# ATTITUDE AND ANGULAR RATE ESTIMATION FOR A LOW-COST SATELLITE

**Davi Antônio dos Santos, davists@ita.br**
**Jacques Waldmann, jacques@ita.br**
Instituto Tecnológico de Aeronáutica - Department of Systems and Control - 12.228-900 - São José dos Campos, SP, Brazil

***Abstract.*** *A scheme for attitude and angular rate estimation from measurements of sun sensors and magnetometers on board a low-cost satellite is investigated. Firstly, two recursive algorithms for attitude quaternion estimation are evaluated, both deriving from the extended Kalman filter (EKF). The algorithms differ in that one of them applies a normalization operation to ensure that the unit magnitude property of the rotation quaternion is preserved throughout filter computations. Then angular rate is estimated by a derivative approach and an EKF is used to suppress the high-frequency noise introduced by the differentiation. Angular rate estimates are used in the propagation stage of the attitude estimator, whereas attitude estimates are employed in the angular rate estimator. Thus, both attitude and angular rate estimators are coupled, and their performances become mutually dependent. Results are presented for simulated measurements based on models of the sun position and the geomagnetic field. The results show that quaternion normalization improves both estimation convergence and accuracy. The attitude determination scheme is intended for future use in the ITASAT student satellite.*

***Keywords****: Attitude determination, sensor fusion, quaternion, extended Kalman filter, student satellite.*

## 1. INTRODUCTION

Real-time attitude determination is an important task in a spacecraft control system as it produces attitude and angular rate estimates required by control laws, and thus impose limits on the pointing accuracy.

The problem of optimal attitude estimation from vector observations is defined as follows. Let $S_B$ denote a Cartesian coordinate system attached to the spacecraft body and $S_R$ some reference Cartesian coordinate system. A set of *generalized vectors* (Choukroun, 2003) $\mathbf{X}(t)=\{\mathbf{x}_i(t)\in R^3,\ i: 1,2,...,m\}$ observed at time instant $t$ resolved in $S_B$ and $S_R$ gives rise to the sets $\mathbf{B}(t)=\{\mathbf{b}_i(t)\in R^3,\ i: 1,2,...,m\}$ and $\mathbf{R}(t)=\{\mathbf{r}_i(t)\in R^3,\ i: 1,2,...,m\}$, respectively, where $m$ is the number of pairs $(\mathbf{b}_i(t),\mathbf{r}_i(t))$ simultaneously measured. Using $\mathbf{B}(t)$ and $\mathbf{R}(t)$, it is desired to find an optimal estimate for the orthonormal matrix $\mathbf{A}(t;\mathbf{p})$ that rotates from $S_R$ to $S_B$, where $\mathbf{p}\in R^n$ is some attitude parameterization (Shuster, 1993).

The solution of the stated problem firstly requires choosing $\mathbf{p}$. The matrix $\mathbf{A}$ belongs to the three-dimensional special orthogonal group SO(3) and then the dimension of $\mathbf{p}$ needs to be $n \geq 3$ (Stuelpnagel, 1964). It is well-known that minimal three-dimensional parameterizations are singular for certain attitudes (Bar-Itzhack and Idan, 1987), but have the advantage of independent components (not constrained). On the other hand, the nine-dimensional fundamental parameterization $\mathbf{A}$ does not have singularities, but care need to be taken to ensure its orthogonality (Bar-Itzhack and Reiner, 1984). The four-dimensional quaternion of rotation $\mathbf{q}$ (Wertz, 1978) has the lowest dimensionality possible for a globally nonsingular representation of SO(3) (Markley, 2004), has a linear kinematic equation (Wertz, 1978), and then it is a very popular choice for $\mathbf{p}$ (Shuster and Oh, 1981; Bar-Itzhack and Oshman, 1985; Markley, 2004; Choukroun *et al.*, 2002; Lefferts *et al.*, 1982). In this work, $\mathbf{p} = \mathbf{q}$.

