OBTAINING MEAN FREQUENCY RESPONSE FUNCTIONS IN THE PRESENCE OF UNCERTAINTY

Lázaro Valentim Donadon, lazaro@ufmg.br
Departamento de Engenharia Mecânica, Escola de Engenharia, Universidade Federal de Minas Gerais, Avenida Antônio Carlos 6627, 31.270-901 – Belo Horizonte, Brazil

Diego Azevedo Siviero, diegosiv@fem.unicamp.br
Departamento de Mecânica Computacional, Faculdade de Engenharia Mecânica, Universidade Estadual de Campinas, Caixa Postal 6122, 13.083-970 – Campinas, Brazil

José Roberto de França Arruda, arruda@fem.unicamp.br
Departamento de Mecânica Computacional, Faculdade de Engenharia Mecânica, Universidade Estadual de Campinas, Caixa Postal 6122, 13.083-970 – Campinas, Brazil

Abstract. *This work deals with how to obtain average Frequency Response Functions (FRF) from a set o measurements of nominally identical structures or a set of measurements of the same structure under varying environmental conditions for robust forced response predictions or control synthesis. It is shown that if the averages are performed over the FRF values directly the results are wrong in the sense that the order of the model is artificially increased. Instead, the averages must be computed in the modal domain, i.e., over the natural frequencies, modal damping and mode shapes. The method proposed here consists of performing a modal parameter estimation using the Orthogonal Chebyshev Polynomials Method and then computing the average of the modal parameters. The average FRF obtained by synthesis using the average modal parameters preserve the same order of each sample of the measured FRF. The method is first illustrated using a simple numerical example and then applied to a real plate for which an active noise controller is being implemented.*

Keywords*: Modal Analysis, Modal Synthesis, and Orthogonal Chebyshev Polynomials*

1. INTRODUCTION

In some cases, when performing system identification procedures, it is observed some variation in the Frequency Response Function (FRF). These variations can show up due to small variation in the system measurement, such as sensor and actuator positions or setup changes from one experiment to another. Besides, depending on the system, it is possible to observe variation from one measurement to another even when there are no changes in the experimental setup, which possibly means that the environmental conditions may affect the measurements. The variations mentioned above are reflected in the FRF amplitudes, natural frequencies, and damping factors, which generate FRFs slightly different from one another.

Another useful aspect is in industrial manufacturing, where it is necessary to identify experimental FRFs in order to validate theoretical models. However, when treating series manufactured products, which means that thousands of samples are being produced, variations are found from one product to another due to assembling tolerances, temperature variations, changes in supplied raw materials, etc.

In some cases it is advantageous to access an average FRF. In this case, to take an average directly over the FRF will not produce a reasonable result because the small variation in the natural frequencies will be seen in the average as an increase in the damping factor or an increase in the order of the system. One solution is to perform an identification of the modal parameters for each sample, and perform an average in the modal parameter - instead of performing directly in the FRFs - and then to reconstruct the FRF from the averaged modal parameter. This technique is discussed in this work.

2. ORTHOGONAL CHEBYSHEV POLYNOMIALS METHOD

Among the available modal parameter extraction methods, the Orthogonal Chebyshev Polynomials method, ORTPOLY, seems to be one of the most appropriate, as it is a frequency domain method which does not require equally-spaced frequency lines. Therefore, the user may choose the number and value of the frequencies where the response is measured, thus reducing the computational effort.

Assuming that the experimental FRF matrix $H(\omega)$, are measured for *m* inputs and *p* outputs, the dimension of the matrix $H(\omega)$ is $(p\times m)$. The modal parameter extraction method starts with the assumption that the FRF, matrix $H(\omega)$, can be expressed as a ratio of the matrix polynomials,

$$[H(\omega)] = \left[\sum_{k=0}^{s}[b_k]\phi_k(i\omega)\right]^{-1}\left[\sum_{k=0}^{r}[a_k]\phi_k(i\omega)\right] \tag{1}$$

where ϕ_k is the kth Chebyshev polynomial, $[a_k]$ is the k^{th} polynomial matrix coefficient, of same dimension as H, and $[b_k]$ is the k^{th} denominator polynomial matrix coefficient, which is a scalar for single excitation and a matrix (m×m) in the case of multiple excitation. Without loss of generality, making b_0 = I and rearranging Eq. (1) as,

$$\sum_{k=1}^{s-1} H(\omega)^H \phi_k^* b_k^H - \sum_{k=0}^{r}\phi_k^* I a_k^H = -\phi_s^* H(\omega)^H \tag{2}$$

where ^{H} represents the conjugate transpose.

