USE OF A THERMAL RADIATION INVERSE ANALYSIS TO ESTIMATE TEMPERATURE FIELD IN SEMITRANSPARENT MEDIA

Carlos T. Salinas
Department of Mechanical Engineering – University of Taubaté. Rua Daniel Danelli s/n – Taubaté (SP) 12060-440 – Brazil.
csalinas_99@yahoo.com; csalinas@unitau.br

Gilmar Guimarães
School of Mechanical Engineering – Federal University of Uberlandia. Av. João Naves de Avila 2121, Campus Santa Monica Block M, Uberlandia.
gguima@mecanica.ufu.br

Abstract. *The inverse analysis in thermal radiation problems has many applications in engineering as, for example, the temperature field prediction in furnaces. It can be observed that in the radiative transport equation, the heat source, the media properties and the boundary conditions are known in direct radiation problems. In this case, the intensity field needs to be estimated. By the other hand, in inverse problems media properties or boundary conditions are unknown. This work presents an inverse analysis for temperature field estimation in a gray parallel media with transparent boundary surfaces and without external incident radiation. The inverse problem is formulated as an optimization problem that minimizes the error between the calculated and the simulated measurement of radiation intensity leaving of the media. The conjugate gradient minimization method is used. The inverse analysis steps: (1) direct problem; (2) sensibility problem; and (3) the gradient equation are presented. The numerical results are obtained by considering simulated data with and without noise. The font term and the temperature field have been estimated with success*

Keywords*: Inverse radiation analysis, conjugate gradient method, temperature field.*

1. INTRODUCTION

Radiant enclosures are encountered in many different industrial settings. In most cases, the enclosure contains participating real gases or heater that irradiates a product located into the medium. The inverse analysis of radiation in participating medium has a broad range of engineering applications, for example, the remote sensing of the atmosphere, determination of the radiative properties of medium, the prediction of temperature flame, design of combustion systems such as furnaces, combustors and large-scale jet flames. Inverse radiation problems have defined a subject of interest for the past 30 or so years and there exists a considerable body of knowledge surrounding the subject that has been extensively reviewed in a series of papers by McCormick (1981, 1984, 1986, 1992), by works of Howell and collaborators, and Ozisik and collaborators.

Direct radiation exchange problems are those in which radiative properties of the gas filling the enclosure as well as those of bounding surfaces along with boundary conditions are all known. This types of problems are mathematically “well posed”; and, therefore, can be solved with straight forward and well-established mathematical procedures.

Inverse radiation exchange problems are, however, those in which either of the mentioned data are unknown; instead, some additional information, such as measured or specified temperatures, are available. The use of the inverse techniques often leads to a mathematically ill-posed problem, because there may exist multiple solutions that produce the specified or desired conditions over the boundary surface or on part of the domain, or no viable solution whatsoever. Consequently, standard analysis techniques will fail to produce a useful solution to problem and special methods are required to treat the ill-conditioned nature of equations. Inverse radiation problems may be classified into two categories of “identification” and “design” problems. In identification problems, temperature or radiative intensity is measured at some locations within the enclosure and based on these measurements and using inverse algorithms, the unknown parameters such as the emissive power of the enclosure boundaries or temperature distribution in the gas filling the enclosure are estimated.

Inverse problems that deal with the prediction of the temperature distribution in a medium from either simulated or experimental radiation measurements have been reported by many researchers. Yi *et al.* (1992), Li and Ozisik (1992, 1993), Siewert (1993, 1994), Li (1994, 1997) and Liu *et al.* (1998, 2000) have reconstructed the temperature profiles or source terms in one-dimensional plane-parallel, spherical and cylindrical media by inverse analysis from the simulated data of the radiation intensities exiting the boundaries.

In most of the above works, the discrete ordinates method (DOM), the discrete transfer method (DTM), the zonal method or the Monte Carlo method were employed to solve the direct and the sensitivity problems.

In high temperature process, thermal radiation is often the dominating mode of heat transfer in the system. For a system governed by radiation, the inverse problem is represented by a set of Fredholm equations of the first kind, which are known to be ill-posed. If the resulting system is solved by simple techniques, like Gauss elimination or Gauss-Siedel iteration, the ill-posed character of the governing equations leads to non-physical solutions that are highly affected by small perturbations in the input. Therefore, in order to produce physically reasonable, yet accurate solutions the system must be regularized. Regularization leads to a set of solutions that ignores some information that is the source of the ill-posed character. Consequently, the solutions are subject to different levels of errors that result from ignored information, and an optimal solution must be sought that satisfies the physical requirements of the problem with an acceptable accuracy.

