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# OPTIMUM WORKSPACE FOR PARALLEL MANIPULATORS

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***Abstract.*** *Manipulators with parallel architecture have inherent advantages in some applications with respect to serial manipulators, like high stiffness, accurate positioning and high movement velocities. Therefore, they address great interest in some industrial applications and medical fields. The volume of the workspace is one of the most important aspects during the manipulator project because it determines the geometrical limits of the task that can be performed. In this work an optimization procedure is proposed to maximize the workspace volume for CaPaMan (Cassino Parallel Manipulator) structure considering different geometries such as parallelepiped, sphere and cylinder as objective function. The design variables are obtained from the manipulator structure, taking into account constraints such as joint limits and link interference, providing a feasible optimal solution. The proposed methodology can be extended to consider the design of different robotic architectures.*

***Keywords****: Parallel Manipulators, workspace optimization, nonlinear programming.*

## 1. INTRODUCTION

The parallel manipulators in general, address great interest because they show better stiffness and payload capacity with respect to the serial architectures and can operate at high velocities and accelerations. Furthermore, the errors in the joints are not additive which contributes for its overall accuracy. Due to their characteristics they have been studied extensively both from theoretical and practical viewpoints whose prototypes have been conceived and built together with the development of theoretical investigations on kinematics and dynamics. The attention are focused to a number of possible industrial applications such as manipulation (Stewart, 1965), packing (Clavel, 1988), assembly and disassembly machines (Pierrot *et al.*, 1991), motion simulation (Ceccarelli, 1997), milling machines (Coelho *et al.*, 2001). However, they have some disadvantages such as small workspace and the complexity of their forward kinematics.

For evaluating performances of parallel manipulators most of the work has been done regarding to the workspace. For the parallel robot the constant-orientation workspace, the reachable workspace and the dexterous workspace can be defined, all of them can be represented in 3-D Cartesian space (Merlet, 1994).

Optimization methodologies have long been applied to mechanism synthesis in order to obtain high performances and suitable mechanism dimensions. Several performance criteria could be taken into account for design purposes, as for example workspace, singularities, stiffness, and dexterity.

Obtaining high performances requires the choice of suitable mechanism dimensions especially as there is much larger variation in the performances of parallel architectures according to the dimensions than for classical serial ones.

Indeed, with the development of manipulators for performing a wide range of tasks, the introduction of performance indices or criteria, which are used to characterize the manipulator, has, became very important. A number of different optimization criteria for manipulators may be appropriate depending on the resources and general nature of tasks to be performed. Consequently, one of the problems facing the designer is how to choose performance criteria and justify the optimality of different designs (Tsai and Huang, 2000).

Few researchers have addressed the optimization of the workspace of manipulators. In fact, it is one of the most important properties because workspace determines geometrical limits on the task that can be performed. Most of works are related to maximize the position workspace (Gosselin and Guillot, 1991), or try to obtain a position workspace as close as possible to a prescribed one (Boudreau and Gosselin, 1998), taking into account singularities (Schonherr, 2000). A formulation for optimum design of the CaPaMan architecture is presented in Ottaviano and Ceccarelli (2001), in order to obtain designed parameters of a robot whose position workspace is suitably prescribed. Another work focuses the optimization of the orientation workspace of CaPaMan (Ottaviano and Ceccarelli, 2002). Orientation workspace is probably the most difficult characteristic of a manipulator to determine and represent.

In this paper is presented a formulation for optimum design of parallel structures and this methodology is applied on CaPaMan (Cassino Parallel Manipulator) architecture, a 3-DOF spatial parallel manipulator, in order to obtain design parameters of a robot whose position and orientation workspaces are suitably prescribed.