Optimal attitude estimation algorithms have been developed following two main approaches (Choukroun, 2003). (1) The first one is *constrained least squares*, which is based on the so-called Wahba problem (Wahba, 1965). *Single-frame* methods belonging to this approach use single time vector pairs and thus require a minimum of two non-collinear generalized vectors at each *t*. Examples are the q-method (Wertz, 1978), and QUEST (Shuster and Oh, 1981). *Multi-frames* methods use information contained in past measurements and require the angular velocity of $S_B$ with respect to $S_R$. Examples are Filter QUEST (Shuster, 1989), REQUEST (Bar-Itzhack, 1996), extended QUEST (Psiaki, 2000), and optimal REQUEST (Choukroun *et al.*, 2001). (2) The second approach is *minimum variance*, which in many applications resort to the extended Kalman filter (EKF) (Gelb, 1974; Jaswinski, 1970). The EKF is the workhorse of satellite attitude determination (Markley *et al.*, 2005), and has the advantage of embedding covariance analysis, is naturally multi-frame and recursive, and it is easily augmented to include parameters others than attitude (Choukroun, 2003). A survey about the methods mentioned above is presented in (Markley *et al.*, 2005). The second approach will be focused here.

There are several different implementations of minimum variance algorithms for attitude determination, depending on both the attitude parameterization used in the state vector and on the observation model (Markley *et al.*, 2005). The most commonly used forms are the additive EKF (AEKF) (Bar-Itzhack and Oshman, 1985) and the multiplicative EKF (MEKF) (Lefferts *et al.*, 1982). In AEKF, the four components of the quaternion are treated as independent parameters, i.e., the normalization condition is relaxed, and the quaternion error is additive. In this case, the normalization needs to be ensured by some additional mechanism, and the most direct response to this question is *"brute force"*, i.e. the division of the updated quaternion estimate by its Euclidian norm (Bar-Itzhack and Oshman, 1985). Markley (2004) presents two others alternative forms to guarantee normality. Recently, Choukroun *et al.* (2002) proposes a quaternion linear Kalman filter (KF) that does not enforce the normality, but the estimate converges to the right direction in $R^4$. The quaternion MEKF algorithms represent the attitude as the product of an estimated attitude and a deviation from that estimate (Markley *et al.*, 2005). The deviation is the quaternion error written in function of some three-dimensional vector which is indeed estimated (Markley, 2003). In this method, the estimate is a unity quaternion by definition (Markley, 2004) and the three-dimensional vector does not approach a singularity because it represents a small attitude error.

The time propagation stage of minimum variance algorithms requires knowing $\boldsymbol{\omega}=\omega_B^{BR}$, the angular velocity of $S_B$ with respect to $S_R$ described in $S_B$. This angular velocity can be obtained by direct measurement of a triad of mechanical rate gyros, but such devices are expensive, consume a significant amount of energy, and have significant mass, which usually go against the requirements of a low-cost satellite. Therefore, methods have been developed to estimate angular velocity without using gyros. Bar-Itzhack (2001) classifies the methods that are based on vector measurements or attitude estimates in two categories. (1) Firstly, the *derivative approach*, which takes the time derivative of either attitude or vector measurements. Because of noise amplified by the differentiatian, some filtering is needed as an additional stage of this approach; (2) Secondly, the *estimation approach*, which relies on the satellite's dynamic equation, which includes uncertainty in the inertia parameters and disturbance torques. Nonlinear estimators like the EKF, PSELIKA, or SDARE are alternatives that can be used either in the second approach or in the filtering stage of the first one (Harman and Bar-Itzhack, 1999). Psiaki and Oshman (2003) present two algorithms that use geomagnetic field measurements to estimate the angular velocity.

In this work, it will be considered as vector observations: (1) the direction and magnitude of the local geomagnetic field; and (2) the unit vector pointing to the sun. Events like eclipses will not be analyzed here. Figure 1 illustrates the devised scheme for attitude and rate estimation. For the *angular velocity estimator (rate estimator)* block, a derivative approach using both vector measurements and attitude estimates will be considered. Two different *attitude estimators* (Bar-Itzhack and Oshman, 1985) are investigated for use in the scheme: (1) An AEKF without normalization; (2) An AEKF with "brute force" normalization.