There exist a number of regularized solution techniques that have been used for solution of similar problems, including truncated singular value decomposition (TSVD) (França *et al.*, 2002), the conjugate gradient method (CGM) (Ozisik and Orlande, 2000), the bi-conjugate gradient method (BiCGM) that is a method based on the CGM and improves the method to solve any arbitrary MxN system (Ertürk *et al.*, 2002) the Tikhonov regularization method, that was first proposed by Tikhonov (1975) for solving inverse conduction problems. A review of regularization methods is provided by Hansen (1998), Vogel (2002), Björck (1996) and Daun and Howell (2005).

Despite the relatively large interest expressed in inverse radiation problems of temperature distributions in media, most of the works use simple spatial interpolation schemes as step and diamond scheme, and low order of angular quadratures for DOM or DTM, and then less accurate solutions for the direct problem were used. The inverse radiation problem considered in this paper is concerned with the estimation of the source term distribution or temperature profile in participating media systems which contains absorbing, emitting, scattering gray medium from the knowledge of the radiative intensities exiting in some points of boundary surfaces which simulates dates of sensing devises. The radiative properties such as single scattering albedo and scattering phase function of the medium, are assumed to be uniform everywhere. The boundaries are considered to be either transparent or opaque. The inverse problem is formulated as an optimization problem and the conjugate gradient method is used for its solution. In this work, the discrete ordinates method is used to solve the direct and sensitivity problem and different angular quadratures are used to examine the accuracy of the estimation. The analysis consists of the direct problem, the gradient equation and the sensitivity problem. The procedure for each of these steps is described and then presents an algorithm for the solution of the inverse radiation problem. Finally, several inverse problems of source term in one-dimensional are investigated to demonstrate the computational accuracy and efficiency of the inverse analysis method presented in this paper.

2. REGULARIZATION METHOD

The basic definitions of the Conjugate Gradient Method (CGM) are explained in the following

Conjugate gradient regularization is based on a minimization technique. However, unlike Newton's method, which tries to find the vector $\boldsymbol{p}^k$ that satisfies $\boldsymbol{\Phi}^k + \boldsymbol{p}^k = \boldsymbol{\Phi}^*$ at each iteration, conjugate gradient minimization instead works by generating a set of *mutually conjugate* search directions, $\{\boldsymbol{p}\}$, that are "non-interfering" in the sense that progress made by minimizing in one direction is not subsequently "spoiled" by minimizing in another direction.

CGM is an N-step solution technique, which provides the exact solution x_e to a system such as ($A_{i,j}\, x_j = b_i$), where N is the number of unknowns. For a system defined by a symmetric and positive-definite $\mathbf{A}$, CGM can be interpreted as a way of minimizing the functional, $H(x) = \mathbf{A}(x_e - x)(x_e - x)$, which reaches zero at its minimum when $x = x_e$. In order to minimize the functional, the gradient that is equivalent to the negative of twice the residual should reach zero ($\nabla H = -2\mathbf{r} = 0$). Therefore, CGM and TSVD are very similar methods based on the same objective function and the differences in the solutions are due to the different methodologies and the numerical applications they follow. The CGM can be generalized for any arbitrary system M x N by modifying the system through multiplication of both sides of the equation by $\mathbf{A}^T$.

In the CGM, the solution is defined as a linear combination of A-conjugate vectors, $\mathbf{p}$'s, defined in every step that satisfy the A-conjugacy condition, $(\mathbf{A} \cdot \mathbf{p}^i) \cdot \mathbf{p}^j = 0$ for $i \neq j$. Each step introduces the addition of new A-conjugate vector to the solution of the previous step, providing a monotonic decrease in the residual. The derivation of the method is presented in Hansen (1998) and Beckman (1960) in detail.

In the conjugate gradient minimization algorithm, the set of search directions is generated iteratively by calculating a new search direction $\boldsymbol{p}^{k+1}$, that is conjugate to the current search direction $\boldsymbol{p}^k$, with respect to the Hessian, i.e. $\boldsymbol{p}^{k+1\mathbf{T}} \nabla^2 F(\boldsymbol{\Phi}^k)\, \boldsymbol{p}^k = 0$.

This minimization technique was later developed into a low storage method for solving well-conditioned sets of linear equations, $\boldsymbol{Ax} = \boldsymbol{b}$. It works by minimizing an objective function having a gradient vector equal to the residual, $\nabla \boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{\delta}(\boldsymbol{x}) = \boldsymbol{Ax} - \boldsymbol{b}$; integrating with respect to $\boldsymbol{x}$ yields

$$F(x) = \frac{1}{2} x^T A x - b^T x \tag{1}$$

where $\nabla^2 \boldsymbol{F}(\boldsymbol{x}) = \mathbf{A}$. If A is symmetric and positive definite, then F(x) has one global minimum $\boldsymbol{x}^*$ that solves Ax = b, since $\nabla \boldsymbol{F}(\boldsymbol{x}^*) = \boldsymbol{\delta}(\boldsymbol{x}^*) = 0$. Problems where A is not symmetric are treated by solving $\boldsymbol{A^T A x} = \boldsymbol{A^T b}$. The conjugate set of search directions need to reach $\boldsymbol{x}^*$ is found through recursion (Nash and Sopher, 1996).