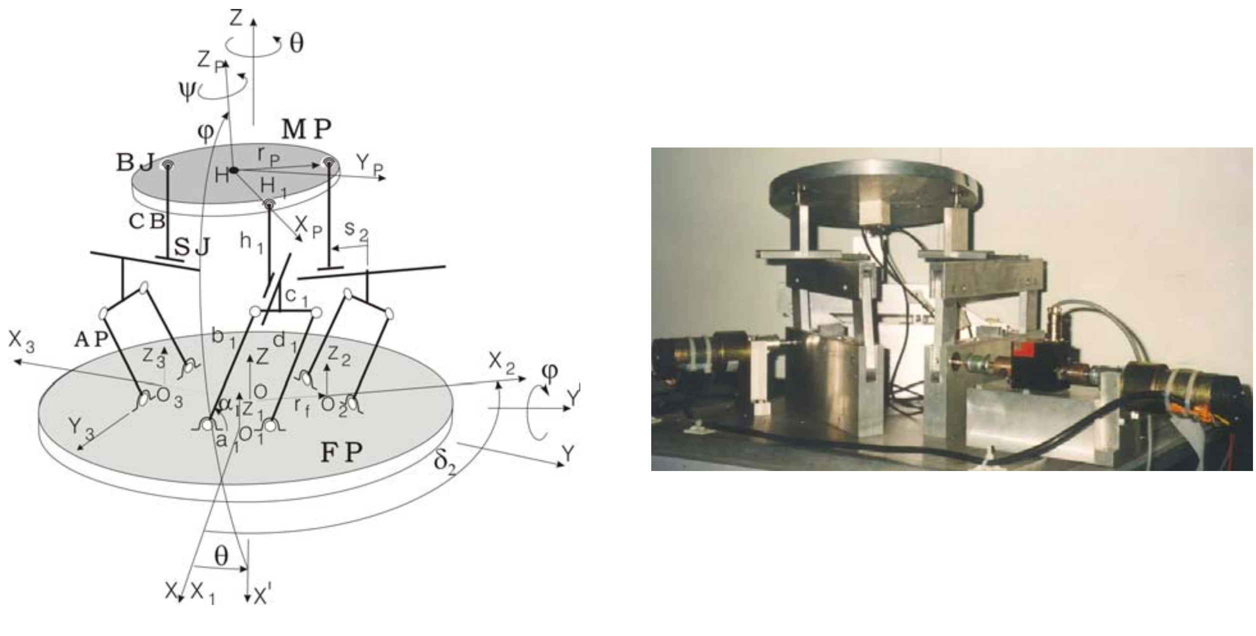
The proposed approach is focused on workspace characteristics, particularly the size of position and orientation workspaces of CaPaMan.

The workspace of parallel kinematic mechanisms has in general a complex volume shape. Discretization algorithms are usually used to determine workspace of manipulators. They consist in discretizing the 3-dimensional space, solving the Inverse Kinematics for each point, and verifying the constraints that limit the workspace (Oliveira, 2006). Such discretization algorithms are used by most of researchers: they are general and can be applied to any type of architecture. The proposed algorithm is based on a suitable approximation of position and orientation workspaces. In particular Ottaviano and Ceccarelli (2001) use a suitable parallelepiped volume to prescribe the size and shape of position and orientation capabilities. In this paper we consider another geometries like cylinder and sphere.

Due to the complexity of position and orientation workspaces evaluation of the robot parallel structure, this workspace has been simplified by using two defined geometries volumes, one that contain the position and another containing orientation workspaces.

## 2. DESCRIPTION OF CAPAMAN ARCHITECTURE

CaPaMan (Cassino Parallel Manipulator) is composed by a fixed plate *FP* that is connected to a movable plate *MP* by means of three leg mechanisms. Each of these is composed by an articulated parallelogram *AP*, a prismatic joint *SJ* and a connecting bar *CB*, Fig.1 (a). *CB* may translate along the prismatic guide of *SJ* keeping its vertical posture while the *BJ* allows the *MP* to rotate in the space. Each *AP* plane is rotated of $\pi/3$ with respect to the neighbor one. A built prototype is shown in Fig.1 (b). Design parameters of a *k* leg mechanism ($k = 1, 2, 3$) are identified through: $a_k$, which is the length of the frame link; $b_k$, which is the length of the input crank; $c_k$, which is the length of the coupler link; $d_k$, which is the length of the follower crank; $h_k$, which is the length of the connecting bar. The kinematic variables are: $\alpha_k$, which is the input crank angle and $s_k$, which is the stroke of the prismatic joint. The size of *MP* and *FP* are given by $r_p$ and $r_f$, respectively, where *H* is the center point of *MP*, *O* is the center point of *FP*, $H_k$ is the center point of the *k BJ* and $O_k$ is the middle point of the frame link $a_k$, Fig.1 (a). *MP* is driven by the three leg mechanisms through the corresponding articulation points $H_1$, $H_2$, $H_3$, so that the device is a *3 DOF* spatial mechanism. In order to describe the motion of *MP* with respect to *FP* a world frame *OXYZ* has been assumed as fixed to *FP* and a moving frame $HX_PY_PZ_P$ has been fixed to *MP*. Particularly, *OXY*Z has been fixed with *Z*-axis orthogonal to the *FP* plane, *X*-axis as coincident with the line joining *O* to $O_1$, and *Y*-axis to give a Cartesian reference frame. The moving frame $HX_PY_PZ_P$ has been fixed in an analogous way to the movable plate *MP* with $Z_P$ orthogonal to the *MP* plane, $X_P$ axis as coincident to the line joining *H* to $H_1$ and $Y_P$ to give a Cartesian frame. Table 1 gives the dimensions of the built prototype of CaPaMan, Fig.1 (b).




(a) (b)

Figure 1. (a) Architecture and design parameters of CaPaMan. (b) A built prototype of CaPaMan.

