STIFFNESS ANALYSIS OF A 6-RSS PARALLEL MANIPULATOR

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Abstract. *This paper describes the stiffness analysis of a 6-RSS parallel manipulator. The 6-RSS kinematic chain is composed by six legs where the axis of the actuators are placed two-by-two along the three axis of a Cartesian reference frame. The manipulation performances of a robot are directly affected by the stiffness structure because external forces and moments may cause large deflections in the links and joints, which are undesirable from the viewpoint of both accuracy and payload performances. The stiffness analysis is based on standard concepts of static elastic deformations. Its analysis can be carried through from the stiffness matrix that can be numerically computed by defining a suitable model of the manipulator, which takes into account the stiffness properties of each element such as links, actuators and joints. In this paper the Jacobian matrix is obtained from the 6-RSS structure kinematic model and then, its stiffness matrix too. A general scheme of the strucuture considering all elements stiffness (links and joints) is presented. In order to simplify the model, the joints stiffness have been neglected. The obtained stiffness matrix is used to map the end-effector displacement when external wrench are applied on it. The used methodology can be applied to analyze the stiffness of other robots structures.*

Keywords*: Parallel Manipulators, stiffness analysis, compliance matrix.*

1. INTRODUCTION

A load applied on a body produces changes on its geometry that are known as deformations or compliances displacements. Stiffness can be defined as the capacity of a mechanical system to sustain loads without excessive changes of its geometry (Rivin, 1999). Moreover, the stiffness of a body can be defined as the amount of force that can be applied per unit of compliance displacement of the body (Nof, 1985), or the ratio of a steady force acting on a deformable elastic medium to the resulting displacement (AccessScience, 2003).

Compliance displacements in a parallel robotic system improve for mechanical float of the end-effector relative to the fixed base. This produces negative effects on static and fadigue strength, wear resistance, efficiency (friction losses), accuracy, and dynamic stability (vibration). The increasing importance of high accuracy and dynamic performance for parallel robotic systems has increased the use of high strength materials and lightweight designs by giving significant reduction of cross-sections and weight. Nevertheless, these solutions also increase structural deformations and may result in intense resonance and self-excited vibrations at high speed (Rivin, 1999). Therefore, a study of stiffness becomes of primary importance in the design of multibody robotic systems in order to properly choose materials, component geometry, shape and size, and interaction of each component with others. Some examples of design procedures based on stiffness analysis can be found in (Liu *et al.*, 2000) (Simaan and Shoham, 2002) (Carbone *et al.*, 2003). In particular, Liu et al. propose an optimum design procedure for 3 degree of freedom spherical parallel manipulators as based on the computation of conditioning and stiffness indices. Simaan and Shoham (2002) propose an optimum design procedure for the synthesis of a planar robot having variable geometry as based on stiffness performance. Carbone et al. (2003) propose a procedure for optimal design of a biped leg module for humanoid robots by considering stiffness performance and lightweight design.

A stiffness analysis can be also very useful for estimating the expected performance of a system in term of payload and accuracy and for verifying the feasibility of specific tasks as proposed for example in (Pai and Leu, 1991) (Carbone *et al.*, 2002) and (Ceccarelli *et al.*, 2002).

The overall stiffness of a manipulator depends on several factors, including the size and material used for the links, the mechanical transmission mechanisms, the actuators, and the controller (Tsai, 1999). In general, to realize a high stiffness mechanism, many parts should be large and heavy. However, to achieve high speed motion, these should be small and light. Moreover, one should point out that the stiffness is greatly affected by both the position and the values of the mechanical parameters of the structures parts (Yoon *et al.*, 2004).

For the stiffness analysis is necessary obtaining the stiffness model of the parallel structure. In the literature there are three main methods have been used to derive the stiffness model of parallel manipulators (Deblaise *et al.*, 2006). These methods are respectively based on: the calculation of the Jacobian matrix (Gosselin, 1990) (Khasawneh and Ferreira, 1999) (Zhang *et al.*, 2004) (Zhang, 2004) (Gosselin and Zhang, 2002) (Majou *et al.*, 2004) (Company *et al.*, 2005); the Finite Element Analysis (Corradini *et al*., 2004) (Bouzgarrou *et al.*, 2004) and the matrix structural analysis (Huang *et al.*, 2002) (Li *et al.*, 2002) (Martin, 1966) (Wang, 1966) (Imbert, 1979) (Clinton *et al.*, 1997) (Dong *et al.* ,2005)

The methods based on calculation of the Jacobian matrix are simple and they supply one initial estimate of the stiffness matrix. One of the first stiffness analysis has been conducted by Gosselin (1990) and the stiffness of a Parallel Kinematic Machines is mapped onto its actuated joints whereas links are supposed as perfectly rigid. This approach was also involved by Khasawneh and Ferreira (1999) and Zhang et al. (2004). To take into account the link flexibility, this first model was supplemented by the lumped model developed by Zhang (2004). Flexible links are then replaced by rigid beams mounted on revolute joints plus torsional spring located at the existing joints.