3. ANALYSIS

The analysis consists of the direct problem, the gradient equation, and the sensitivity problem. It is noted that the analysis of practical applications of inverse radiation problems must, in general, be based on more complex model than the one considered in this work, for example, multidimensional problems with non-gray and scattering medium. Of course, more complex models may require strictly more complicated numerical methods of solution, and thus the simplified model considered here is to be understood as a first attempt to provide a more deterministic algorithm for the problem.

3.1 Direct Problem

Consider the radiative transfer process in an absorbing, emitting, scattering, gray, plane parallel medium. The boundary surfaces are considered to be transparent. There is no external incident radiation. The radiative properties, such as single scattering albedo ω and scattering phase function Φ(s´,s) of the medium, are assumed to be uniform everywhere. The direct problem of concern here is to find the radiative intensities exiting the boundaries for the known source term distribution and radiative properties.

The radiative transport equation for an absorbing, emitting gray gas with isotropic scattering can be written as Siegel and Howell (1992),

$$\left(\Omega.\nabla\right)\boldsymbol{I}(\boldsymbol{r},\Omega) = -(\kappa+\sigma)\boldsymbol{I}(\boldsymbol{r},\Omega) + S(\boldsymbol{r}) + \frac{\sigma}{4\pi}\int_{4\pi}\boldsymbol{I}(\boldsymbol{r},\Omega')\boldsymbol{d}\Omega' \tag{2}$$

where $\boldsymbol{I}(\boldsymbol{r},\Omega)$ is the radiation intensity in $\boldsymbol{r}$, and in the direction Ω; $\boldsymbol{I_b}(\boldsymbol{r})$, is the radiation intensity of the blackbody in the position $\boldsymbol{r}$ and at the temperature of the medium; κ is the gray medium absorption coefficient; σ is the gray medium scatter coefficient; and the integration is in the incident direction Ω'. The source term $S(\boldsymbol{r})$ is related to the temperature $T(\boldsymbol{r})$ of the medium by

$$S(\boldsymbol{r}) = \frac{(1-\omega)\overline{n}^2\overline{\sigma}\boldsymbol{T}^4(\boldsymbol{r})}{\pi} \tag{3}$$

Here, $\overline{n}$ is the refractive index and $\overline{\sigma}$ is the Stefan Boltzmann constant.

For diffusely reflecting surfaces the radiative boundary condition for Eq. (2) is

$$\boldsymbol{I}(\boldsymbol{r},\Omega) = \varepsilon\boldsymbol{I_b}(\boldsymbol{r}) + \frac{\rho}{\pi}\int_{\boldsymbol{n}.\Omega'<0}|\,\boldsymbol{n}.\Omega'|\,\boldsymbol{I}(\boldsymbol{r},\Omega')\boldsymbol{d}\Omega' \tag{4}$$

where $\boldsymbol{r}$ lies on the boundary surface Γ, and Eq. (4) is valid for $\boldsymbol{n}.\Omega>0$. $\boldsymbol{I}(\boldsymbol{r},\Omega)$ is the radiation intensity leaving the surface at the boundary condition, ε is the surface emissivity, ρ is the surface reflectivity and $\boldsymbol{n}$ is the unit vector normal to the boundary surface.

In the method of discrete ordinates, the equation of radiation transport is substituted by a set of M discrete equations for a finite number of directions Ω_m, and each integral is substituted by a quadrature series (Fiveland W., 1984),

$$\left(\Omega_m.\nabla\right)\boldsymbol{I}(\boldsymbol{r},\Omega_m) = -\beta\boldsymbol{I}(\boldsymbol{r},\Omega_m) + \boldsymbol{S}(\boldsymbol{r}) + \frac{\sigma}{4\pi}\sum_{m=1}^{M}\boldsymbol{w}_m\boldsymbol{I}(\boldsymbol{r},\Omega_m) \tag{5}$$

This angular approximation transforms the original equation into a set of coupled differential equations, with $\beta = (\kappa+\sigma)$ as the extinction coefficient.

$$\boldsymbol{S}_m = \frac{\sigma}{4\pi}\sum_{m=1}^{M}\boldsymbol{w}_m\boldsymbol{I}(\boldsymbol{r},\Omega_m) \tag{6}$$

where S_m represents the entering scattering source term, $\mathbf{w}_m$ are the ordinates weight, M is the number of directions Ω_m of the angular quadrature and $\mathbf{I}_m$ is obtained by solving the radiative transport equation in discrete ordinates.