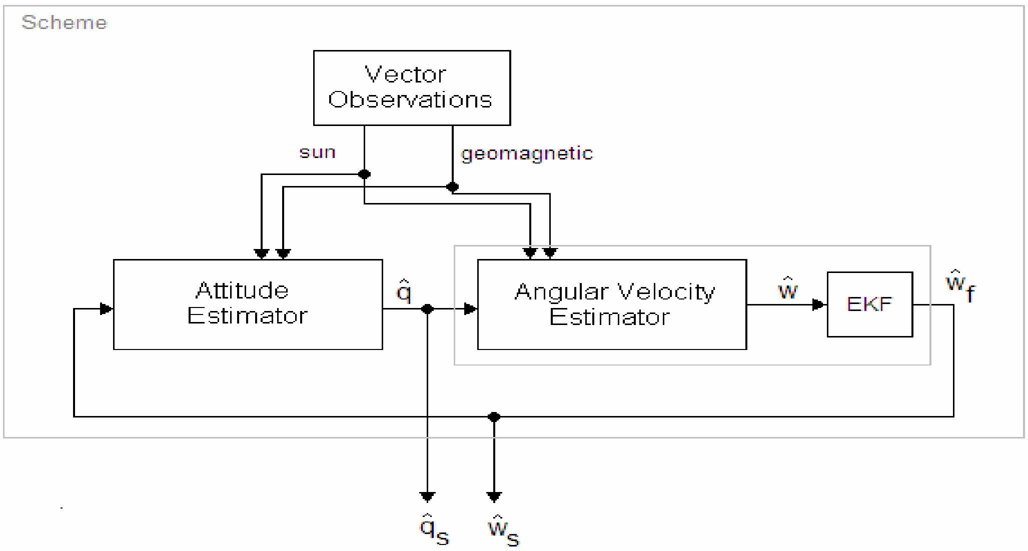


Figure 1 – Intertwined scheme for attitude/rate estimation.

## 2. INTERTWINED ATTITUDE AND RATE ESTIMATION

### 2.1. Attitude Estimation with the Additive EKF

#### 2.1.1. Measurement model

In general, the vectors in reference frame, $\mathbf{R}(t)$, are known quite accurately from models, while the body vectors, $\mathbf{B}(t)$, are corrupted by measurement errors with magnitudes considerably more significant than those of the errors in $\mathbf{R}(t)$, which may be neglected (Choukroun, 2003). As a result, the discrete-time measurement model is described by the following equation:

$$\mathbf{b}_i(t_{k+1}) = \mathbf{A}(\mathbf{q}(t_{k+1})).\mathbf{r}_i(t_{k+1}) + \delta\mathbf{b}_i(t_{k+1}) \quad (1)$$

The subscripts $i$ in Eq. (1) identify each vector measurement and, in this work, it assumes value $i = 1$ for the geomagnetic field vector and $i = 2$ for the sun direction. The quadratic relation between the attitude matrix and the quaternion of rotation is (Wertz, 1978):

$$\mathbf{A}(\mathbf{q}) = (\lambda^2 - \mathbf{e}^T\mathbf{e})\mathbf{I}_3 + 2\mathbf{e}\mathbf{e}^T - 2\lambda[\mathbf{e}\times] \tag{2}$$

where $\lambda$ and $\mathbf{e}$ are the scalar and the vector part of $\mathbf{q}$, respectively, and $[\mathbf{e}\times]$ is the cross-product matrix.

Let $\hat{\mathbf{q}}_{k+1|k}$ be the estimate of the true quaternion $\mathbf{q}_{k+1}$ given the previous measurements until the time instant $t_k$, and let the true quaternion at time $t_{k+1}$ be expressed in additive form as follows:

$$\mathbf{q}_{k+1} = \hat{\mathbf{q}}_{k+1|k} + \delta\mathbf{q}_{k+1|k} \tag{3}$$

where $\delta\mathbf{q}_{k+1|k}$ is the additive error that will be indeed estimated by the quaternion filter. Substituting Eq. (3) in Eq. (1), results:

$$\mathbf{b}_i(t_{k+1}) = \mathbf{A}(\hat{\mathbf{q}}_{k+1|k} + \delta\mathbf{q}_{k+1|k}).\mathbf{r}_i(t_{k+1}) + \delta\mathbf{b}_i(t_{k+1}) \tag{4}$$