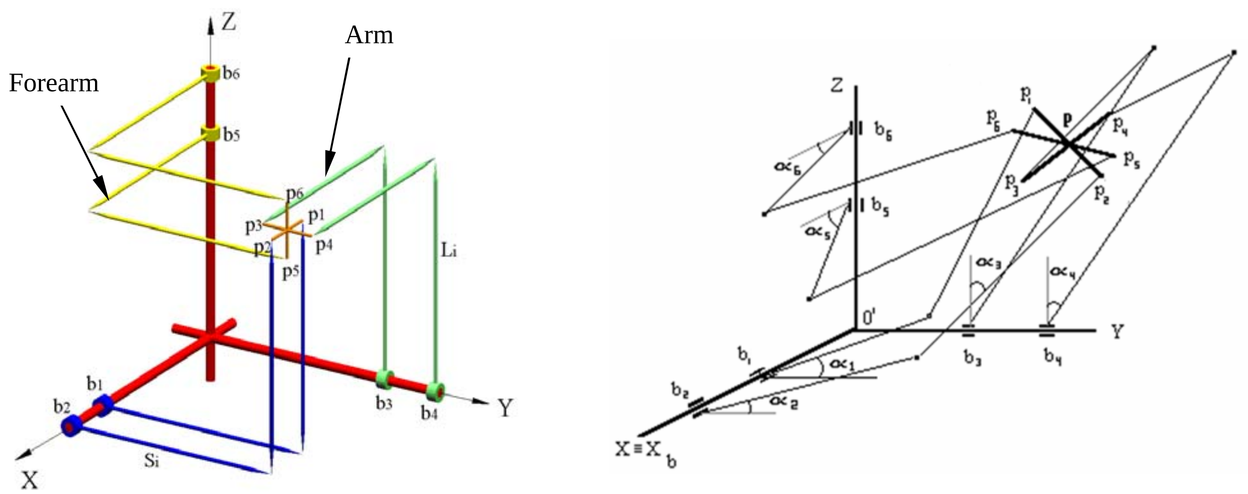
The uses of Finite Element Analysis models are reliable, but these models have to be remeshed over again, these results in very tedious and time-consuming routines (Huang *et al.*, 2002). However these models are well adapted to validate analytical models (Khasawneh and Ferreira, 1999) (Li *et al.*, 2002), or some experimental results (Corradine *et al.*, 2004). Bouzgarrou *et al.* (2004) used the Finite Element Analysis model to study static rigidity and natural frequencies of the T3R1 parallel robot.

Another methods based on matrix structural analysis are not so often used. Clinton *et al.* (1997) used this approach to derive the stiffness matrix for each element of a Stewart platform and then assemble individual ones into a system stiffness matrix. This approach is also used in Huang *et al.* (2002) to give the stiffness model of a machine frame considered as a substructure. The superposition principle is used to achieve the stiffness model of the machine structure as a whole.

In this paper is presented the calculated stiffness matrix of the 6-RSS parallel manipulator. This matrix can be computed based on lumped stiffness parameters of the links. The Jacobian matrix is obtained from the kinematic model of the 6-RSS parallel manipulator and then the stiffness matrix is calculated. The end-effector output forces are related to its deflections by the stiffness matrix. With the stiffness matrix one do the map of the compliance displacement of end-effector as function of the external efforts applied on its center, and in this paper it is carried through for different configurations of 6-RSS parallel manipulator.

2. THE 6-RSS PARALLEL ARCHITECTURE

The 6-RSS parallel architecture is a 6 degree of freedom manipulator, which is characterized by a base and a platform (end-effector), connected by six *RS-SS* segments, where the R-joint is on the Cartesian axes, two joints by axis, as shown in Fig. 1. The Cartesian system is the base of the structure where the actuators are mounted, reducing dead loads. The links can be constructed by light and resistant materials that make then rigid, with low inertia and in a modular construction. The *S*-joints on the other extremity of the segments are connected at the end-effector, consisting in a virtual cube where the *S*-joints are tied on the center of its faces, having then, three crossed segments. Kinematics variables are the input angles α_i (*i=1 to 6*) in the *R*-joints. The studied structure has the *RS*-segment and the *SS*-segment the same length and: $|b_1b_2| = |b_3b_4| = |b_5b_6| = |p_1p_2| = |p_3p_4| = |p_5p_6|$, Fig. 1.