Arranging Eq. (2) in matrix form for a varying ω, yields an overdetermined linear system of equations, which can be solved for $[a_k]$ and $[b_k]$ in a least-squares sense. It is important to mention that the frequency ω should be normalized in order to improve the condition of the least-squares problems, [Kelly 1967].

After the polynomial fit has been performed, it is proposed to form a companion matrix problem in the orthogonal polynomial basis can be assembled; see Arruda et al. (1996) for more details. The modified companion matrix can be formed combining the characteristic equation, which in the case of multiple references is given by,

$$\left[\sum_{k=0}^{s-1}[b_k]\phi_k(i\omega)\right]\{V\} = \{0\} \tag{3}$$

The formula to generate the orthogonal Chebyshev polynomials is defined as,

$$\phi_k(i\omega) = 2i\omega\phi_{k-1}(i\omega) - \phi_{k-2}(i\omega) \tag{4}$$

Solving the combined eigenproblem formed in the usual way, a companion matrix is formed, and the eigenvalues s_r, r=1,...,sm are obtained. The first m elements of the eigenvectors of the associated problem, V_r, are the so-called modal participation vectors L_r of the partial expression of $H(\omega)$, given by,

$$H(\omega) = \sum_{r=1}^{Nm}\psi_r\left(\frac{L_r^T}{i\omega - s_r}\right) + \psi_r^*\left(\frac{L_r^H}{i\omega - s_r^*}\right) \tag{5}$$

where $Nm = sm$ is the number of modes and ψ_r are the mode shapes.

Once the modal participation vectors and the eigenvalues are known, computing the mode shapes is straightforward. Eq. (5) can be rearranged for each element of matrix H, corresponding to each degree of freedom, varying with frequency arranged in vector form to yield,

$$\tilde{H}_0 = \psi_0 B^T \tag{6}$$

where

$$B = \begin{bmatrix} LS_1 \\ LS_2 \\ \vdots \\ LS_{ns} \end{bmatrix} \;;\; L = \begin{bmatrix} L_1 & L_2 & \cdots & L_{Nm} \end{bmatrix} \tag{7}$$

$$S_i = diag\left[(i\omega_i - s_1)^{-1} \quad (i\omega_i - s_2)^{-1} \quad \cdots \quad (i\omega_i - s_{Nm})^{-1}\right] \tag{8}$$

where ω_i is the i^{th} frequency line, ns is the number of frequency lines and ψ_0 is a vector with components corresponding to the 0th degree of freedom of all the identified mode shapes. Solving the linear system of equations in Equation (6) for each element of matrix $H(\omega)$ yields the mode shapes.

The final result of this procedure is to extract from the FRFs in matrix $H(\omega)$ the eigenfrequencies, $\omega_r = |S_r|$, $r=1\ldots,sm$, the modal damping factors $\xi_r = \Im(S_r)/|S_r|$, $r=1,\ldots,sm$ and the eigenmodes ψ_r, $r=1,\ldots,sm$. Repeated eigenvalues with multiplicity up to m may be identified.

3. PERFORMING THE AVERAGE

The first step to perform an average in the FRFs is to perform a modal parameter estimation using the method described in the previous section for each FRF separately. Once performed the modal parameter estimation, and consequently having accessed the mode shapes ψ_r, the natural frequencies and the modal damping, both represented by S_r, and the modal participation vectors L_r, the average is performed for each mode. Supposing N FRFs, the averages are done by,

$$\tilde{\psi}_r = \frac{1}{N}\sum_{i=1}^{N}\psi_r \; ; \; \tilde{S}_r = \frac{1}{N}\sum_{i=1}^{N}S_r \; ; \; \tilde{L}_r = \frac{1}{N}\sum_{i=1}^{N}L_r \tag{9}$$

