheses must be considered: 1) the system is treated as a rigid and symmetrical body to the long of the longitudinal axle, possessing three degrees of freedom, 2) the resultant lateral force exerted by the ground is perpendicular to the plan of the wheel and acts directly on the projection of the wheel's center, 3) the slip angle (α_i) is small and assumes a linear relation between the slip angle and the friction force.

These hypotheses became possible that the differential equation of motion is linear and no much information will be lost with this simplification. Thus the motion of the system can be considered as rigid body in plan x-y.

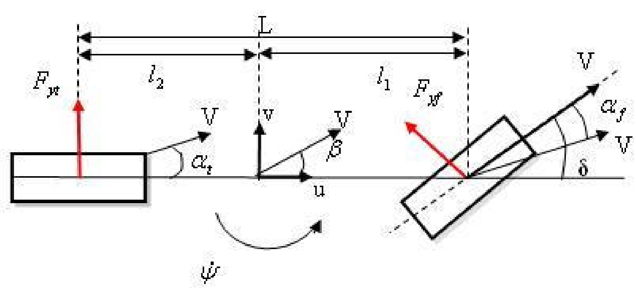


Figure 1. Force acting between road and wheel

Where:
F_{yt} and F_{yf} = lateral force on the rear axle and the front axle (N)
β = sideslip angle of the vehicle (rad)

δ = steering angle (rad)
α_t and α_f = slip angle of the tire on the rear and front axle (rad)
$\dot{\psi}$ = yaw velocity (m/s^2)
V= vehicle speed (m/s^2)
v_x and v_y = components of the velocity along *x* and *y* (m/s^2)
v_t and v_f = velocity on the rear and front (m/s^2)
L= wheelbase (m)
l_1 = distance between center of mass and front axle (m)
l_2 = Distance between center of mass and rear axle (m)

2.1. Lateral Force

The lateral tire force is related with the slip angle, the load force and the pressure in the tire. The velocity does not influence significantly in the lateral tire force in steady state. For values of the slip angle below of 10°, the lateral force is approximately proportional to the slip angle, presenting a linear relation. Thus the lateral force (F_y) can be express as (Genta, 1997):

$$F_y = -C_\alpha \alpha \tag{1}$$

Where C_α is the cornering stiffness coefficient.

A positive slip angle produces a negative force in the tire, implying that the cornering stiffness coefficient will be negative. The cornering stiffness coefficient is dependent of some variable like the size of the tire, the type of the tire, the width of the wheels. (Genta,1997)

As the load force influences in the lateral tire force, so the lateral force is (Gillespe,1992):

$$F_y = (aF_z - bF_z^{\ 2})\alpha \tag{2}$$

Where *a* and *b* are constant dependents of the property of the wheel.

2.1. Slip Angle

The slip angle is related with lateral tire force. The slip angle (α) is calculated in accordance with velocity equation in the longitudinal and transversal direction of the vehicle.

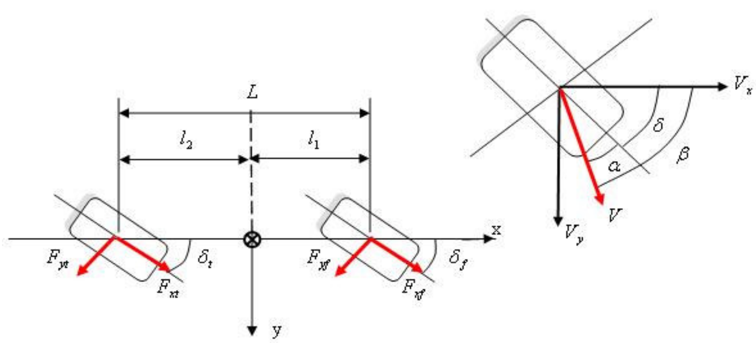


Figure 2. Decomposition of the speeds and forces

The sideslip angle of the vehicle can be expressed in function of the velocity as:

$$\beta_i = \arctan\left(\frac{v_{yi}}{v_{xi}}\right) = \arctan\left(\frac{v_y + \dot{\psi} x_i}{v_x - \dot{\psi} y_i}\right) \tag{3}$$

So the slip angle can be expressed as:

$$\alpha_i = \beta_i - \delta_i = \arctan\left(\frac{v + \dot{\psi} x_i}{u - \dot{\psi} y_i}\right) - \delta_i \tag{4}$$

2.3. Mathematical model of the magic formula

The magic formula (Pacejka,1991) is vastly used for empirical models and simulation. The magic formula can be used in tire to characterize the relation between the lateral force and the slip angle.