Table 1. Sizes of design parameters of the built prototype of CaPaMan.

| $a_k=c_k$ (mm) | $b_k=d_k$ (mm) | $h_k$ (mm) | $r_p=r_f$ (mm) | $\alpha_k$ (deg) | $S_{kmax}$ (mm) |
|---|---|---|---|---|---|
| 200.00 | 80.00 | 96.00 | 109.50 | 45;135 | 50.00 |

From kinematic analysis of CaPaMan the position $(x, y, z)$ of point *H* of the *MP* and the *MP* orientation $(\psi, \theta, \varphi)$ are given by Ceccarelli (1997):

$$x = \frac{y_3 - y_2}{\sqrt{3}} - \frac{r_p}{2}(1 - \sin\varphi)\cos(\psi - \theta) \tag{1}$$

$$y = y_1 - r_p(\sin\psi\cos\theta + \cos\psi\sin\varphi\sin\theta) \tag{2}$$

$$z = \frac{z_1 + z_2 + z_3}{3} \tag{3}$$

$$\psi = \tan^{-1}\left[\sqrt{3}\,\frac{z_3 - z_2}{2z_1 - z_2 - z_3}\right] \tag{4}$$

$$\theta = \sin^{-1}\left[2\,\frac{y_1 + y_2 + y_3}{3r_p(1 + \sin\varphi)}\right] - \psi \tag{5}$$

$$\varphi = \cos^{-1}\left[\pm\frac{2}{3r_p}\sqrt{z_1^2 + z_2^2 + z_3^2 - z_1 z_2 - z_2 z_3 - z_1 z_3}\right] \quad \text{If } ( z \geq z_1 \rightarrow +; z < z_1 \rightarrow - ) \tag{6}$$

where the $(y_k, z_k)$ coordinates are:

$$y_k = b_k \cos\alpha_k \tag{7}$$

$$z_k = b_k \sin\alpha_k + h_k \tag{8}$$

and the radius of the *MP* is

$$r_p = \left(\frac{a_k}{2} + b_k\right)/\sqrt{3} \tag{9}$$

The optimization problem enable to obtain the design parameters $a_k = c_k$, $b_k = d_k$, $h_k$, for three legs and $r_p$ for the *MP* of CaPaMan when the size and shape of orientation and position workspace are suitably prescribed (Ceccarelli, 1997).

## 3. A FORMULATION FOR AN OPTIMUM DESIGN

In this section is discussed the objective to be optimized that consists of the computation of the workspace volume though different geometries.

In a previous paper Ottaviano and Ceccarelli (2001) was presented a formulation for an optimum design for CaPaMan architecture when the workspace is suitably prescribed. A formulation that uses the volume of a geometric parallelepiped is presented where a fixed volume is a goal to be achieved through a classical nonlinear programming methodology. The applied strategy considers the workspace volume and a local optimization.

A similar formulation for an optimum design of the same architecture was presented in Ottaviano and Ceccarelli (2002), considering that the orientation workspace is suitably specified. By using a nonlinear programming methodology, the workspace orientation was considered.

The above discussed papers are summarized in Ceccarelli and Ottaviano (2000), where the workspace positioning and orientation are considered simultaneously. All the approaches use the parallelepiped as reference geometry.

In the present paper these analysis are extended in two different ways: the proposed geometry and the optimization procedure.

For analysis purposes the workspace volume can be approximated by the smallest solid $V_p$, containing the workspace. Thus, the design problem is to find the size of design parameters such that the workspace volume $V_p$, which is a numerical approximation of the real volume, is as close as possible to a prescribed volume $V$, Fig. 2 (a), (b) and (c).

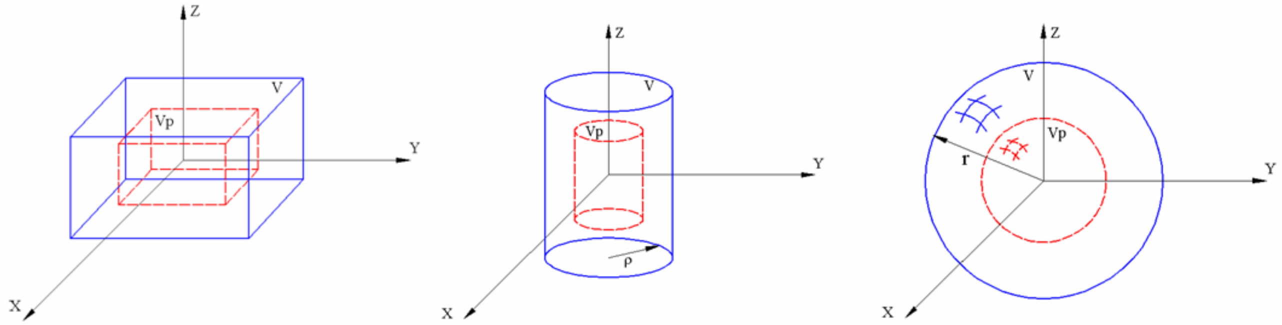
### 3.1 The objective function

The objective will be described by considering different geometries, that is, by parallelepiped, by cylindrical, and by spherical geometries.

The objective that describes the volume to be achieved through parallelepiped geometry is given by the equation

$$f_1 = \left(1 - \frac{|(2x)(2y)(2z)|}{V}\right)^2 \tag{10}$$

where *x*, *y* and *z* represent the half length of each side of the solid, as shown in Fig. 2.