Thus, the average FRF is given by Eq. (5) in the following form,

$$\tilde{H}(\omega) = \sum_{r=1}^{Nm}\tilde{\psi}_r\left(\frac{\tilde{L}_r^T}{i\omega - \tilde{s}_r}\right) + \tilde{\psi}_r^*\left(\frac{\tilde{L}_r^H}{i\omega - \tilde{s}_r^*}\right) \tag{10}$$

4. THEORETICAL MODEL

The theoretical model consists of an aluminum clamped-clamped beam, Figure 1, Elasticity Modulus E = 70 GPa, density ρ = 2800 kg/m^3, modal damping coefficient η = 0.005, which was assumed equal for all modes and introduced in the model by making the Elasticity Modulus complex as E(1+jη).The variations in the theoretical model were introduced in the geometrical characteristic, i.e., the base b = 1 mm was constant while the thickness used was h = 3 mm for the first sample, h = 2.95 mm for the second sample, and h = 3.05 mm for the third sample.

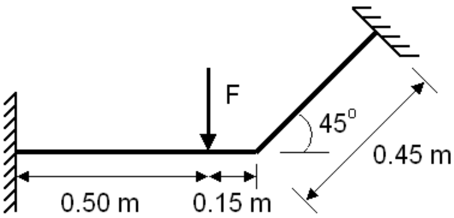

Figure 1: Schematic diagram of the beam used in the simulation

The theoretical model was simulated using the Bernoulli-Euler beam theory whose model was implemented using the Spectral Element Method (SEM), [Doyle 1997], which consists of using the analytical solution of the wave equation written in a matrix form such as the Finite Element Method (FEM). As it uses the analytical solution to the wave equation, it is necessary only one element in between impedance changes. The receptance measured at the driving point is shown in Figure 2.

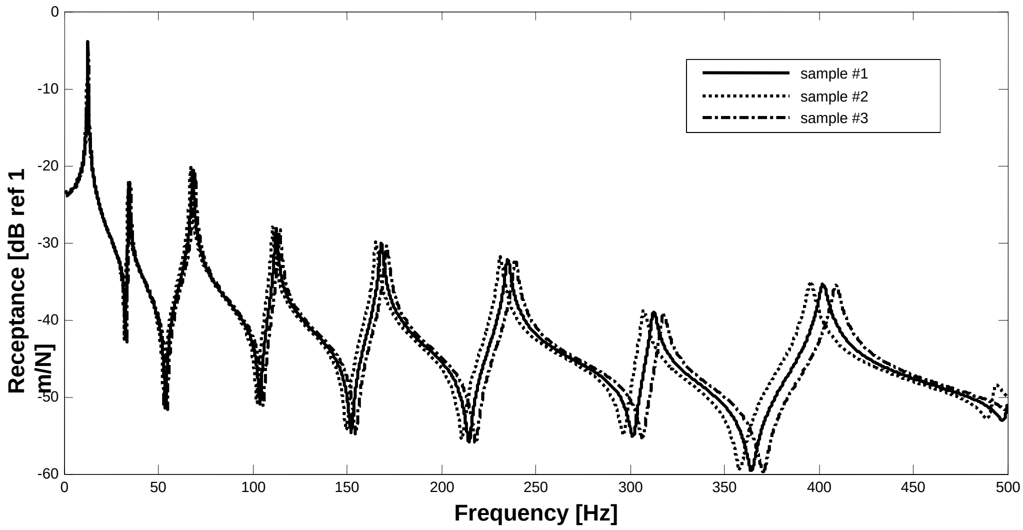

Figure 2: Receptance measured at the same point of the excitation.

The first step to generate the averaged FRF is to perform the individual identification to each sample separately using the procedure presented in section 2 and then, it is performed the average following the procedure presented in section 3. The way that the samples were chosen, the sample #1 can be considered as the mean value and it will be used as the objective of the averaged FRF and at this point it will be named as the nominal FRF. The results of the identifications can be seen in Table 1 and in Figure 3. The identified modal damping is expressed in terms of an equivalent loss factor, which is twice the value of the identified viscous damping factor, i.e., $\eta_r = 2\xi_r$. For small damping factors, this factor is known to be approximately equal to the material loss factor imposed in the complex elasticity modulus.