The magic formula express the lateral force (F_y), the moment of alignment (M_z) and the longitudinal force (F_x) in function of the slip angle and longitudinal slip. The general formula is:

$$y(x) = D\sin\left[C\arctan\left\{Bx - E\left(Bx - \arctan Bx\right)\right\}\right] \tag{5}$$

Where:

$$Y(x) = y(x) + S_v \tag{6}$$

$$x = X + S_h \tag{7}$$

Thus to use the magical formula, it is necessary to get some parameters for the modeling and simulation, as shown:

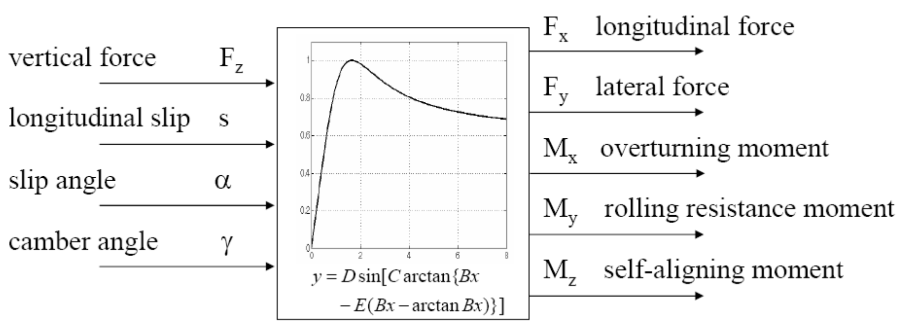


Figure 3. Representation of the parameters of exit and entrance of magic formula

The lateral force is:

$$F_y = D\sin\left(C\arctan\left\{B(1-E)(\alpha + S_h) + E\arctan\left[B(\alpha + S_h)\right]\right\}\right) + S_V \tag{8}$$

The shape factor (C) practically appears to be independent of the normal load (F_z):

$$C = a_0 \tag{9}$$

The peak factor (D) is express in function of the normal load as:

$$D = \mu_{yp} F_z \tag{10}$$

Where:

$$\mu_{yp} = a_1 F_z + a_2 \tag{11}$$

The curvature factor (E) in function of the normal load is given by:

$$E = a_6 F_z + a_7 \tag{12}$$

And the cornering stiffness:

$$BCD = a_3 \sin\left[2\arctan\left(\frac{F_z}{a_4}\right)\right](1 - a_5|\gamma|) \tag{13}$$

The camber angle (γ) has an influence in the relation between the lateral force and the slip angle in the form of the horizontal and vertical displacement. The additional displacement can be express as:

$$S_h = a_8\gamma + a_9 F_z + a_{10} \tag{14}$$

$$S_v = a_{11}\gamma F_z + a_{12} F_z + a_{13} \tag{15}$$

It is possible to obtain the lateral force function through determining the a_i coefficients using the physical significance of B, C, D, E parameters. (Beato, 2000)

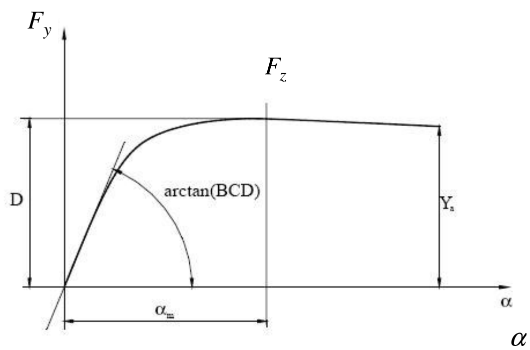


Figure 4. The lateral force (Beato, 2000)

Using the figure 4 can notice that D represents the peak value while BCD is the slope of the curve. Considering some relationships:

$$2 - C = \frac{2}{\pi}\arcsin\frac{Y_a}{D} \tag{16}$$

$$E = \frac{B\alpha_m - \tan(\pi/2C)}{B\alpha_m - \arctan(B\alpha_m)} \tag{17}$$

Thus the parameters C and E can be determinate.