Expanding the attitude matrix of Eq. (4) in a Taylor series until the first-order terms yields:

$$\mathbf{b}_i(t_{k+1}) = \mathbf{W}_{k+1}.\mathbf{r}_i(t_{k+1}) + \delta\mathbf{b}_i(t_{k+1}) \tag{5}$$

where

$$\mathbf{W}_{k+1} = \mathbf{A}(\hat{\mathbf{q}}_{k+1|k}) + \sum_{j=1}^{4} \left.\frac{\partial\mathbf{A}}{\partial q_j}\right|_{\hat{\mathbf{q}}_{k+1|k}} .\delta\mathbf{q}_{k+1|k,j} \tag{6}$$

Define now:

$$\mathbf{z}_i(t_{k+1}) = \mathbf{b}_i(t_{k+1}) - \mathbf{A}(\hat{\mathbf{q}}_{k+1|k}).\mathbf{r}_i(t_{k+1}) \tag{7}$$

$$\mathbf{h}_i^j(t_{k+1}) = \left.\frac{\partial\mathbf{A}}{\partial q_j}\right|_{\hat{\mathbf{q}}_{k+1|k}} .\mathbf{r}_i(t_{k+1}) \tag{8}$$

$$\mathbf{H}_i(t_{k+1}) = \begin{bmatrix} {}^1\mathbf{h}_i(t_{k+1}) & {}^2\mathbf{h}_i(t_{k+1}) & {}^3\mathbf{h}_i(t_{k+1}) & {}^4\mathbf{h}_i(t_{k+1}) \end{bmatrix} \tag{9}$$

It can be easily shown that, using Eq. (7)-(9), Eq. (5) takes the following form:

$$\mathbf{z}_i(t_{k+1}) = \mathbf{H}_i(t_{k+1}).\delta\mathbf{q}_{k+1|k} + \delta\mathbf{b}_i(t_{k+1}) \tag{10}$$

which is the sought linearized *measurement model*. The noise term $\delta\mathbf{b}_i(t_{k+1})$ is assumed to be zero-mean and white process with covariance matrix given by:

$$E[\delta\mathbf{b}_i(t_{k+1}).\delta\mathbf{b}_i^T(t_{k+1})] = \mathbf{R}_i(t_{k+1}) \tag{11}$$

**2.1.2. State model**

The state model that will be derived here consists of the kinematic equation for quaternions discretized in time and considering velocity additive error.

For propagation of the estimates between sample instants, the angular velocity of the satellite or its estimate needs to be used. The true angular velocity of $S_B$ with respect to $S_R$ is denoted by:

$$\boldsymbol{\omega}(t) = [\omega_x(t)\ \ \omega_y(t)\ \ \omega_z(t)]^T \tag{12}$$

The quaternion kinematic equation is given by (Wertz, 1978):

$$\dot{\mathbf{q}}(t) = \boldsymbol{\Omega}(t).\mathbf{q}(t) \tag{13}$$

where

$$\boldsymbol{\Omega}(t) = \frac{1}{2}\begin{bmatrix} 0 & -\boldsymbol{\omega}^T(t) \\ \boldsymbol{\omega}(t) & -[\boldsymbol{\omega}(t)\times] \end{bmatrix} \tag{14}$$

Integrating Eq. (13) from $t_k$ to $t_{k+1}$, results:

$$\mathbf{q}(t_{k+1}) = \boldsymbol{\Phi}(t_{k+1}, t_k).\mathbf{q}(t_k) \tag{15}$$

Assuming that $\boldsymbol{\omega}(t)$ is constant during the time interval $\Delta t = t_{k+1} - t_k$, it is well-known that the state-transition matrix results:

$$\boldsymbol{\Phi}(t_{k+1}, t_k) = e^{\boldsymbol{\Omega}(t_k).\Delta t} \tag{16}$$

The true angular velocity is unknown and thus it will be indeed estimated. In this work, it is assumed that the estimated angular velocity is modeled by:

$$\hat{\boldsymbol{\omega}}(t_k) = \boldsymbol{\omega}(t_k) + \boldsymbol{\varepsilon}(t_k) \tag{17}$$

where the noise term is a white process and has the following properties:

$$E[\boldsymbol{\varepsilon}(t_k)] = 0 \tag{18}$$

$$E[\boldsymbol{\varepsilon}(t_k).\boldsymbol{\varepsilon}^T(t_k)] = \mathbf{Q}^{\varepsilon}(t_k) \tag{19}$$