Table 1: Comparison of the modal parameters in between the nominal FRF and the reconstructed one.

Nominal FRF (sample #1)		**Average Identified Model**	
Natural Frequency [Hz]	Structural Damping coefficient	Natural Frequency [Hz]	Structural Damping coefficient
12.69	0.005	12.69	0.0051
34.67	0.005	34.67	0.0049
68.31	0.005	68.30	0.0049
112.53	0.005	112.66	0.0047
168.27	0.005	168.11	0.0051
235.27	0.005	235.30	0.0049
312.29	0.005	312.28	0.0050
402.38	0.005	402.37	0.0051

As can be seen in Table 1, the modal parameters of the Nominal FRF, sample #1, are practically the same parameters identified in the Averaged Model which means that the method used here can be applied in practical situations for extracting averaged modal parameters. Besides, Figure 3 shows that numerical FRF of the Nominal FRF (sample #1) matches in amplitude and phase with the Average Identified Model. Therefore, it is expected that the method proposed here can perfectly predicted an averaged model for experimental data.

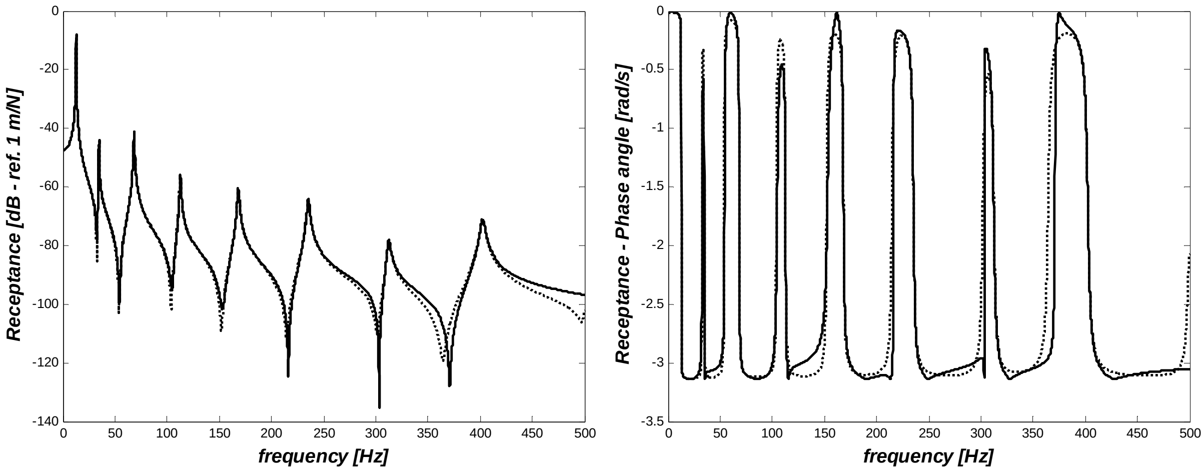

Figure 3: Comparison of the receptances: “ ··” nominal model (sample #1) and “---” reconstructed one.

In order to demonstrate the method, in Figure 4 it is shown a “zoom” to closely see what happens if the average is performed directly in the FRFs. Clearly it is seen that this approach is not reasonable due to the fact that the average directly over the FRFs leads to three peaks and three phase changes in the average FRF in contrast to the method proposed here that leads to practically the correct FRF.