(a) (b) (c)

Figure 2. (a) The parallelepiped geometry , (b) cylindrical geometry and (c) spherical geometry .

This formulation is motivated by the formulation proposed in Ceccarelli and Ottaviano (2000). Here, the solid is supposed to be centered at the origin of the Cartesian planes, and therefore preserves symmetry regarding the reference plane.

The objective function for the analysis through the cylindrical geometry, Fig. 2 (b), is given by

$$f_2 = \left(1 - \frac{\pi\rho^2 z}{V}\right)^2 \tag{11}$$

At last, to consider the spherical geometry, Fig. 2(c), the following objective function is proposed:

$$f_3 = \left(1 - \frac{\frac{4.\pi.r^3}{3}}{V}\right)^2 \tag{12}$$

where $\rho = \sqrt{x^2 + y^2}$ ; $r = \sqrt{x^2 + y^2 + z^2}$ , *V* is the required volume and the *x*, *y* and *z* variables are given by Eqs. (1), (2) and (3), respectively.

The above formulation is used to analyze the positioning in the workspace. To consider the orientation by means of parallelepiped geometry, the objective functions is given by

$$f_4 = \left(1 - \frac{|(2\varphi)(2\psi)(2\theta)E_k|}{V_o}\right)^2 \tag{13}$$

When considering the cylindrical geometry, the objective function is given by

$$f_5 = \left(1 - \frac{\pi\rho_o^2\theta}{V_o}\right)^2 \tag{14}$$

where $\rho_o = \sqrt{E_k(\varphi^2 + \psi^2)}$, $V_o$ is the required volume to be achieved, and $\psi$, $\theta$, and $\varphi$ are given by Equations (4), (5) and (6) respectively. The proposed formulation allows the use of a scale factor $E_k$, and computes a squared difference.

Finally, the spherical geometry of the orientation workspace is represented through the equation

$$f_6 = \left( 1 - \frac{\frac{4.\pi.r_o^3}{3}}{V_o} \right)^2 \quad (15)$$

where $r_o = \sqrt{E_k(\varphi^2 + \psi^2 + \theta^2)}$.

### 3.2. Multiobjective Programming Approach

In problems with multiple criteria one deals with a design variable vector *x*, which satisfies all constraints and makes as small as possible the scalar performance index that is calculated by taking into account the *m* components of an objective function vector *f(x)*. This goal can be achieved by the vector optimization problem:

$$\min_{x\in\Omega} \{ f(x) \mid h(x)=0, g(x) \le 0 \} \quad (16)$$

where $\Omega \subset R^n$ is the domain of the objective function (the design space), *h(x)* and *g(x)* represent the equality and inequality constraints, respectively.

An important feature of such multiple criteria optimization problems is that the optimizer has to deal with objective conflicts (Deb, 2001). Other authors discuss the so-called compromise programming, because there is no unique solution to the problem (Eschenauer *et al.*, 1990).

With the aim to evaluate the performance of different objectives (position and orientation), for each of the proposed geometries the optimization is performed. Weighting objectives is one of the most usual (and simple) substitute models for multiobjective optimization problems. It permits a preference formulation that is independent from the individual minimum for positive weights.

For the first geometry, the parallelepiped, the scalar objective function is defined as

$$f_{par} = \gamma_1 f_1 + \gamma_2 f_4 \quad (17)$$

In a similar way, the objectives

$$f_{cyl} = \gamma_1 f_2 + \gamma_2 f_5 \quad (18)$$

and

$$f_{sph} = \gamma_1 f_3 + \gamma_2 f_6 \quad (19)$$

define the cylindrical and spherical geometries.

Based on the built prototype, the associated constraints are given by

$$|x| - 40 < 0 \, ; |y| - 40 < 0 \, ; \; |z| - 180 < 0 \quad (20)$$

$$|\varphi| - 100° < 0 \, ; \; |\psi| - 100° < 0 \, ; \; |\theta| - 230° < 0 \, ; \quad (21)$$

$$a_k - 2\sqrt{3r_p} - b_k \le 0 \quad (22)$$

$$b_k > 0 \, ; \quad h_k > 0 \, ; \; a_k > 0 \, ; \quad (23)$$

Equation (20) defines de bound constraint in *x*, *y* and *z*. These values are allowed to be positive or negative, since the center of the considered geometry is positioned in the origin of the Cartesian reference system. The analogous variables that define the orientation are constrained by Equation (21). Equation (22) is a physical constraint that avoids collision between the legs of the manipulator basis. Equation (23) is a constraint to ensure positive values of the design parameters.

### 3.3 The optimization algorithm

When dealing with nonlinear and non convex objective functions, direct search approached is a suitable tool. There are several strategies, as those proposed by Bunday and Garside (1987), Gill *et al.* (1981), Hook and Jeeves (1969), the genetic algorithms (Goldberg, 1989), Differential Evolution (Storn and Price, 1997) (Price *et al.*, 2005), and the evolutionary algorithms (Rechenberg, 1973) (Voigt, 1992) (Viana and Steffen Jr, 2006), among others.