Substituting Eq. (17) in Eq. (16), results:

$$\boldsymbol{\Phi}(t_{k+1}, t_k) = e^{[\hat{\boldsymbol{\Omega}}(t_k) - \boldsymbol{\Omega}_{\varepsilon}(t_k)]\Delta t} = e^{\hat{\boldsymbol{\Omega}}(t_k)\Delta t}.e^{-\boldsymbol{\Omega}_{\varepsilon}(t_k)\Delta t} \tag{20}$$

where $\boldsymbol{\Omega}_{\varepsilon}(t_k)$ is given by Eq. (14), but inputting $\boldsymbol{\varepsilon}(t_k)$ instead of $\boldsymbol{\omega}(t)$. Using the series definition for the second exponential in the right hand side of Eq. (20), truncating the series after the first-order term, and substituting the result in Eq. (15), the following approximation results:

$$\mathbf{q}(t_{k+1}) = e^{\hat{\boldsymbol{\Omega}}(t_k)\Delta t}.\mathbf{q}(t_k) - e^{\hat{\boldsymbol{\Omega}}(t_k)\Delta t}.\boldsymbol{\Omega}_{\varepsilon}(t_k).\Delta t.\mathbf{q}(t_k) \tag{21}$$

After manipulations of the second term of Eq. (21) (see appendix A.1 of (Choukroun, 2003)), and replacing the quaternion by its estimate in the noise intensity matrix, the design state model is given by:

$$\mathbf{q}(t_{k+1}) = e^{\hat{\boldsymbol{\Omega}}(t_k)\Delta t}\mathbf{q}(t_k) - \frac{\Delta t}{2}e^{\hat{\boldsymbol{\Omega}}(t_k)\Delta t}.\hat{\boldsymbol{\Xi}}(t_k).\boldsymbol{\varepsilon}(t_k) \tag{22}$$

Using Eq. (18)-(19), the covariance of the noise term of Eq. (22) is computed by:

$$\mathbf{Q}^{q}(t_k) = \frac{\Delta t}{2} e^{\hat{\mathbf{\Omega}}(t_k)\Delta t}.\hat{\mathbf{\Xi}}(t_k).\mathbf{Q}^{\varepsilon}(t_k).\left(\frac{\Delta t}{2} e^{\hat{\mathbf{\Omega}}(t_k)\Delta t}.\hat{\mathbf{\Xi}}(t_k)\right)^T \tag{23}$$

Taking the expectation of Eq. (22) conditioned on all the past measurements until $t_k$, the optimal estimate at $t_{k+1}$ is given by:

$$\hat{\mathbf{q}}_{k+1|k} = e^{\hat{\mathbf{\Omega}}_k \Delta t}\hat{\mathbf{q}}_{k|k} \tag{24}$$

Now, substituting Eq. (22) and Eq. (24) in Eq. (3) and isolating the quaternion error, results:

$$\delta\mathbf{q}_{k+1|k} = e^{\hat{\mathbf{\Omega}}_k \Delta t}\delta\mathbf{q}_{k|k} - \frac{\Delta t}{2} e^{\hat{\mathbf{\Omega}}_k \Delta t}.\hat{\mathbf{\Xi}}_k.\boldsymbol{\varepsilon}_k \tag{25}$$

which is the sought linear discrete-time state model. The covariance of the noise term of Eq. (25) is given by Eq. (23).

Markley (2004) shows that if the true quaternion has unit norm and the quaternion error is not zero, then the norm of the estimate must be less than unity. A simple way to treat this problem is realized simply dividing the estimate by its Euclidian norm after measurement updates. It can be shown that, for a first-order approximation, the covariance of the estimation error is not affected by such operation (Bar-Itzhack and Oshman, 1985).