(a) (b)

Figure 1. (a) Initial configuration of the 6-RSS parallel architecture; (b) Generic configuration.

Figures 2 and 3 show the prototype constructed at the Laboratory of Automation and Robotics in Uberlândia/Brazil.

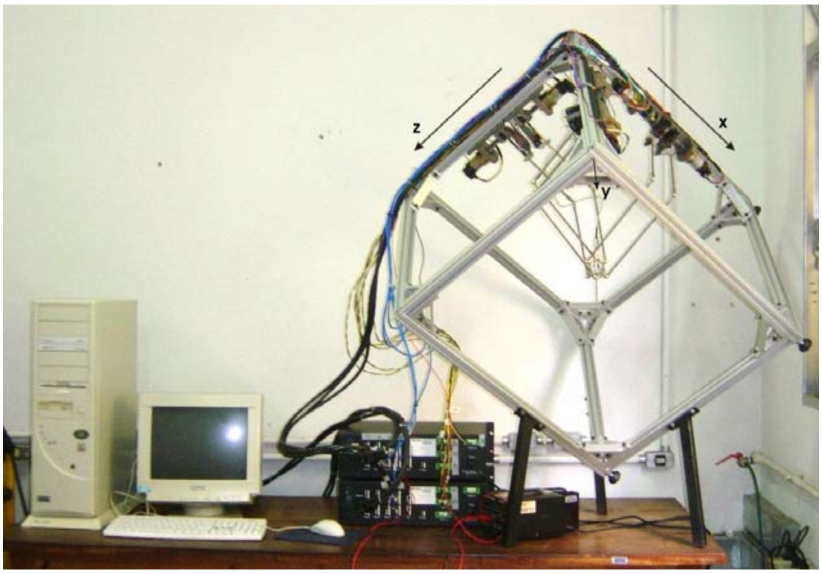
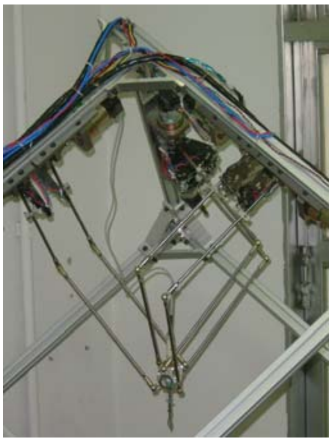
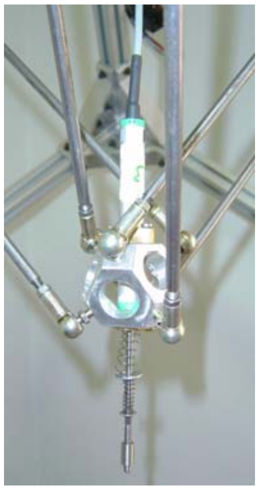


Figure 2. The built prototype of the 6-RSS Cartesian parallel manipulator.



(a) Detail of legs and actuators.



(b) End-effector with a linear transducer.

Figure 3. Detail of the prototype of a 6-RSS Parallel Manipulator.

2.1. Inverse Kinematic Model

Kinematic equations can be obtained by using a suitable analysis procedure with a vector and matrix formulation (Bezerra, 1996). Equations can be written by considering a generic reference frame fixed to the base (0_F, x_F, y_F, z_F) and another at the platform (0_M, x_M, y_M, z_M) as shown in Fig. 4. The parameters are: α_i are the generalized coordinates; $\boldsymbol{b}_i$ the position vector of the base points b_i (position of the R-joint in the base), $\boldsymbol{S}_i$ and $\boldsymbol{L}_i$ the arm and the forearm vectors, all referred to the base reference frame, and S_i and L_i their lengths; $\boldsymbol{p}_i$ the position vector of the platform points p_i (position of the S-joint at the mobile platform), referred to the mobile reference frame. All parameters for *i=1 to 6*.