The Differential Evolution is a heuristic method which utilizes a parameter vector as a population for each generation. This vector does not change during the minimization process. The initial population is chosen randomly. By using the default options, it is assumed a uniform probability distribution for all random decisions. The main idea proposed by the method is a scheme for generating trial parameter vectors. The new vectors are generated by adding the weighted difference vector between two population members to a third member. If the resulting vector yields a lower objective function value than a previous population member, the newly generated vector replaces the comparing vector.

In contrast, a classical nonlinear programming method requires information about the gradient of the objective function, and provides a local minimum at a low computational cost.

Aiming to obtain the best of each methodology, in this paper a hybrid strategy is adopted. At first, a Differential approach is applied to each problem. The design obtained from this process is then used as initial guess for a nonlinear programming based algorithm.

## 4. NUMERICAL RESULTS

According the original purpose of the weighting objectives (Deb, 2001), the relation $\gamma_1 + \gamma_2 = 1$ must hold. To analyze the importance of each objective in the overall performance index, different values for $\gamma_1$ and $\gamma_2$ where considered.

Where performed 300 runs for each geometry, with $\gamma_1 = i, \gamma_2 = 1 - i$, for *i=0.1, 0.2, …0.9*. Therefore, 2700 numerical problems where solved by the optimization process. In the first phase of the process, a heuristic optimization was performed through the Differential Evolution strategy. Then, the obtained solution was used as initial guess for a Quadratic Sequential Programming strategy.

The 3 best results obtained by using the parallelepiped geometry are summarized in Table 2. One contribution of the present strategy is that the same optimal design is achieved when different weight parameters are considered.

Table 2. Optimal results for parallelepiped.

| Objective value | $b_k$[mm] | $h_k$ [mm] | $r_p$ [mm] | $a_k$ [mm] |
|---|---|---|---|---|
| 0 | 16.74281 | 153.3647 | 42.88132 | 115.0596 |
| 0.000001 | 30.26156 | 146.1346 | 38.88597 | 74.18182 |
| 0.000001 | 29.91899 | 152.7558 | 137.7999 | 417.5149 |

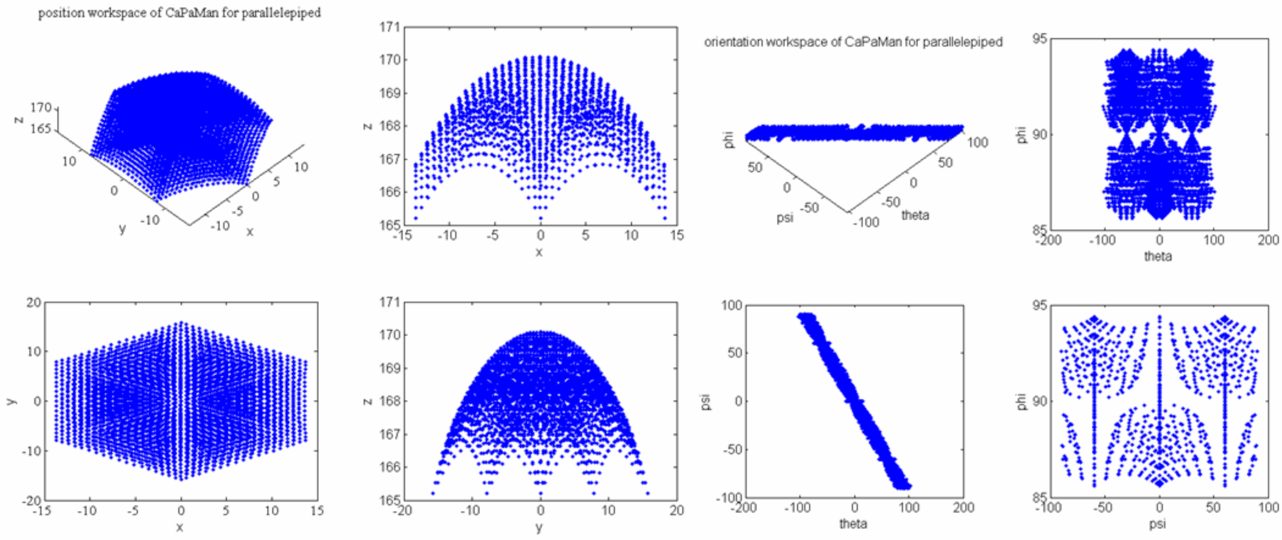
By comparing the optimal values of the table, is remarkable that there are different designs with a similar index performance. This result leads to conclude that there are several geometries that satisfy the required value of the performance index. The statistical information is presented in Table 3.

Table 3. Statistical analysis of optimal results for Parallelepiped.

| Parallelepiped | $b_k$ [mm] | $h_k$[mm] | $r_p$ [mm] | $a_k$ [mm] |
|---|---|---|---|---|
| Average | 49.2183 | 94.2645 | 95.4948 | 232.367 |
| Standard deviation | 41.8397 | 53.5875 | 66.2927 | 196.963 |

The smallest objective value is zero that means the required volume was fulfilled. The largest objective value is 0.578393 in the worst case. A feature of the processed data is a relative large standard deviation. It empathizes that there are several distinct designs that lead to the optimal required volume. This is a common behavior of the problem, and happens due the intrinsic complexity of the structure.