### 2.2. Angular Velocity Estimator

From vector kinematics, it is well-known that the time derivatives of the true vectors $\mathbf{b}_i^o$ and $\mathbf{r}_i^o$ are related by:

$$\mathbf{A}(\mathbf{q}).\dot{\mathbf{r}}_i^o = \dot{\mathbf{b}}_i^o + \boldsymbol{\omega} \times \mathbf{b}_i^o \tag{26}$$

where $\boldsymbol{\omega}$ and $\mathbf{A}(\mathbf{q})$ are true values of the angular velocity and attitude matrix of $S_B$ with respect to $S_R$, respectively. This expression may be rewritten in the following form:

$$\dot{\mathbf{b}}_i^o = [\mathbf{b}_i^o \times]\boldsymbol{\omega} + \mathbf{A}(\mathbf{q}).\dot{\mathbf{r}}_i^o \tag{27}$$

Considering the two vector observations used in this work (direction of the sun and geomagnetic field), the augmented form of Eq. (27) at time $t_k$ is given by:

$$\dot{\mathbf{s}}^o(t_k) = \mathbf{B}^o(t_k).\boldsymbol{\omega}(t_k) + \mathbf{U}^o(t_k) \tag{28}$$

where

$$\mathbf{s}^o(t_k) = \begin{bmatrix} \mathbf{b}_1^o(t_k) \\ \mathbf{b}_2^o(t_k) \end{bmatrix} \tag{29}$$

$$\mathbf{B}^o(t_k) = \begin{bmatrix} [\mathbf{b}_1^o(t_k)\times] \\ [\mathbf{b}_2^o(t_k)\times] \end{bmatrix} \tag{30}$$

$$\mathbf{U}^o(t_k) = \begin{bmatrix} \mathbf{A}(\mathbf{q}(t_k)).\dot{\mathbf{r}}_1^o(t_k) \\ \mathbf{A}(\mathbf{q}(t_k)).\dot{\mathbf{r}}_2^o(t_k) \end{bmatrix} \tag{31}$$

Now, considering the measured vectors $\mathbf{b}_i$ and $\mathbf{r}_i$, rather than their true values, and further assuming non-colinearity (Bar-Itzhack, 2001) of the generalized vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, then one estimate of the angular velocity are given by:

$$\hat{\boldsymbol{\omega}}(t_k) = \mathbf{B}^{\#}(t_k).[\,\dot{\mathbf{s}}(t_k) - \mathbf{U}(t_k, \mathbf{q}(t_k))\,] \tag{32}$$

where $\mathbf{B}^{\#}(t_k) = (\mathbf{B}^T(t_k).\,\mathbf{B}(t_k))^{-1}.\,\mathbf{B}^T(t_k)$ is the pseudo-inverse of $\mathbf{B}(t_k)$.

The estimate dependence on the unknown true attitude is explicitly shown in Eq. (32). In this case, the estimated value will be used. Another observation about this equation is that the time derivatives of the noisy vectors introduce a significant amount of high-frequency noise in the estimate, which needs to be suppressed. This will be done here by an EKF.

In this case, the measurement model is simply that given in Eq. (17), which is rewritten as:

$$\hat{\boldsymbol{\omega}}(t_{k+1}) = \mathbf{I}_3.\hat{\boldsymbol{\omega}}(t_{k+1}) + \boldsymbol{\varepsilon}(t_{k+1}) \tag{33}$$

where $\boldsymbol{\varepsilon}(t_{k+1})$ is the error related to the estimates given by Eq. (32), which is modeled here by a zero-mean white noise process.

The state model consists of the dynamic equations of motion (Wertz, 1978), which for a rigid satellite without wheels is as follows:

$$\dot{\boldsymbol{\omega}} = \mathbf{J}^{-1}.[\mathbf{J}\boldsymbol{\omega}\times].\boldsymbol{\omega} + \mathbf{J}^{-1}.\mathbf{T} + \mathbf{n} \tag{34}$$

where $\mathbf{J}$ is the inertia tensor, $\mathbf{T}$ is the known control torque, and $\mathbf{n}$ is the state noise that emerges due to unmodeled disturbance torques. This noise will be modeled as white process with mean and covariance given by:

$$E[\,\mathbf{n}(t)] = 0 \tag{35}$$

$$E[\,\mathbf{n}(t).\mathbf{n}(t)^T\,] = \mathbf{Q}^n(t) \tag{36}$$

## 3. SIMULATION RESULTS

The results were obtained from simulated measurements. The vectors in the reference frame, $\mathbf{R}(t)$, were calculated using a Keplerian model of the sun position and a forth-degree model of the geomagnetic field (Wertz, 1978). The true position of the satellite was propagated using the SGP4 model (Hoots and Roehrich, 1980), while the correspondent design model was a simple Keplerian propagator (Wiesel, 1997). The following indices were used to evaluate performance in terms of convergence and orthogonality:

$$J_{q,k} = trace\{(\mathbf{A}(\hat{\mathbf{q}}_{k|k}) - \mathbf{A}(\mathbf{q}(t_k)))^T.(\mathbf{A}(\hat{\mathbf{q}}_{k|k}) - \mathbf{A}(\mathbf{q}(t_k)))\} \tag{37}$$

$$J_{w,k} = (\boldsymbol{\omega}(t_k) - \hat{\boldsymbol{\omega}}(t_k))^T.(\boldsymbol{\omega}(t_k) - \hat{\boldsymbol{\omega}}(t_k)) \tag{38}$$

$$F_{q,k} = trace\{(\mathbf{A}(\hat{\mathbf{q}}_{k|k})^T.\mathbf{A}(\hat{\mathbf{q}}_{k|k}) - \mathbf{I}_3)^T.(\mathbf{A}(\hat{\mathbf{q}}_{k|k})^T.\mathbf{A}(\hat{\mathbf{q}}_{k|k}) - \mathbf{I}_3)\} \tag{39}$$

where $J_{q,k}$ measures attitude convergence, $J_{w,k}$ measures angular velocity convergence, and $F_{q,k}$ refers to the normality of the quaternion estimates.

The dynamics of the simulated attitude motion refers to a rigid cube satellite modeled as in Eq. (34). The inertia tensor was assumed as $\mathbf{J}$ = *diag{[6.5 6.5 8.0]}* $kg.m^2$ for design model and $\mathbf{J}$ = *diag{[6.4 6.6 7.8]}* $kg.m^2$ for the true model. For simplicity, the control torque $\mathbf{T}$ was considered null. The disturbance torques due to gravity-gradient and residual magnetism was simulated in the true model, while in the design dynamic model the disturbances were considered a white noise, as mentioned before.

A Monte-Carlo simulation consisting of 100 runs was carried out for each of the two algorithms. Ensemble averages of the above indices were computed and are presented in the figures below. In each run, the initial velocity estimate was a Gaussian distributed random vector in $R^3$ with null mean and covariance given by $diag\{[0.25\ 0.25\ 0.25]\}\ (rad/s)^2$ and the initial attitude was obtained by normalizing the sum of $\mathbf{q}_m=[0\ \ 0.7071\ \ 0\ \ 0.7071]^T$ and a Gaussian vector in $R^4$ with null mean and covariance *diag{[0.5 0.5 0.5 0.5]}*. On the other hand, the initial true velocity was $\boldsymbol{\omega}_o= [0.5\ 0.4\ {-0.1}]^T$ *rad/s* and the initial true attitude $\mathbf{q}_o=[1\ 0\ 0\ 0]^T$ for every run. The motion is simulated for a low earth orbit with zero eccentricity and 25 degree of inclination.

Figure 2 shows that the algorithm using the so called "brute-force" normalization is slightly better in terms of convergence rate and accuracy. The averaged angular velocity index is presented in Fig. 3, which shows again a better performance of the normalized quaternion algorithm. Finally, as expected, the normality index for the normalized algorithm is significantly smaller, as shown in Fig. 4 and Fig. 5.

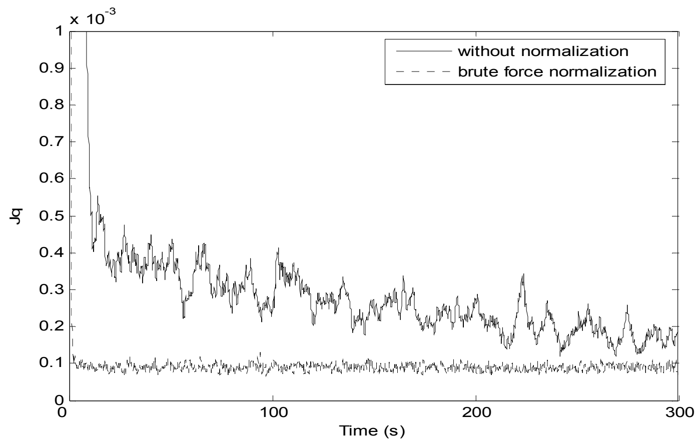


Figure 2. Attitude convergence index for both algorithms: with and without normalization