In the Cartesian ordinate system, the one-dimensional radiative transport equation in the ***m*** direction for an emitting, absorbing and non-scattering medium is

$$\mu_m \frac{d\boldsymbol{I}_m}{d\boldsymbol{x}} = -\beta \boldsymbol{I}_m + \boldsymbol{S}(\boldsymbol{x}) \tag{7}$$

where μ_m, is the directional cosine of Ω_m. The boundary condition in discrete ordinates can be written as

$$\boldsymbol{I}_m = 0; \quad \mu_m > 0 \text{ in } x \in \Gamma \tag{8}$$

The discretization in finite volumes can be obtained by multiplying Eq. (7) by dx and integrating over a control volume *(i)*

$$\mu_m d\boldsymbol{x}(\boldsymbol{I}^m_{i+\frac{1}{2}} - \boldsymbol{I}^m_{i-\frac{1}{2}}) = V_i(\boldsymbol{S}(\boldsymbol{x}) - \beta \boldsymbol{I}^m)_i \tag{9}$$

where V_i is the control volume i in m^3.

Assuming that the boundary conditions are given, the system of equations is closed and defines an interpolation system relating the intensities at the face to the nodal values. To discretize the radiative transport equation, one can rewrite Eq. (9) based upon the method outlined in Ismail and Salinas (2004) as,

$$(\boldsymbol{I}^m_i)^{n+1} = \frac{V_i(\boldsymbol{S}(\boldsymbol{x}))^n + |\mu_m| d\boldsymbol{x}(\boldsymbol{I}^m_{i-\frac{1}{2}})^{n+1} + \boldsymbol{S}^n_{df}}{|\mu_m| d\boldsymbol{x} + \beta V_i} \tag{10}$$

where $\boldsymbol{S}_{df}$ is the deferred factor correction as calculate in Ismail and Salinas (2004)

$$\boldsymbol{S}^n_{df} = |\mu_m| d\boldsymbol{x}(\boldsymbol{I}^m_i - \boldsymbol{I}^m_{i+\frac{1}{2}})^n \tag{11}$$

The values of the intensities in the faces $(\boldsymbol{I}^m_{i+\frac{1}{2}})^n$ are interpolated using the CLAM scheme while the values of the intensities $(\boldsymbol{I}^m_{i-\frac{1}{2}})^{n+1}$ are interpolated using a step scheme.

3.2 Inverse Problem

For the inverse problem, the source term distribution is regarded as unknown, but the other quantities in Eqs. (7) and (8) are known. In addition, measured radiative intensities existing boundaries are considered available. In the inverse analysis, the source term distribution is estimated by the measured data of exit radiative intensities.

The source term can be represented by a polynomial as

$$\boldsymbol{S}(\boldsymbol{x}) = \sum_{n=0}^{N} \boldsymbol{a}_n (\beta \boldsymbol{x})^n \quad \text{or} \quad \boldsymbol{S}(\tau) = \sum_{n=0}^{N} \boldsymbol{a}_n (\tau)^n \tag{12}$$

where $\tau = \beta x$ is the optical thickness variable, N is the order of the source term polynomial expansion.

The inverse radiation problem can be formulated as an optimization problem. We wish to minimize the objective function

$$\boldsymbol{J}(\tilde{\boldsymbol{a}}) = \sum_{\mu_i<0} \boldsymbol{w}_i [\boldsymbol{I}(0, \mu_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Y}(\mu_i)]^2 + \sum_{\mu_i>0} \boldsymbol{w}_i [\boldsymbol{I}(\boldsymbol{x}_L, \mu_i, \tilde{\boldsymbol{a}}) - \boldsymbol{Z}(\mu_i)]^2 \tag{13}$$

where $Y(\mu_i)$ and $Z(\mu_i)$ are measured exit radiation intensities at the boundaries of x = 0 and x = x_L, respectively; $\boldsymbol{I}(0, \mu_i, \tilde{\boldsymbol{a}})$ and $\boldsymbol{I}(\boldsymbol{x}_L, \mu_i, \tilde{\boldsymbol{a}})$ are estimated exit radiation intensities at x = 0 and x = x_L, respectively, for the estimated vector $\tilde{\boldsymbol{a}} = (a_o, a_1, \ldots, a_N)^T$. The problem is then to find the vector $\tilde{\boldsymbol{a}}$ which minimizes the function ***J***.

The computational algorithm of this minimization procedure consists of two main modules; the direct radiation computation and the search modules. As explained previously, for the first module the Discrete Ordinates Method (DOM) is employed in this work, while for the latter the Conjugate Gradient Method (CGM) is used as the basic search method to minimize the function ***J***.