The optimal geometry, obtained when using the configuration $b_k$ = *16.74281*; $h_k$ = *153.3647*; $r_p$ = *42.88132*; $a_k$ = *115.0596*, is presented in Figure 3(a). The corresponding orientation is shown in Figure 3(b).




(a) (b)

Figure 3. (a) Optimal positioning geometry for parallelepiped. (b) Optimal orientation geometry for parallelepiped.

Should be pointed out that many designs take to similar results. From the geometrical point of view, the main difference between different results is the change in the corresponding scale.

In the following, the 3 best results obtained by using the cylindrical geometry are summarized in Table 4.

Table 4. Optimal results for Cylinder.

| Objective value | $b_k$ [mm] | $h_k$ [mm] | rp [mm] | $a_k$ [mm] |
|---|---|---|---|---|
| 0 | 15.36028 | 174.9813 | 16.45805 | 26.29341 |
| 0 | 17.47014 | 136.3981 | 44.65914 | 119.7679 |
| 0 | 15.60572 | 173.3254 | 110.6967 | 352.264 |

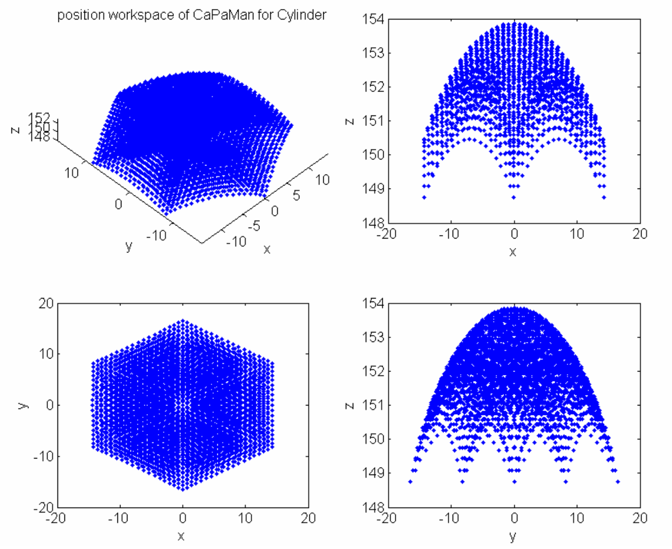
As in the preceding case, is remarkable that there are different designs with the same index performance. The statistical information is presented in Table 5.

Table 5. Statistical analysis of optimal results for Cylinder.

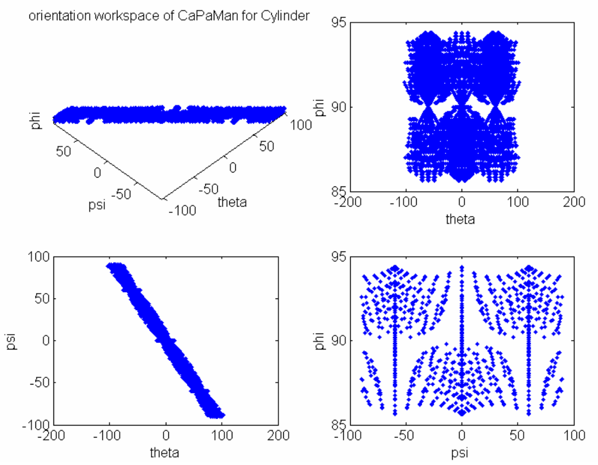
| Cylinder | $b_k$ [mm] | $h_k$ [mm] | rp [mm] | $a_k$ [mm] |
|---|---|---|---|---|
| Average | 36.83547 | 93.28925 | 83.70154 | 216.288 |
| Standard deviation | 36.56697 | 42.87543 | 66.14095 | 201.3851 |

In this set of experiments was also observed a relative large standard deviation. Therefore, is possible to conclude that for this geometry, there are several distinct designs that lead to the optimal required volume.

By using this geometry, the smallest objective value is zero and the largest objective value is *0.751899*. The optimal geometry, obtained when using the configuration $b_k$ = *17.47014*; $h_k$ = *136.3981*; $r_p$ = *44.65914* and $a_k$ = *119.7679*, is presented in Figure 4(a). The corresponding orientation is shown in Figure 4(b).




(a)




(b)

Figure 4. (a) Optimal positioning geometry for cylinder. (b) Optimal orientation geometry for cylinder.

In the following, the 3 best results obtained by using the spherical geometry are summarized in Table 6.

Table 6. Optimal results for sphere.