2.4. Simulation of the wheelchair

The wheelchair was represented in the Working Model® as:

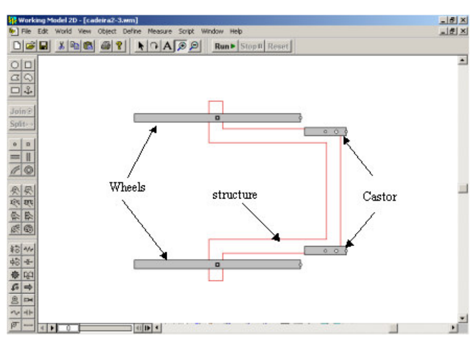


Figure 5. The wheelchair

2.4.1 Steering angle (δ)

To simulate the lateral tire force, it was necessary, in the first place, to calculate the steering angle (δ). For this it was made using two points of references; the first point was put in the center of the wheel and another in the extremity of the wheel.

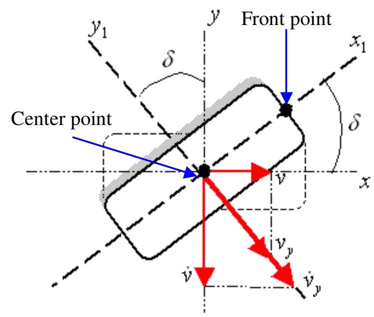


Figure 6. Decomposition of the velocity and acceleration

Then:

$$\delta = \arctan\left(\frac{y_2 - y_1}{x_2 - x_1}\right) \tag{18}$$

Where
y_1= position in y of the center point
y_2= position in y of the front point
x_1= position in x of the center point
x_2= position in x of the front point

Thus making the decomposition of the velocity showed in Fig.6:

$$v_{x1} = v_x \cos(\delta) + v_y \sin(\delta) \tag{19}$$

$$v_{y1} = v_y \cos(\delta) + v_x \sin(\delta) \tag{20}$$

Making the same decomposition for the acceleration, results as:

$$\dot{v}_{x1} = \dot{v}_x \cos(\delta) - \dot{v}_y \sin(\delta) \tag{21}$$

$$\dot{v}_{y1} = \dot{v}_y \cos(\delta) - \dot{v}_x \sin(\delta) \tag{22}$$

The coefficients a_i used in the simulation are: (Genta, 1997)

Table 1. Coefficients a_i

Coeficientes			
a0	1,3	a7	1
a1	-53,31	a8	0
a2	1190	a9	0
a3	588,6	a10	0
a4	2,5315	a11	0
a5	0	a12	0
a6	-0,5178	a13	0

Thus the simulation is:

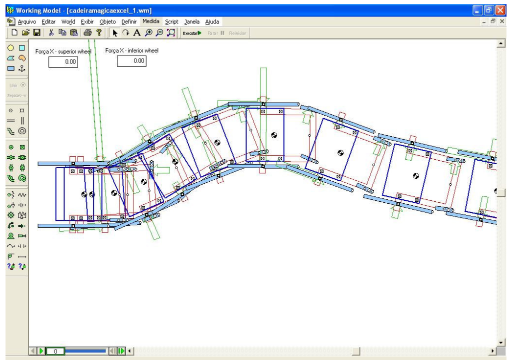


Figure 7. Simulation of the magic formula

Figure 7 shows a simulation of the top view of the wheelchair where was applied different force of propulsion in the rear wheel to make this trajectory.

In this figure perceives that the lateral force (showed by the green arrow) tries to correct the movement and notices that the lateral force is bigger when the wheelchair realizes one curve or changes the trajectory.

3. EXPERIMENTAL STUDY OF THE LATERAL TIRE FORCE OF WHEELS

The lateral tire force is one of the most important parameters of the tire and it is crucial for the stability and the handling of the vehicle. For the mathematical model of the contact between wheel and road was necessary to construct an experiment.

This experiment has an articulated table, a system that connects the wheel and allows the variation of the slip angle (α) and the slide bars for the movement of the wheel on the table.

Figure 8. The Configuration of the experiment

3.1. Measure

The software used for the data acquisition was the LabVIEW®. The measurements were made varying the angle of the wheel (0°, 5° 10° and 15°) and for each angle three different loads were applied ($F_1 < F_2 < F_3$).