3.3 Conjugate Gradient Method of Minimization

The minimization of the objective function with respect to the desired vector is the most important procedure in solving the inverse problem. The Conjugate Gradient Method to determine the unknown temperature distribution is used in this work. Iterations are built in the following manner (Li and Ozisik, 1992; Liu *et al.*, 2000):

$$\mathbf{a}^{k+1} = \mathbf{a}^k - \alpha^k \mathbf{d}^k \tag{14}$$

where α^k is the step size, $\mathbf{d}^k$ is the direction vector of descent given by

$$\mathbf{d}^k = \nabla \boldsymbol{J}(\mathbf{a}^k) + \beta^k \mathbf{d}^{k-1} \tag{15}$$

and the conjugate coefficient β^k is determined from

$$\beta^{\boldsymbol{k}} = \frac{\nabla \boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}}) \nabla \boldsymbol{J}^{\boldsymbol{T}}(\boldsymbol{a}^{\boldsymbol{k}})}{\nabla \boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}-1}) \nabla \boldsymbol{J}^{\boldsymbol{T}}(\boldsymbol{a}^{\boldsymbol{k}-1})} \qquad \beta^0 = 0 \tag{16}$$

where the row vector $\nabla \boldsymbol{J}$ defined by

$$\nabla \boldsymbol{J} = \left(\frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_0}, \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_1}, \ldots\ldots.., \quad \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_{\boldsymbol{N}}} \right) \tag{17}$$

is the gradient of the objective function. Its components are defined as

$$\begin{aligned} \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_{\boldsymbol{n}}} = {} & 2\sum_{\mu_i<0} \boldsymbol{w}_i [\boldsymbol{I}(0,\mu_i;\tilde{\boldsymbol{a}}) - \boldsymbol{Y}(\mu_i)] \frac{\partial \boldsymbol{I}(0,\mu_i;\tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{\boldsymbol{n}}} + \\ & 2\sum_{\mu_i>0} \boldsymbol{w}_i [\boldsymbol{I}(\boldsymbol{x}_{\boldsymbol{L}},\mu_i;\tilde{\boldsymbol{a}}) - \boldsymbol{Z}(\mu_i)] \frac{\partial \boldsymbol{I}(\boldsymbol{x}_{\boldsymbol{L}},\mu_i;\tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{\boldsymbol{n}}} \end{aligned} \tag{18}$$

In principle, the step size of the k th iteration α^k can be determined by minimizing the function, $\boldsymbol{J}(\mathbf{a}^k - \alpha^k \mathbf{d}^k)$, for the given $\mathbf{a}^k$ and $\mathbf{d}^k$ in the following manner:

$$\frac{\partial \boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}} - \alpha^{\boldsymbol{k}} \boldsymbol{d}^{\boldsymbol{k}})}{\partial \alpha^{\boldsymbol{k}}} = 0 \tag{19}$$

Since $\boldsymbol{J}(\mathbf{a}^k - \alpha^k \mathbf{d}^k)$ is the implicit function of α^k, the exact step is difficult to solve. As first- order approximation, we make the first-order Taylor expansion of the function with respect to α^k. Using Eq. (19), we have

$$\begin{aligned} \alpha^{\boldsymbol{k}} = {} & \left\{ \sum_{\mu_i<0} \boldsymbol{w}_i [\boldsymbol{I}(0,\mu_i;\tilde{\boldsymbol{a}}^{\boldsymbol{k}}) - \boldsymbol{Y}(\mu_i)][\nabla \boldsymbol{I}(0,\mu_i;\tilde{\boldsymbol{a}}^{\boldsymbol{k}}).\boldsymbol{d}^{\boldsymbol{k}}] + \right. \\ & \left. \sum_{\mu_i>0} \boldsymbol{w}_i [\boldsymbol{I}(\boldsymbol{x}_{\boldsymbol{L}},\mu_i;\tilde{\boldsymbol{a}}) - \boldsymbol{Z}(\mu_i)][\nabla \boldsymbol{I}(\boldsymbol{x}_{\boldsymbol{L}},\mu_i;\tilde{\boldsymbol{a}}^{\boldsymbol{k}}).\boldsymbol{d}^{\boldsymbol{k}}] \right\} \\ & \Big/ \left\{ \sum_{\mu_i<0} \boldsymbol{w}_i [\nabla \boldsymbol{I}(0,\mu_i;\tilde{\boldsymbol{a}}^{\boldsymbol{k}}).\boldsymbol{d}^{\boldsymbol{k}}]^2 + \sum_{\mu_i>0} \boldsymbol{w}_i [\nabla \boldsymbol{I}(\boldsymbol{x}_{\boldsymbol{L}},\mu_i;\tilde{\boldsymbol{a}}^{\boldsymbol{k}}).\boldsymbol{d}^{\boldsymbol{k}}]^2 \right\} \end{aligned} \tag{20}$$

Here the row vector

$$\nabla \boldsymbol{I} = \left(\frac{\partial \boldsymbol{I}}{\partial \boldsymbol{a}_0}, \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{a}_1}, \ldots\ldots, \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{a}_N} \right) \tag{21}$$

is the sensitivity coefficient vector, which is essential in the solution procedure of inverse problems.