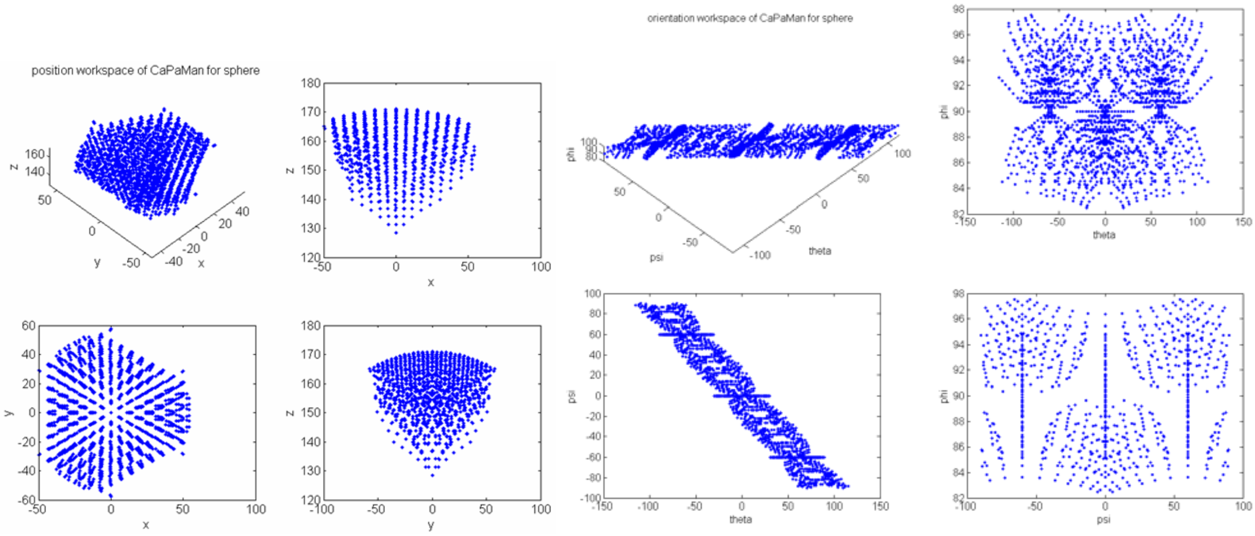
| Objective value | $b_k$ [mm] | $h_k$ [mm] | $r_p$ [mm] | $a_k$ [mm] |
|---|---|---|---|---|
| 0 | 145.1331 | 25.88811 | 203.2356 | 413.7825 |
| 0 | 137.3821 | 104.1929 | 190.8649 | 386.4302 |
| 0 | 154.4617 | 112.1805 | 214.5954 | 434.4778 |

In the following, Table 7 presents statistical information about the optimal results. As a remark, for this geometry the choice between different designs that lead to similar performance index also exists.

Table 7. Statistical analysis of optimal results for sphere.

| Sphere | $b_k$ [mm] | $h_k$ [mm] | $r_p$ [mm] | $a_k$ [mm] |
|---|---|---|---|---|
| Average | 44.80083 | 31.36688 | 50.64686 | 85.8492 |
| Standard deviation | 59.55588 | 20.0241 | 78.67626 | 158.9529 |

The optimal geometry, obtained when using the configuration $b_k = 145.1331$; $h_k = 25.88811$; $r_p = 203.2356$; $a_k = 413.7825$, is presented in Figure 5(a). The smallest objective value is zero and the largest objective value is *0.01966*. The corresponding orientation is shown in Figure 5(b).




(a) (b)

Figure 9. (a) Optimal positioning geometry for sphere. (b) Optimal orientation geometry for sphere.

Although the larger variation of the design parameters value presented on Tables 2, 4 and 6 one can observe that using the sphere as an objective workspace function, the performance of the optimization process is improved. It can be found by the smaller value of the worst case, when compared with those of other geometries.

## 5. CONCLUSION

In this paper a contribution about the optimal design of a CaPaMan parallel robotic structure is presented.

Initially, the subject of parallel robotic structures, the concept of workspace positioning and orientation, and a review about some existing methodologies to obtain the optimal design are considered.

The presented formulation extends previous works, since it considers the use of different geometries to obtain the required volume. Therefore, one contribution of this paper is the analysis of the influence about distinct geometries in the optimal design. Another contribution is the use of a hybrid optimization strategy to obtain a global optimum.

Through a large number of numerical tests is possible to conclude that the weight factors do not affects significantly the optimum design.

Therefore, this methodology is useful to establish the optimal design. Due the good results achieved, the authors believe that the strategy can also be useful to consider other decision criteria, e.g., singularities and stiffness, when computing the optimal design.

As a result, the method was shown to be appropriate in exploring the highly nonlinear nature of the structure, once it finds several solutions with similar performance index.

Finally, due to the good performance presented by the numerical experiments, the authors believe that this methodology can also be useful to consider other decision criteria, e.g., singularities and stiffness, when computing the optimal design, and also can be efficiently used to analyze different parallel robotic structures.

## 6. ACKNOWLEDGMENTS

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## 7. REFERENCES

Boudreau, R. and Gosselin, C. M., 1998, “The Synthesis of Planar Parallel Manipulators with a Genetic Algorithm”, Proceedings of the ASME Design Engineering Technical Conference, Atlanta, Paper DETC2000/MECH-5957.