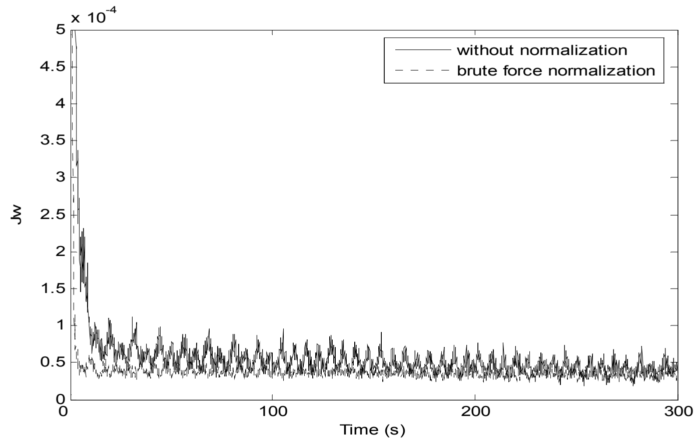


Figure 3. Angular velocity convergence index for both algorithms: with and without normalization

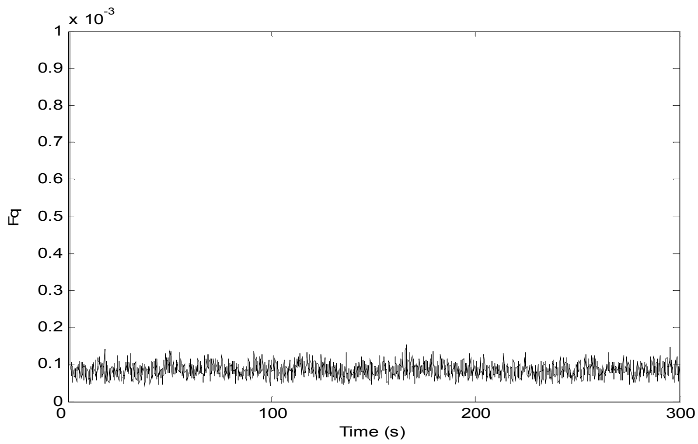


Figure 4. Attitude orthogonality index for the algorithm without normalization

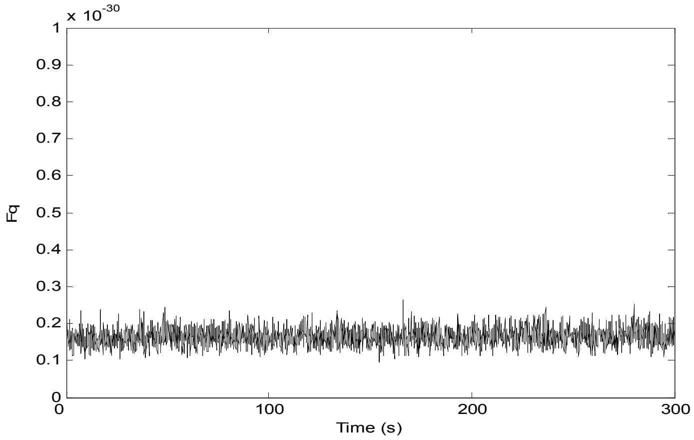


Figure 5. Attitude orthogonality index for the algorithm with brute force normalization

## 4. CONCLUSION

An intertwined scheme for attitude and angular velocity estimation from observations of sun and geomagnetic field vectors was investigated. It was shown that the "brute force" normalization used in the attitude estimator improves quaternion normality, convergence rate and accuracy. Moreover, the intertwined estimation scheme showed robustness with respect to the initialization of the estimation process, which obviated the need for a customized initialization.

Control systems in low-cost satellites avoid using gyros and then rely only on vector measurements for attitude determination. In this way, the scheme presented here is intended for use in the ITASAT student satellite.

## 5. ACKNOWLEDGEMENTS

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