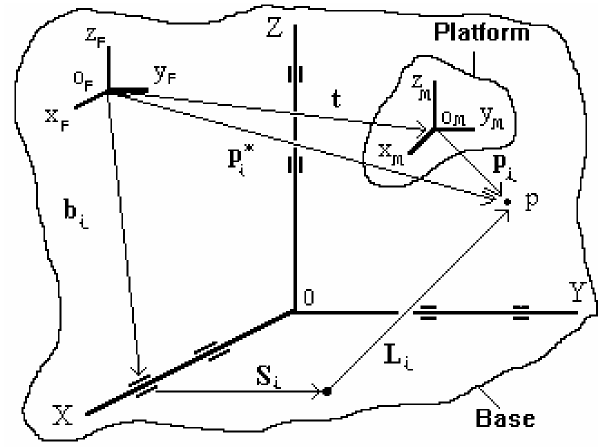


Figure 4. Vectors for modeling the mechanism.

From Figure 4 one can obtain:

$$\mathrm{L_i} = [R]\mathrm{p_i} + \mathrm{t} - \mathrm{b_i} - \mathrm{S_i} \qquad \text{for i=1 to 6} \tag{1}$$

where

$$\mathrm{b_i} = \{b_{ix} \quad b_{iy} \quad b_{iz}\}^t$$

$$\mathrm{p_i} = \{p_{ix} \quad p_{iy} \quad p_{iz}\}^t$$

$$\mathrm{S_i} = \{S_{ix} \quad S_{iy} \quad S_{iz}\}^t$$

$$\mathrm{L_i} = \{l_{ix} \quad l_{iy} \quad l_{iz}\}^t$$

$$S_i = |\mathrm{S_i}| = (S_{ix}^2 + S_{iy}^2 + S_{iz}^2)^{1/2}$$

$$L_i = |\mathrm{L_i}| = (l_{ix}^2 + l_{iy}^2 + l_{iz}^2)^{1/2}$$

$$[R] = [\mathrm{n} \quad \mathrm{o} \quad \mathrm{a}] = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix} \qquad \text{for i=1 to 6} \tag{2}$$

In Equation (2) only the length L_i of the vector $\boldsymbol{L_i}$ is known. Then, one can write:

$$\mathrm{L}_i^2 = ([R]\mathrm{p_i} + \mathrm{t} - \mathrm{b_i} - \mathrm{S_i})^2 \qquad \text{for i=1 to 6} \tag{3}$$

Developing Eq. (3), one have:

$$\begin{aligned}
&2p_{ix}[(n_x t_x + n_y t_y + n_z t_z - n_x S_{ix} - n_y S_{iy} - n_z S_{iz})] + 2p_{iy}[(o_x t_x + o_y t_y + o_z t_z - o_x S_{ix} - o_y S_{iy} - o_z S_{iz})] + \\
&2p_{iz}[(a_x t_x + a_y t_y + a_z t_z - a_x S_{ix} - a_y S_{iy} - a_z S_{iz})] + 2b_{ix}[(S_{ix} - t_x - n_x p_{ix} - o_x p_{iy} - a_x p_{iz})] + \\
&2b_{iy}[(S_{iy} - t_y - n_y p_{ix} - o_y p_{iy} - a_y p_{iz})] + 2b_{iz}[(S_{iz} - t_z - n_z p_{ix} - o_z p_{iy} - a_z p_{iz})] + \\
&(t_x^2 + t_y^2 + t_z^2) + \mathrm{p}_i^2 + \mathrm{S}_i^2 + \mathrm{b}_i^2 - 2(t_x S_{ix} + t_y S_{iy} + t_z S_{iz}) - \mathrm{L}_i^2 = 0
\end{aligned} \qquad \text{for i=1to 6} \tag{4}$$

Choosing proper reference frames, Eq. (4) can be simplified. In this work the used reference frames are: (X_b, Y_b, Z_b) at the base, where the origin coincides with point O; (X_p, Y_p, Z_p) at the center of mobile platform as represented in Fig. 1. The matrix of rotation [R] is given by the Euler angles (rotation ϕ_1 about x, rotation ϕ_2 about y and rotation ϕ_3 about z):

$$[R_{Euler}] = \begin{bmatrix} c\phi_2 c\phi_3 & -c\phi_2 s\phi_3 & s\phi_2 \\ s\phi_1 s\phi_2 c\phi_3 + c\phi_1 s\phi_3 & -s\phi_1 s\phi_2 s\phi_3 + c\phi_1 c\phi_3 & -s\phi_1 c\phi_2 \\ -c\phi_1 s\phi_2 c\phi_3 + s\phi_1 s\phi_3 & c\phi_1 s\phi_2 s\phi_3 + s\phi_1 c\phi_3 & c\phi_1 c\phi_2 \end{bmatrix} \tag{5}$$