Byun, Y.K. and Cho, H.S., 1997, “Analysis of a Novel 6-dof, 3 PPSP Parallel Manipulator”, The Int. J. of Robotics Research, Vol.16, No.6, pp.859-872.

Ceccarelli, M. and Ottaviano, E., 2000, “An Analytical Design for CaPaMan with Prescribed Position and Orientation”, Proceedings of the ASME Design Engineering Technical Conference and Computers and Information in Engineering Conference, Baltimore, Paper DETC2000/MECH-14099.

Ceccarelli, M., 1997, “A New 3 dof Spatial Parallel Mechanism”, Mechanism and Machine Theory, Vol.32, No.8, pp. 895-902.

Clavel, R., 1988, “DELTA: A Fast Robot with Parallel Geometry”, Proc. of 18th Int. Symp. on Industrial Robots, Lausanne, pp.91-100.

Coelho, T.A.H., Batalha, G.F., Moraes, D.T.B. and Boczko, M., 2001, “A Prototype of a Contour Milling Machine Based on a Parallel Kinematic Mechanism”. Proc. of the 32nd Int. Symposium on Robotics, Seoul, Korea.

Deb, K., 2001, “Multi-Objective Optimization using Evolutionary Algorithms”, John Wiley, New York.

Eschenauer, H., Koski, J. and Osyczka, A., 1990, “Multicriteria Design Optimization, Berlin”, Springer-Verlag.

Gill, P. E.,. Murray, W and Wright, M. H., 1981, “Practical Optimization”, Academic Press, New York.

Goldberg, D.E., 1989, “Genetic Algorithms in Search”, Optimization & Machine Learning, Addison-Wesley.

Gosselin, C. M. and Guillot, M., 1991, “The Synthesis of Manipulators with Prescribed Workspace”, Journal of Mechanical Design, Vol.113, pp.451-455.

Hook, R. and Jeeves, T., 1969, “Direct search solution of numerical and statistical problems”, J. ACM, 7, 212-229.

Merlet, J.-P., 1994, “Determination de l’Espace de Travail d’un robot parallele pour une orientation constante”, Mechanism and Machine Theory, Vo.29, No.8, pp.1099-1113.

Oliveira., G. T., 2006, “Estudo e Aplicações da Evolução Diferencial”, Dissertação (Mestrado em Curso de Pos Graduação Em Engenharia Mecânica) - Universidade Federal de Uberlândia, Coordenação de Aperfeiçoamento de Pessoal de Nível Superior. Orientador: Sezimária de Fátima Pereira Saramago.

Ottaviano, E. and Ceccarelli, M., 2001, “Optimal Design of CaPaMan (Cassino Parallel Manipulator) with Prescribed Workspace”, Proceedings of Computational Kinematics, Seoul.

Ottaviano, E. and Ceccarelli, M., 2002., “Optimal Design of CaPaMan (Cassino Parallel Manipulator) with a specified orientation workspace”, Robotica, Vol. 20, pp. 159-166.

Pierrot, F., Dauchez, P. and Fournier, A., 1991, “Hexa: A Fast Six-Dof Fully Parallel Robot”, Proc. of Int. Conference on Advanced Robotics , Pisa, pp.1159-1163.

Price, K., Storn, R. and Lampinen, J., 2005, “Differential Evolution - A Practical Approach to Global Optimization”, Springer, Berlin.

Rechenberg, I., 1973, “Evolutionsstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution”. Frommann-Holzboog, Stuttgart.

Schonherr, J., 2000, “Evaluation and Optimum Design of Parallel Manipulators Having a Defined Workspace”, Proceedings of the ASME Design Engineering Technical Conference and Computers and Information in Engineering Conference, Baltimore, Paper DETC2000/MECH-14092.

Stewart, D., 1965, “A Platform with Six Degrees of Freedom”, Proc. of Institution of Mechanical Engineers, Vol.180, No.15, part I, pp.371-386.

Storn, R. and Price, K., 1997, “Differential Evolution - a Simple and Efficient Heuristic for Global Optimization over Continuous Spaces”, Journal of Global Optimization, Kluwer Academic Publishers, Vol. 11, pp. 341 - 359.

Tsai, K. Y. and Huang K. D., 2000, “The Design Considerations of Manipulators”, Proceedings of the ASME Design Engineering Technical Conference and Computers and Information in Engineering Conference, Baltimore, Paper DETC2000/MECH-14089.

Viana, F. A. C. and Steffen Jr, V., 2006, “Ant Colony Optimization Applied to Discrete-Continuous Problems”, In: VII Simpósio de Mecânica Computacional 2006, Araxá. Anais do VII Simpósio de Mecânica Computacional, p. 1-10.

Voigt, H. M., 1992, “Fuzzy Evolutionary Algorithms”, Technical Report TR-92-038 at ICSI, <ftp.icsi.berkeley.edu>.

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