3.4 Sensitivity Problem

To obtain the sensitivity coefficients, we substitute Eq. (12) into Eq. (7) and differentiate the direct problem defined by Eqs. (7) and (8) with respect to a_n. The equations of sensitivity coefficients can be written as

$$\mu \frac{\partial}{\partial \boldsymbol{x}} \left(\frac{\partial \boldsymbol{I}(\boldsymbol{x}, \mu)}{\partial \boldsymbol{a}_n} \right) + \beta \left(\frac{\partial \boldsymbol{I}(\boldsymbol{x}, \mu)}{\partial \boldsymbol{a}_n} \right) = (\beta \boldsymbol{x})^{\boldsymbol{n}} \qquad n = 1, 2, \ldots\ldots, N \tag{22}$$

With the boundary conditions

$$\left(\frac{\partial \boldsymbol{I}(0, \mu)}{\partial \boldsymbol{a}_n} \right) = 0 \qquad \mu > 0 \qquad n = 1, 2, \ldots\ldots, N \tag{23a}$$

$$\left(\frac{\partial \boldsymbol{I}(\boldsymbol{x}_L, -\mu)}{\partial \boldsymbol{a}_n} \right) = 0 \qquad \mu > 0 \qquad n = 1, 2, \ldots\ldots, N \tag{23b}$$

A similar numerical iteration procedure as the direct problem is used for the solution of the sensitivity problem and will not be repeated here. Because the sensitivity coefficient vector $\nabla \boldsymbol{I}$ is independent of the vector **a**, the estimation of the source term distribution is linear, and only it is necessary to solve once at first.

3.5 Stopping Criterion

The stopping criterion of the iteration is selected in the following manner: if the problem contains no measurement error, the following condition

$$\boldsymbol{J}(\mathbf{a}^k) < \delta^* \tag{24}$$

is used for terminating the iterative process, where δ^* is a small specified positive number; otherwise, the condition

$$\left| \frac{2\sigma_o^2}{\boldsymbol{J}(\boldsymbol{a}^k)} - 1 \right| < \delta_1^* \tag{25}$$

are used as the stopping criterion, where δ_1^* is a small specified positive number, where σ_o is the standard deviation.

3.6 Computational Algorithm

The computational algorithm for the solution of the inverse radiation problem can be summarized as follow
Step 1. Pick the initial guesses of $\mathbf{a}^o$. Set k = 0.
Step 2. Solve the sensitivity problem and compute the sensitivity coefficient vector $\nabla \boldsymbol{I}$.
Step 3. Solve the direct problem and compute the exit radiation intensities $\boldsymbol{I}(0, \mu_m)$ and $\boldsymbol{I}(x_L, \mu_m)$.
Step 4. Calculate the function objective $\boldsymbol{J}(\mathbf{a}^k)$. Terminate the iteration process if the specified stopping criterion is satisfied. Otherwise, go to step 5.
Step 5. Compute the gradient of the function objective $\nabla \boldsymbol{J}(\mathbf{a}^k)$.
Step 6. Knowing $\nabla \boldsymbol{J}(\mathbf{a}^k)$, compute the conjugate coefficient β^k from Eq.(16); then compute the direction vector of descent $\mathbf{d}^k$ from Eq.(15).
Step 7. Knowing $\nabla \boldsymbol{I}$, $\boldsymbol{I}(0, \mu_m)$ and $\boldsymbol{I}(x_L, \mu_m)$, compute the step size α^k from Eq. (20).

Step 8. Compute the new estimated vector $\mathbf{a}^{k+1}$ from Eq. (14).
Step 9. Set k = k +1, and go to step 3.
To start the iteration, the initial guesses $\mathbf{a}^{o} = \mathbf{0}$ is used.

4. RESULTS AND DISCUSSION

Based on the theoretical and numerical analysis described earlier, a computer code has been developed to solve the inverse radiation problem of source term in one- dimensional plane parallel media. To examine the accuracy of the method presented in this paper, two different test cases are considered. In the first case, assuming the measurement data exit radiation intensities have no errors, the temperature profile is determined. In the second case, the effects of random measurement errors on the estimation are analyzed. The optical thickness of the slab is chosen to be 1.0. To simulate the measured exit radiation intensities, Y and Z, containing measurement errors, random errors of standard deviation σ_o are added to the exact exit radiation intensities computed from the solution of the direct problem. Thus we have

$$\boldsymbol{Y}_{\text{measured}} = \boldsymbol{Y}_{\text{exact}} + \sigma_o \zeta \tag{26a}$$

$$\boldsymbol{Z}_{\text{measured}} = \boldsymbol{Z}_{\text{exact}} + \sigma_o \zeta \tag{26b}$$

where ζ is a normal distribution random variable with zero mean and unit standard deviation. There is a 99% probability of ζ lying in the range $-2.567 < \zeta < 2.567$ (Li and Ozisik, 1992). For all the results presented in this work, it is assumed that the exit radiation intensities are available at the quadrature points for different angular quadratures of DOM.