The inverse kinematic model of the 6-RSS parallel manipulator obtains the co-ordinates α_i (i=1 to 6) when the position (t_x, t_y, t_z) and orientation (ϕ_1, ϕ_2, ϕ_3) of the platform are known. Equation (4) constitute a system with six variables: α_1, α_2, α_3, α_4, α_5 and α_6, representing the inverse kinematic model of the 6-RSS parallel manipulator. In this system, vectors $\boldsymbol{p_i}$, $\boldsymbol{b_i}$ and $\boldsymbol{t}$, the matrix [R] and lengths of $\boldsymbol{S_i}$ and $\boldsymbol{L_i}$ vectors are known. Only the generalized coordinates α_i (i=1 to 6) are unknown.

This system has 64 solutions in general. But the system solution has repeated roots, including configurations not possible to be attained by the mechanism, having then only one set of roots α_i (i=1 to 6) valid (Bezerra, 1996) (Carvalho, et al., 2001).

3. STIFFNESS ANALYSIS OF 6-RSS PARALLEL MANIPULATOR

3.1. Stiffness Model

The stiffness properties can be defined through a 6 x 6 matrix that is called as stiffness matrix K. This matrix gives the relationship between the vector of the compliance displacements $\Delta S = \{\Delta x, \Delta y, \Delta z, \Delta\phi_1, \Delta\phi_2, \Delta\phi_3\}^T$ occurring to a frame fixed at the end of the kinematic chain when a static wrench $W = \{Fx, Fy, Fz, T_x, T_y, T_z\}^T$ acts upon it (Tsai, 1999). The relationship between the vector of the compliant displacement ΔS and external wrench W can be written in the form, Eq. (6):

$$W = K\ \Delta S \tag{6}$$

The stiffness matrix can be numerically computed by defining a suitable model of the robot, which takes into account the stiffness properties of each element of the robot and, in particular, the lumped stiffness parameters are modeled as linear and torsion springs. A simplified stiffness model of 6-RSS Parallel Manipulator can be defined as show in Fig. 5. It is worthy to note that the stiffness matrix is configuration dependent since it is a function of the input angles. Therefore, the solution of the inverse kinematics is necessary.

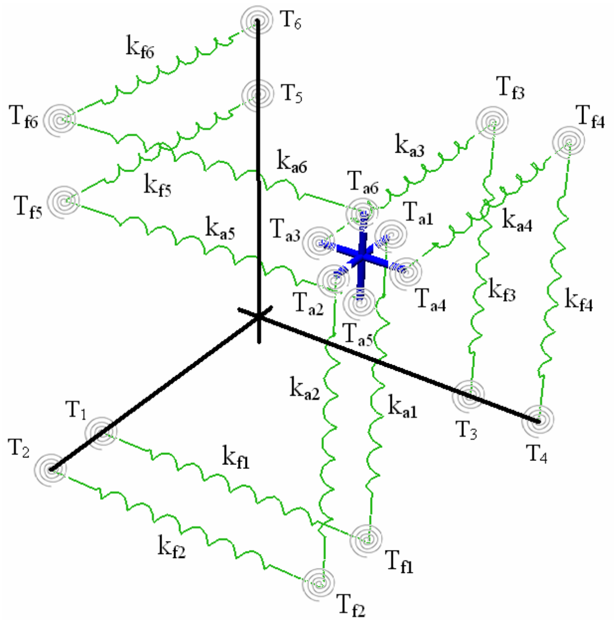


Figure 5. Stiffness Model for the 6-RSS Parallel Manipulator.

In Figure 5 the spherical joints are changed for torsion spring, where T_i (i = *1 to 6*) are the torsion springs in place of revolute joints; k_{fi} and k_{ai} (i = *1 to 6*) are linear spring that represent the forearm and arm, respectively. Finally T_{fi} and T_{ai} (i = *1 to 6*) represent the torsion spring of the spherical joints at the extremity of segments *RS* and *SS* respectively.