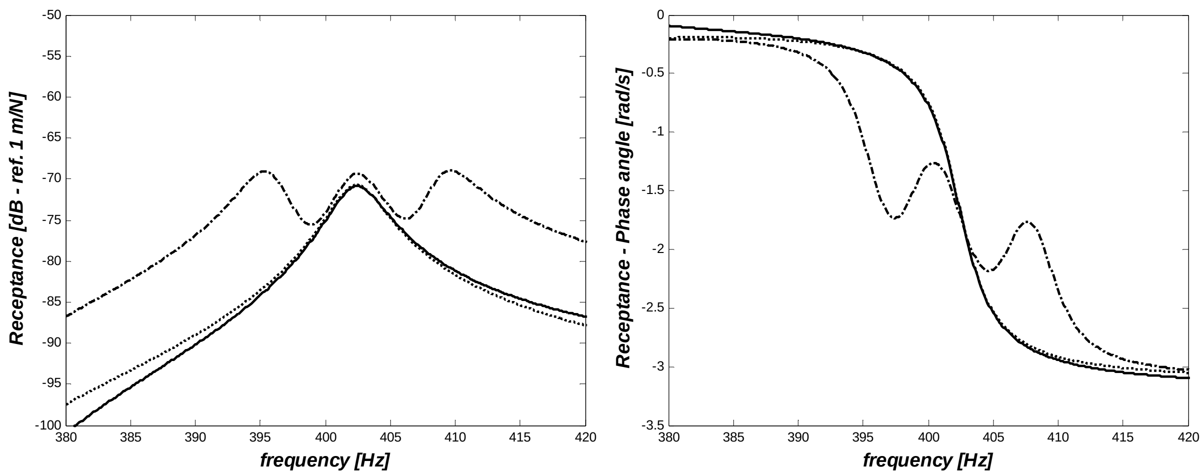

Figure 4: “Zoom” in the comparison of the receptances between the “···”nominal model (sample #1), “---” the reconstructed one, and the “-·-·-·” average model with averaged FRF amplitudes.

5. EXPERIMENTAL MODEL

Now, the method will be applied in a more realistic system, which consists of a clamped panel made of Lexan, fixed in a rigid baffle excited by a punctual force radiating sound to free-field. A microphone is placed 20 mm far away from the panel in order to measure the pressure at this point, Figure 5 and Figure 6. For more explanation about this system, see Donadon et al. 2006.

The punctual force was applied by the shaker model V201/3 and a power amplify model PA25E, both from LDS. The force was measured by a force transducer, model 208A02 from PCB. The pressure was measured by an omni directional microphone of ¼”, model 130A10, using a pre-amplifier model 130P10, both from PCB. The Frequency response functions were performed using a spectral analyzer, model 35650A from Hewlett Packard, and the data was acquired using 30 averages, Hanning window, acquisition frequency of 1024 Hz with 4096 frequency lines, and the it was used the H1 estimator (Ewins, 1988).

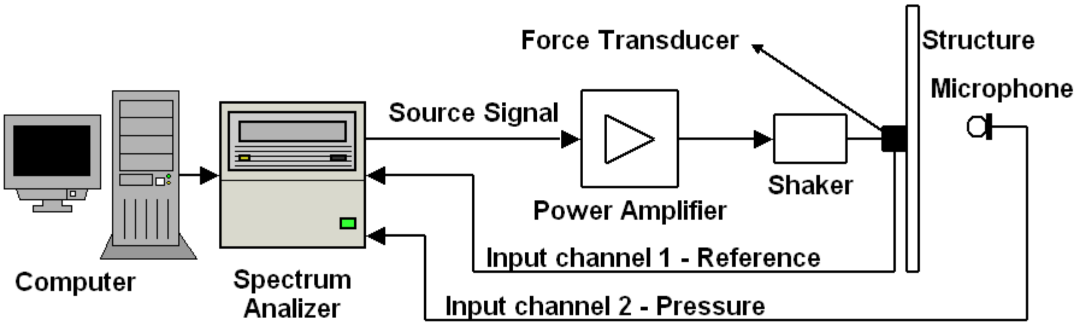

Figure 5: Schematic diagram of the experimental setup

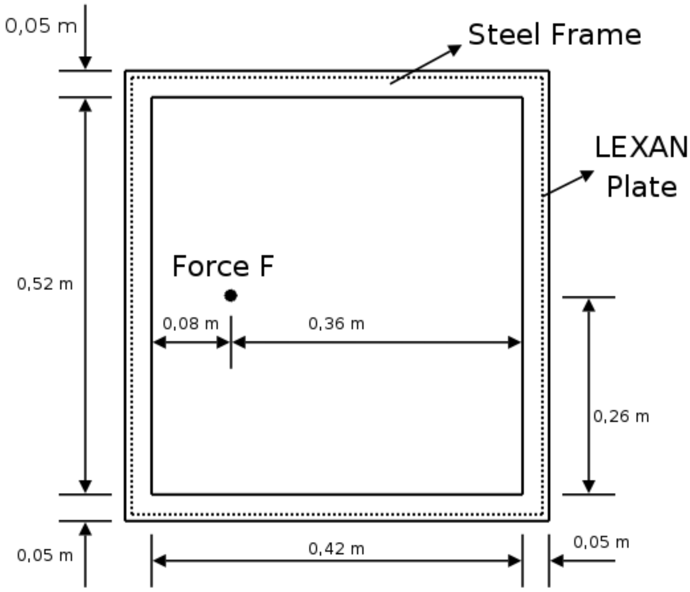

Figure 6: Schematic diagram of the lexan panel.