Where:

- $F_1 \approx 30$ N
- $F_2 \approx 110$ N
- $F_3 \approx 210$ N

These measurements were repeated also for two different velocities:

- $V_1 \approx$ 0,002 m/s
- $V_2 \approx$ 0,004 m/s

For the measures were used three load cells fixed in the table. The localization of the load cells is shown in the figure below:

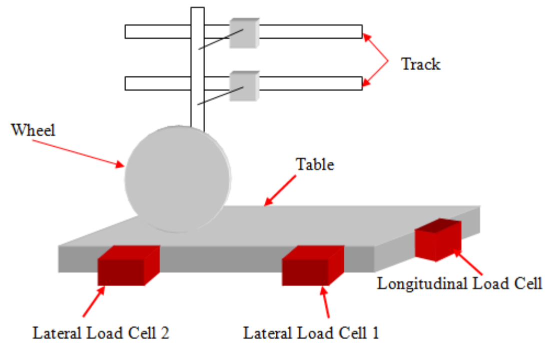
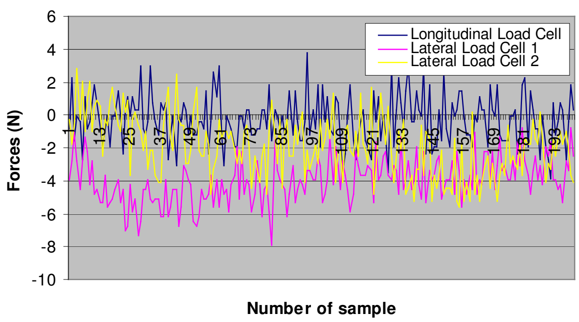
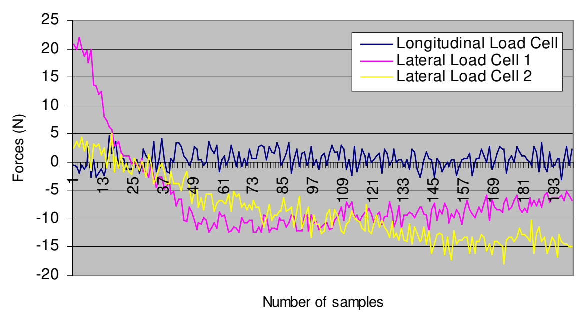


Figure 9. The position of the load cells

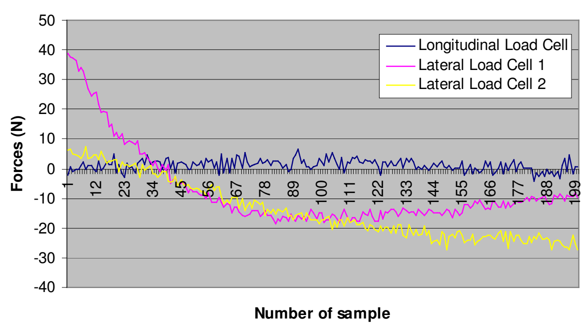
Making the measurements for the angle of the wheel equal to 0° and for three different loads (F_1, F_2 and F_3) had been gotten the following values:



(a)



(b)



(c)

Figure 10. Graph of the Values Measured for an angle equal to 0° (a) Load Force F_1, (b) Load Force F_2 and (c) Load Force F_3

Repeating this exactly procedure for all the angles (5°, 10° and 15°) and for the two values of velocities (V_1 and V_2) it was possible to make the graph of the lateral force versus the slip angle.

The lateral force was gotten through the averages of the lateral forces measured by the lateral load cells 1 and the lateral load cell 2, thus we had a measured value of the lateral force for each angle of the wheel and for different load forces.