3.2. Jacobian Matrix

Several different approaches can be used for deriving the stiffness matrix. A first approach is based on the computation of the Jacobian matrix for the given multibody robotic system. With the inverse kinematic model of the architecture it can write:

$$\boldsymbol{\alpha} = \mathbf{G}\ \mathbf{S} \tag{7}$$

where $\boldsymbol{\alpha}$ is the vector of the joint coordinates and $\mathbf{S} = [t_x\ t_y\ t_z\ \phi_1\ \phi_2\ \phi_3]^T$ is the vector expressing position and orientation of a reference frame attached to the end-effector with respect to the fixed reference frame; **G** is a matrix expressing the inverse kinematics. By using Taylor series expansion for Eq. (7) and considering only its first term, one can write:

$$\Delta\boldsymbol{\alpha} = \mathbf{J_x}\ \Delta\mathbf{S} \tag{8}$$

where $\Delta\boldsymbol{\alpha}$ is the vector of increments of the joint coordinates, $\Delta\mathbf{S}$ is the vector of increment of the t_x,t_y, t_z, ϕ_1, ϕ_2 and ϕ_3 and $\mathbf{J_x}$ is the Jacobian matrix relative of the inverse kinematic model. The Jacobian matrix can be written in form:

$$\mathbf{J_x} = [\mathbf{J_1}, \mathbf{J_2}, \ldots, \mathbf{J_6}] \tag{9}$$

where

$$\begin{aligned}
&\mathbf{J_1} = \left(\frac{\partial\alpha_1}{\partial t_x},\ldots,\frac{\partial\alpha_6}{\partial t_x}\right)^T \quad \mathbf{J_2} = \left(\frac{\partial\alpha_1}{\partial t_y},\ldots,\frac{\partial\alpha_6}{\partial t_y}\right)^T \quad \mathbf{J_3} = \left(\frac{\partial\alpha_1}{\partial t_z},\ldots,\frac{\partial\alpha_6}{\partial t_z}\right)^T \\
&\mathbf{J_4} = \left(\frac{\partial\alpha_1}{\partial\phi_1},\ldots,\frac{\partial\alpha_6}{\partial\phi_1}\right)^T \quad \mathbf{J_5} = \left(\frac{\partial\alpha_1}{\partial\phi_2},\ldots,\frac{\partial\alpha_6}{\partial\phi_2}\right)^T \quad \mathbf{J_6} = \left(\frac{\partial\alpha_1}{\partial\phi_3},\ldots,\frac{\partial\alpha_6}{\partial\phi_3}\right)^T
\end{aligned} \tag{10}$$

By applying the principle of duality between the force/torque and velocity fields (Bashar *et al.*, 1999), one can write:

$$W = J_x^T F \tag{11}$$

where F = $\{F_{xi}, F_{yi}, F_{zi}, T_{xi}, T_{yi}, T_{zi}\}^T$ is the vector of forces acting on each link; J_x^T is the transpose matrix of the Jacobian defined in Eqs. (9) and (10). Moreover, neglecting the effects in the joints and considering Fig. 5

$$F = K_L\ \Delta\alpha \tag{12}$$

where

$$K_L = \begin{bmatrix} k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_6 \end{bmatrix} \tag{13}$$

where k_i (i = 1,…,6) are the lumped stiffness parameters of the *i-th* link. From Equations (8) and (12)

$$F = K_L\ J_x\ \Delta S \tag{14}$$

Multiplying both sides of Eq. (14) by J_x^T and substituting in Eq. (11) one have

$$W = J_x^T \mathrm{K_L} \ \mathrm{J_x} \ \Delta S \tag{15}$$

In this way, from Eqs. (6) and (15) the stiffness matrix of 6-RSS parallel manipulator can be defined as:

$$K = J_x^T \mathrm{K_L} \ \mathrm{J_x} \tag{16}$$

4. NUMERICAL ESTIMATION OF STIFFNESS PERFORMANCES

From Figure 5, the 6-RSS parallel manipulator has six set of equals forearm and arm. Therefore, Eq. (16) can be simplified considering a same lumped stiffness parameters k ($K_L = diag(k)$)

$$K = kJ_x^T \ \mathrm{J_x} \tag{17}$$

Figure 6 show one set of forearm and arm of the 6-RSS parallel manipulator with the lumped stiffness parameters, considering only the stiffness of the forearm and arm:

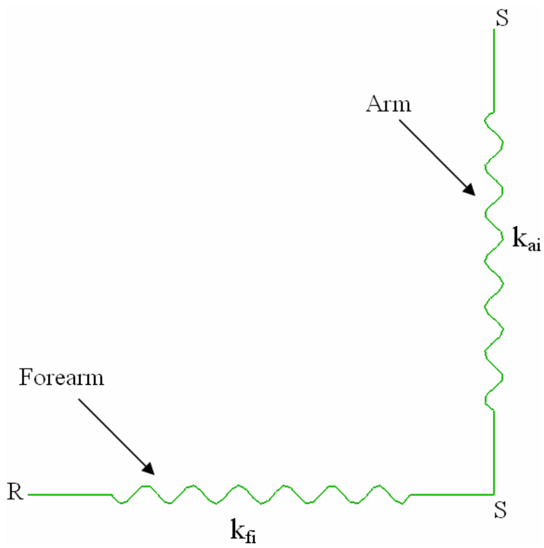


Figure 6. Lumped Stiffness Parameters.