4.1 Test problem 1.

We consider a source term expressed as a polynomial of degree 4 as in Li and Ozisik (1992),

$$S(\tau) = S(\beta x) = 1.0 + 20\tau + 44\tau^2 - 128\tau^3 + 64\tau^4, \qquad \text{W/cm}^2 \tag{27}$$

The estimated values of the source term by inverse analysis are shown in Fig. 1. With no measurement errors, σ_o, no observable difference could be detected between the exact values of the source term and the estimated values when is used the Tn6 angular quadrature. Otherwise, it is observed discrepancies with the exact solution when is used the quadrature S_6. Figure 2, shows the characteristic variation of the objective function with the number of iterations for the case of Tn6 angular quadrature. After several numerical experiments a proper value of δ^* is chosen.

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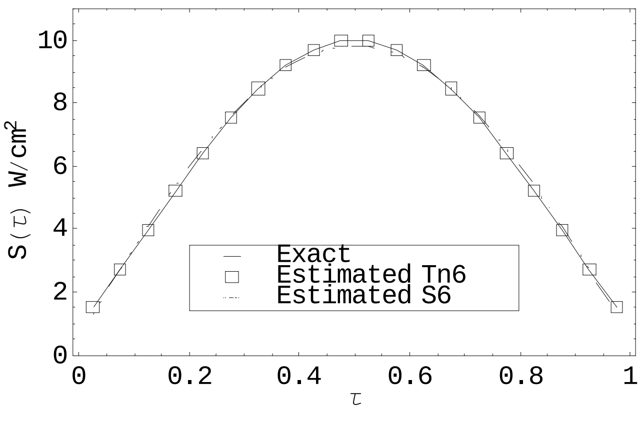

Figure 1. Estimation of the source term using simulated measured exit radiation intensity data with $\sigma_o = 0$ and different angular quadratures of DOM.

4.2 Test problem 2.

In practical process of measurement and inverse problem solution, the measured parameters may have random errors more or less. In order to examine the effect of these errors on the temperature estimation, it is assumed that the simulated experimental data containing random measurement errors of standard deviation $\sigma_o = 0.03$ and $\sigma_o = 0.06$.

Different sets of random number for measures in every boundary were used and different sets of random numbers to repeat the inverse calculations were used. Also, to examine the effects on the accuracy of the estimation, different angular quadratures for DOM were used. Figures 3 and 4 show the results of the estimation of the source term for a random sample and different angular quadratures. It was found that when it is used a Tn6 quadrature the results of the estimation are very accurate with the exact values for σ_o equal to 0.03 and 0.06. When low order quadratures as S_4 and S_6 is used, the estimation is less accurate than when is used the quadrature Tn6 and the estimations with quadrature S_4 are very poor.

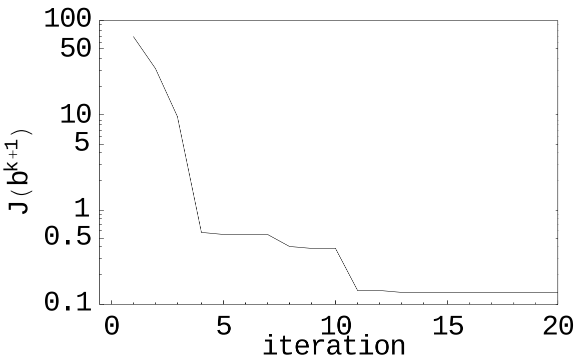

Figure 2. Variation of the objective function.

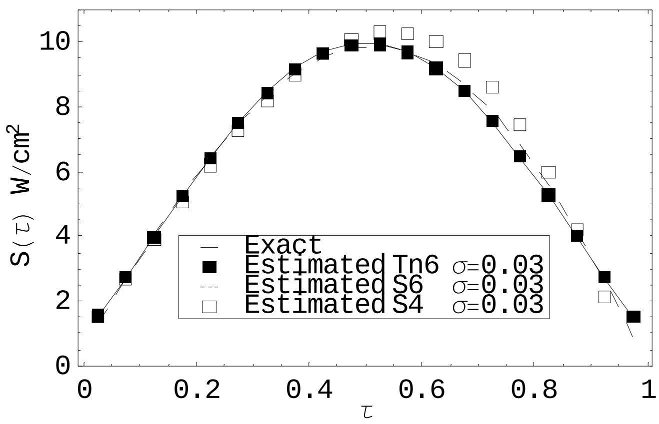

Figure 3. Estimation of the source term using simulated measured exit radiation intensity data with σ_o = 0.03 and different angular quadratures of DOM.