Thus the graph of the lateral force versus the slip angler for different load force and velocity is:

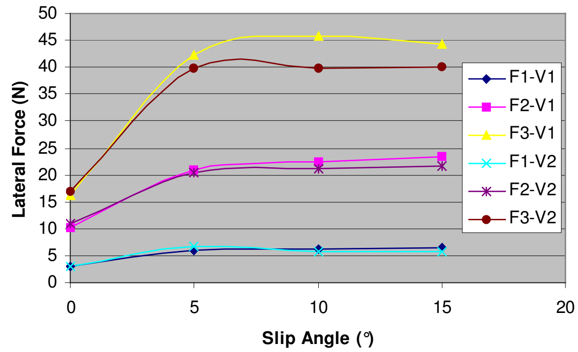


Figure 11. Lateral Force versus slip angle

From the graph of Fig. 11 notices that when the velocity increases the lateral force don't have a large variation, this only happened when the slip angle is superior to 10°. This is more noticed when the normal force F_3 is applied.

Making a comparison with the gotten graph (Fig. 11) and the graph of the magical formula (Fig. 4) notices that the experimental curve followed the graph of the magical formula, that is, the lateral force started to grow until the slip angle reached around 5° and afterward it tended to be constant.

In the experimental graph it showed an imbalance in relation to the origin in comparison to the one of the magical formula. This occurred because of some error of measurements, calibrations or probably of the assembly of the wheel. Therefore the wheel can have presented some Camber angle.

Another experiment was made but now was used two types of wheels. The procedure of the previous experiment was repeated and the type of the tire was modified having one tire more rigid (R_1) and a tire more flexible (R_2).

Thus the graph of the lateral force versus slip angle gotten was:

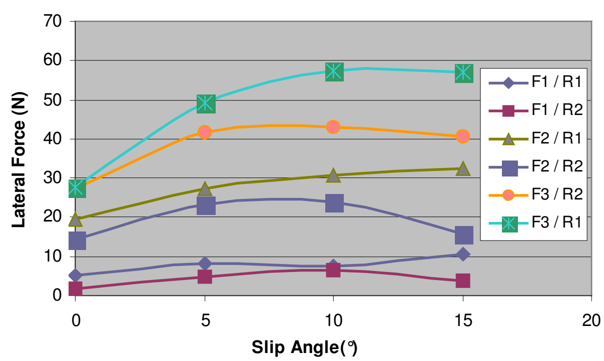


Figure 12. Lateral Force versus slip angle

From Figure 12 notices that the lateral tire force for the rigid tire (R_1) is bigger than the flexible tire (R_2). And the lateral force started to grow until the slip angle reached around 5° and later it tended to be constant or when the tire is more flexible (R_2) the lateral force tended to decrease.

In the experimental graph it showed an imbalance in relation to the origin in comparison to the one of the magical formula. This occurred in same way in the Fig. 12 so we have the same problem in this experiment.

3.2. The a_i coefficients

Using the measure from Figure 11 we could obtain the coefficients a_i:

- a_0=48
- a_1=0,00005238
- a_2=0,20657
- a_3=8,8589
- a_4=355

- a_6=-0,0000167
- a_7=-0,0165

The simulation with all these new coefficients is:

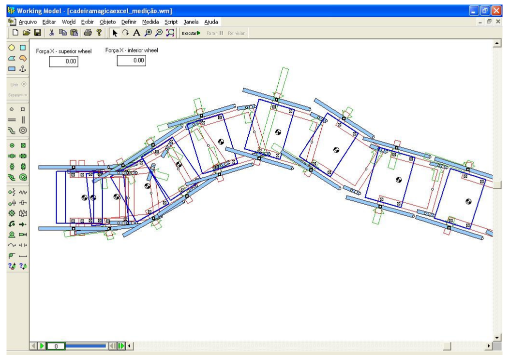


Figure 13. Simulation of the magic formula using the new coefficients

This simulation was closed with the simulation on Fig.7. In this figure notice that the lateral force (the green arrow) tries to correct the movement and it is bigger when the wheelchair realizes one curve. But in the end of the simulation the wheelchair slip a little, this occurred because of some error of measures in the attainment of the parameters.

4. CONCLUSION

The model of the contact between wheel and ground was sufficiently, showing the behavior of the wheelchair. But at some moment this wheelchair suffers a slip due for maybe the parameters used, so was need to do the experiment.

The experiment showed result satisfactory just having some errors of measurements due to the equipment used, so some points must be improved.

A real model of the wheelchair will make possible a better visualization of the dynamic behavior. And with the knowledge of the dynamic behavior is possible to make a better control of navigation in motorized wheelchair taking in consideration the applied lateral force in the vehicle.

5. ACKNOWLEDGEMENTS

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