For the structure 6-RSS the forearm and arm have the same length. Therefore, referring to Fig. 6 the equivalent lumped stiffness parameter for each leg can be write:

$$k_i^{-1} = k_{fi}^{-1} + k_{ai}^{-1} \quad \text{for (i = 1,...,6)} \tag{18}$$

The lumped stiffness parameters k_{fi} and k_{ai} are calculated as:

$$k_{fi} = k_{ai} = \frac{E_i A_i}{L_i} \tag{19}$$

where E_i is the Young's module of the material, A_i is the section of the link and L_i is its length. Based on prototype the scalar lumped stiffness parameter k is *1,056e7 N/m*. Using Equation (16) and (6) the compliance displacement for a given external wrench can be computed as:

$$\Delta S = K^{-1} W \tag{20}$$

By using the proposed formulation the compliances displacements, which are due to the external force and torque can be numerically computed. Tables 1, 2 and 3 show the computed values of the displacements of 6-RSS parallel manipulator in three different configurations. The Figure 7 plots the compliance displacement in function of an applied external wrench. This proposed formulation can be also used to compute the extreme values of the stiffness over the workspace of the manipulator.

Table 1. Displacement of 6-RSS parallel manipulator when $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0$ degree.

$\Delta x_{6\text{-}\underline{R}SS}$	Fe = (100 ,0, 0) Te = (0, 0, 0) [N; Nm]	Fe = (0 ,100, 0) Te = (0, 0, 0) [N; Nm]	Fe = (0 ,0, 100) Te = (0, 0, 0) [N; Nm]	Fe = (100 ,100, 100) Te = (0, 0, 0) [N; Nm]
Δtx [mm]	0.0526	0	0	0.0526
Δty [mm]	0	0.0526	0	0.0526
Δtz [mm]	0	0	0.0526	0.0526
Δϕ1 [º]	0	0	0	0
Δϕ2 [º]	0	0	0	0
Δϕ3 [º]	0	0	0	0

Table 2. Displacement of 6-RSS parallel manipulator when $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 10$ degree.

$\Delta x_{6\text{-}\underline{R}SS}$	Fe = (100 ,0, 0) Te = (0, 0, 0) [N; Nm]	Fe = (0 ,100, 0) Te = (0, 0, 0) [N; Nm]	Fe = (0 ,0, 100) Te = (0, 0, 0) [N; Nm]	Fe = (100 ,100, 100) Te = (0, 0, 0) [N; Nm]
Δtx [mm]	0.0629	-0.0156	-0.0156	0.0317
Δty [mm]	-0.0156	0.0629	-0.0156	0.0317
Δtz [mm]	-0.0156	-0.0156	0.0629	0.0317
Δϕ1 [º]	0	0	0	0
Δϕ2 [º]	0	0	0	0
Δϕ3 [º]	0	0	0	0

Table 3. Displacement of 6-RSS parallel manipulator when $\alpha_1 = 5$; $\alpha_2 = -10$; $\alpha_3 = 3$; $\alpha_4 = 4$; $\alpha_5 = 1$; $\alpha_6 = 2$ degree.

$\Delta x_{6\text{-}\underline{R}SS}$	Fe = (100 ,0, 0) Te = (1, 0, 0) [N; Nm]	Fe = (0 ,100, 0) Te = (1, 0, 0) [N; Nm]	Fe = (0 ,0, 100) Te = (1, 0, 0) [N; Nm]	Fe = (100 ,100, 100) Te = (1, 1, 1) [N; Nm]
Δtx [mm]	-0.0970	-0.1193	-0.1191	0.1566
Δty [mm]	-0.5817	-0.4009	-0.4680	0.4237
Δtz [mm]	-0.5300	-0.4165	-0.3780	0.4006
Δϕ1 [º]	12.8729	10.2236	10.5190	-7.5639
Δϕ2 [º]	6.2878	4.9498	5.1038	-4.1049
Δϕ3 [º]	-18.9019	-14.9422	-15.3615	12.2677

From Table 1 for the initial condition ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0$) the applied Forces produce the same displacement in the direction of the force when other forces are zeros. If the same values of forces are applied the compliance displacement and angular displacement are equals. From Figure 7 if the external forces increase the displacements increase too.