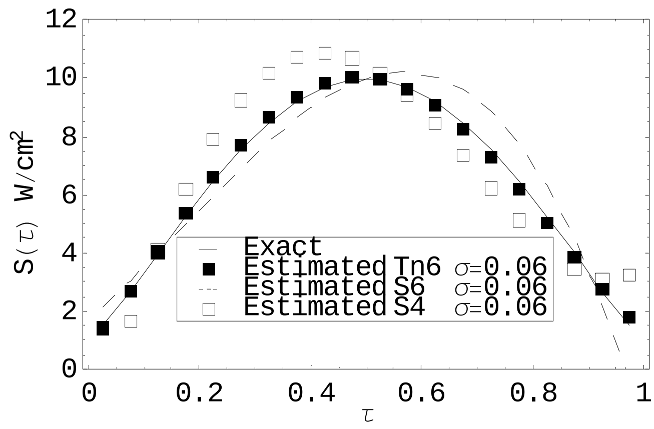

Figure 4. Estimation of the source term using simulated measured exit radiation intensity data with σ_o = 0.06 and different angular quadratures of DOM.

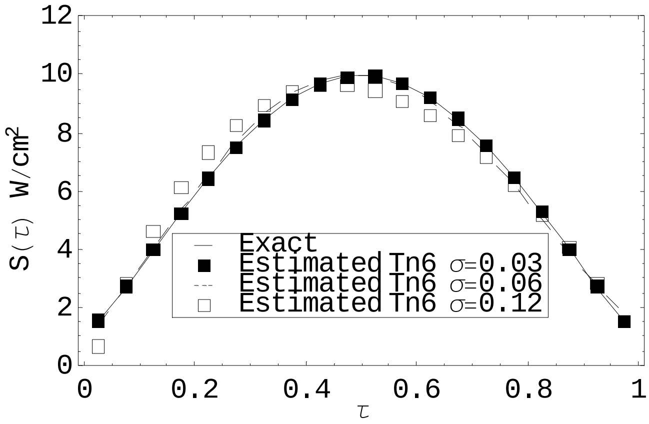

Figure 5. Comparison of the estimation of the source term using simulated measured exit radiation intensity data with angular quadratures Tn6, and σ_o = 0.03 and 0.06.

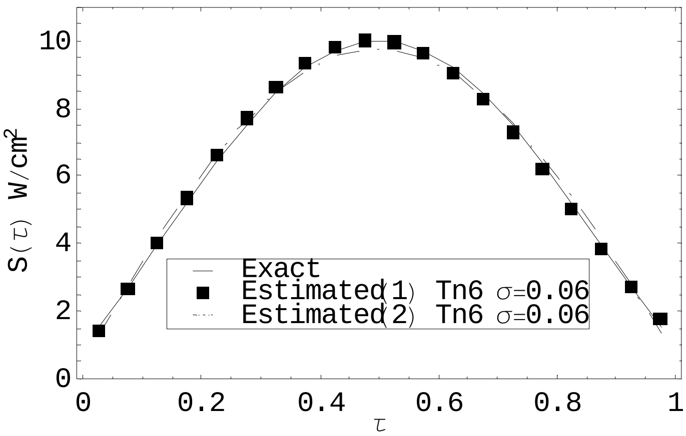

Figure 6. Comparison of the estimation of the source term using two different sets of data of simulated measured exit radiation intensity data with angular quadratures Tn6, and σ_o = 0.06.

Comparing Figs. 3 and 4, we note that when increasing σ_o from 0.03 to 0.06, the accuracy of the source in the estimation decreases when is used the angular quadrature S_6 and also, it can be observed that for σ_o = 0.06, the estimation using angular quadrature S_6 is very poor. The accuracy of the estimation is sensitive to the angular quadrature which was used. Figure 5, shows the results of the source term estimation for the angular quadrature Tn6 and different standard deviations. Clearly, the agreement between the exact and the estimated results for the source term is very good for σ_o equal 0.03 and 0.06. The accuracy decreases when a strong noise with σ_o equal to 0.12 is used, but the results have still good approximation. Figure 6, shows the estimation of the source term for two different sets of random errors and with σ_o =0.06. It can be observed that the results of the estimation are very closed and accurate.

The CPU time required for each sample of estimation calculation varied from less of 10 sec for S_6 to approximately 30 sec for Tn6 on a personal computer with Intel Pentium 4 3.0 processor.

5. CONCLUSIONS

An inverse method is presented for estimation of the temperature distribution for an emitting, non-scattering, gray plane-parallel medium. The exit radiation intensities at the bounding surfaces are assumed known. The inverse problem is solved by using the conjugate gradient method. Noisy input data have been used to test the accuracy of the method which is used. The results show that the temperature profile can be estimated accurately even with noise data when is used high order angular quadrature for DOM as Tn6, but is more sensitive to increases in measurement error when is used less order angular quadratures as S_6 or S_4. The inverse method results are sensitive to the accuracy of the direct problem solution.

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8. RESPONSIBILITY NOTICE