The variations found in the FRFs are related to the variations in the environment temperature, which was controlled between 21^o and 24^o degrees Celsius. This control was performed by the operator observing an electronic thermometer placed near the panel, and more than 30 measurements were performed during 3 months. However, only the 10 most significant FRFs were used in the estimation of an averaged model. It must be mentioned that more accurate was the temperature control, the closer the FRFs were. The results for this example are shown in Figure 7, where it can be seen that the proposed method could identify an average FRF over the experimental measurements. However, it must be said that the weaker the mode coupling, the better the average identification, which can be shown by observing the four peaks between 350 Hz and 650 Hz, in contrast with the peaks around 300 Hz. Another observation is that the average FRF tends to smear out the anti-resonances.

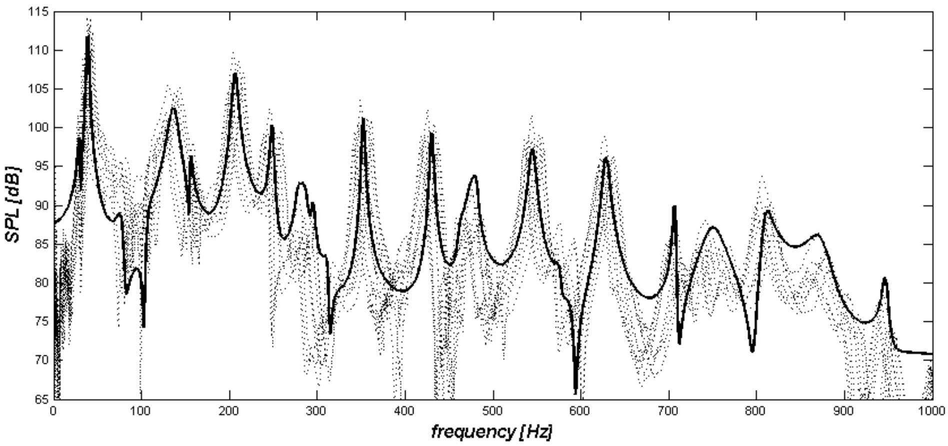

Figure 7: Comparison between ' ···' the measured FRFs and "–" the average FRF.

6. FINAL REMARKS

It was explained how to correctly perform an average in experimental Frequency Response Functions, where it can be done performing a modal parameter estimation and then performing an average in the estimated parameters. However, it can be observed that following the proposed procedure, anti-resonances tend to smear out. Besides, the user must accurately separate the modes, meaning that for large variations in the FRFs, the method will probably not work properly.

7. ACKNOWLEDGEMENTS

The authors wish to thank the CNPq, CAPES, FAPESP, and FAPEMIG for the financial support.

8. REFERENCES

Arruda, JRF, Rio, SAV, Santos, LASB. 1997. "A space-frequency data compression method for spatially dense laser Doppler vibrometer measurements". Acta Acustica, Vol. 83, pp. 465-474

Donadon, LV, Siviero, DA, Camino, JF, Arruda, JRF. 2006. "Comparing a Filtered-X LMS and an H_2 Controller for the Attenuation of the Sound Radiated by a Panel". Proceeding of the International Conference on Noise & Vibration Engineering (ISMA), Leuven, Belgium.

Doyle, JF. 1997. "Wave propagation in structures: spectral analysis using fast discrete Fourier transforms", 2nd ed., Ed. Springer-Verlag.

Kelly, LG 1967. "Handbook of Numerical Methods and Applications". Ed Addison-Wesley.

Ewins, DJ 1988. "Modal Testing: Theory and Practice". Ed Jhon Wiley & Sons.

9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.