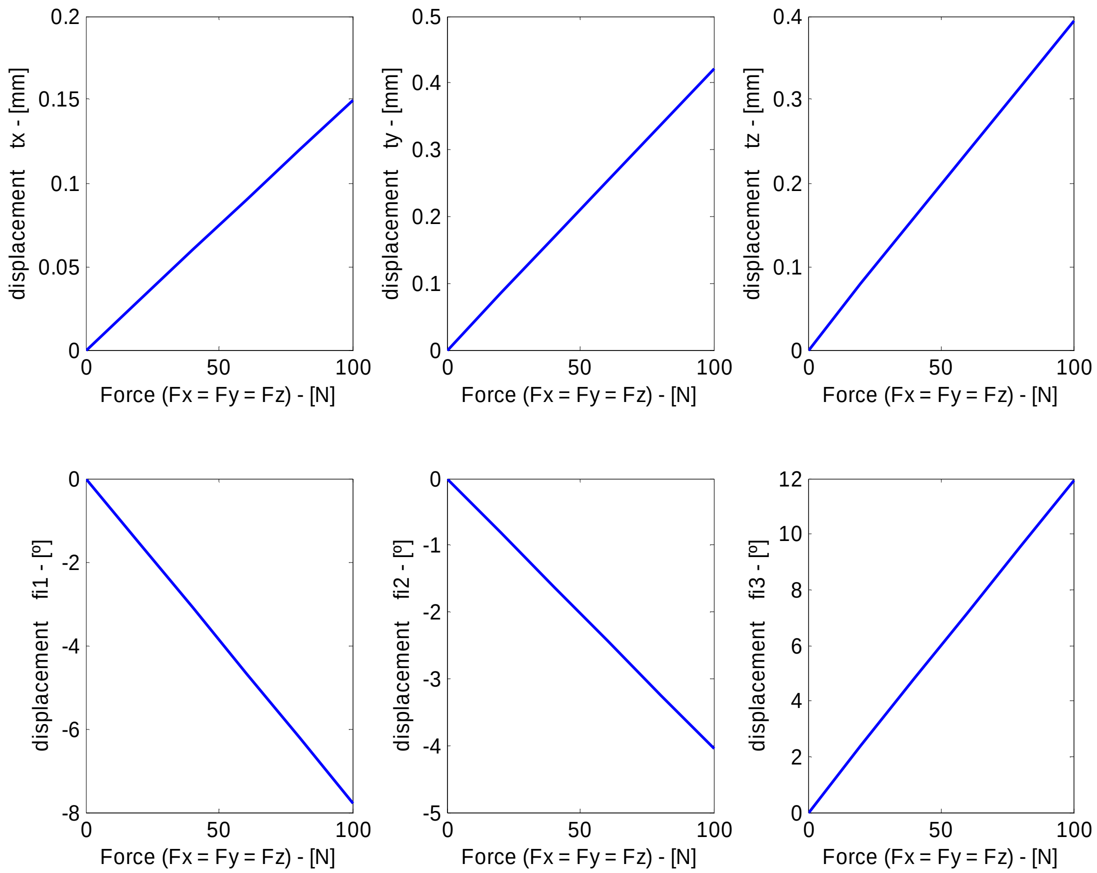


Figure 7. Displacements of 6-RSS parallel manipulator as a function of Fx = Fy = Fz when Tx = Ty = Tz = 0 for $\alpha_1 = 5$; $\alpha_2 = -10$; $\alpha_3 = 3$; $\alpha_4 = 4$; $\alpha_5 = 1$; $\alpha_6 = 2$ degree. (a) Δtx; (b) Δty; (c) Δtz; (d) $\Delta\phi 1$ [º]; (e) $\Delta\phi 2$ [º]; (f) $\Delta\phi 3$ [º]

5. CONCLUSION

In this paper the stiffness of 6-RSS parallel manipulator has been carried out as based on suitable kinematic and stiffness models. The Jacobian matrix is derivate from the inverse kinematic model of the structure. Then the stiffness matrix is calculated considering only the flexibility of the forearms and arms. The stiffness matrix enable to obtain the mobile platform displacement when an external wrench is applied. The procedure was used to map the mobile platform displacement for different configurations of the 6-RSS parallel structure. The used methodology can be applied to analyze the stiffness of other robots structures. Future works concern now in improving the stiffness model considering the joints and experimental test with the prototype.

6. ACKNOWLEDGMENTS

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