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Redundancy and connectivity in kinematic chains

Andrea Piga Carboni, andrea.piga@emc.ufsc.br
Daniel Martins, daniel@emc.ufsc.br
Universidade Federal de Santa Catarina, 88040-900 Florianopolis SC Brazil
daniel@emc.ufsc.br

Abstract. *This paper presents a new methodology for the automatic calculation of the most important parameters of a kinematic chain, namely redundancy, connectivity and degrees-of-control. To the authors' knowledge, this is the first algorithm that accurately calculates redundancy, connectivity and degrees-of-control in all cases, without exceptions. This paper starts by reviewing previous works on the connectivity, degrees-of-control and redundancy calculation of a kinematic chain, identifying the sources of some flaws and showing counterexamples for the algorithms proposed in the literature. Moreover, new definitions of* connectivity *and* degrees-of-control *are introduced, which do not conflict with the previous definitions found in the literature. These new definitions have an algorithmically oriented form and identify a systematic procedure for the calculation of these parameters. Based on these definitions, a new algorithm is proposed, which overcomes the deficiencies of the previous algorithms. Connectivity and redundancy may be used to classify kinematic chains according to the constraints required. In this way, a natural application of the algorithm proposed is in the process of enumeration of kinematic chains, namely* number synthesis*, where an automatic procedure is needed to select the best kinematic chains for a determined task, among a large number of chains generated.*

Keywords: *redundancy, connectivity, kinematic chain, degrees-of-control*

1. Introduction

The choice of the kinematic topology of a mechanism usually depends on the designer's experience and capability. In practice, some fundamental properties of the kinematic chains, such as number of links, number of kinematic pairs, type of joints, and end-effector mobility, are parameters fixed at the earliest stage of the project.

However, the kinematic topology could be chosen through a more systematic approach by taking into account all the constraints that derive from the desired characteristics, such as the kind of task required, the environment, the number of degrees of freedom, the possible redundancy, and so on.

The enumeration of kinematic chains, also known as *number synthesis*, has been used for at least the past four decades, *e.g.* (Davies and Crossley, 1966), as a means of finding better mechanisms for some predefined purpose. In practice, however, enumeration can be difficult to implement since the number of kinematic chains generated is often too large to manually consider the individual merits of each chain. For this reason, the concepts of *connectivity* and *variety* can be used to classify kinematic chains according to the constraints required (Tischler et al., 1995), (Tischler et al., 1998), (Tischler et al., 2001). Other concepts, created and adapted in (Belfiore and Di Benedetto, 2000), such as *degrees-of-control* and *redundancy*, are also important to this individuation process.

One of the main problems of the enumeration of kinematic chains is the selection of suitable robot manipulators. This problem is particularly complex for parallel manipulators especially when redundancy is involved. In these cases, redundancy is one of the most important parameters and gives useful support in the first conceptual phase of the design of the manipulator.

In the field of parallel robots for machine-tools, redundancy has been used to increase the workspace of the robot (such as in the Eclipse parallel robot (Ryu et al., 1998)) and to deal with singularities. Another form of redundancy is the concept of modular robots (Yang et al., 1999) where additional actuators allow the adaption of the geometry of the robot according to the task to be performed.

Redundant robots are used in confined spaces, in order to avoid collisions (Simas et al., 2003) and redundancy is an important parameter in cooperative robots (Dourado, 2005), with the application of virtual chains (Campos et al., 2005), (Campos et al., 2003).

The two main results of this paper are: a redefinition of the concepts of *connectivity* and *degrees-of-control* in an algorithmic form as discussed in Section 5. and a new algorithm to obtain the parameters of a kinematic chain.

Section 6. presents our algorithm for the calculation of degrees-of-control, connectivity and redundancy, entirely based on the previous definitions, which overcomes the deficiencies of the algorithms found in the literature.

To to the authors' knowledge, this is the first algorithm that accurately calculates connectivity and redundancy in all cases, without exception.

2. Basic definitions

Let us first introduce some basic definitions for kinematic chains.

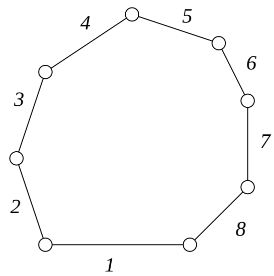


Figure 1: Closed-loop kinematic chain with $M = 5$. For links 1 and 4: $K_{14} = 3$, $C_{14} = 3$, $R_{14} = 0$. For links 1 and 5: $K_{15} = 4$, $C_{15} = 3$, $R_{15} = 1$

Definition 1 *The number of* degrees of freedom*, or* mobility *of a kinematic chain is the number of independent parameters required to completely specify the configuration of the kinematic chain in the space, with respect to one link chosen as the reference.*

The mobility of a kinematic chain, with n links and g single degree of freedom joints, may be calculated by the general mobility criterion (Hunt, 1978) applied to a set of n links and g single degree of freedom joints:

$$M = \lambda(n - g - 1) + g \tag{1}$$

where λ is the order of the screw system to which all the joint screws belong.

For instance, the mobility of the planar closed-loop kinematic chain shown in Figure 1 is, using equation (1), $M = 5$.

Definition 2 *(Hunt, 1978) The* connectivity C_{ij} *between two links i and j of a kinematic chain is the relative mobility between links i and j.*

In other words, the connectivity can be defined as the number of degrees of freedom (DoF) between two specific links in a kinematic chain. The concept of *joint in the bag equivalence*, introduced in (Phillips, 1984), is also useful for the conceptual definition of connectivity. According to such equivalence, all the interposing links and joints between two links i and j may be considered as hidden inside a flexible *black bag*. This bag can be regarded as an equivalent unknown joint between links *i* and *j*, and the DoF of this equivalent joint is a measure of the connectivity between the two joints.

It should be remembered that the DoF of any single joint cannot be greater than the maximum degrees of freedom of a rigid body in the system considered, usually referred to as the *dimension of the screw system* λ. Consequently, the connectivity is upper-bounded by the value of λ. Therefore, it will be less than or equal to 3 in the case of plane or spherical screw systems ($\lambda = 3$) and it will be less than or equal to 6 in the general case of the general spatial motion ($\lambda = 6$).

In (Belfiore and Di Benedetto, 2000) another important concept is introduced: *degrees-of-control*.

Definition 3 *(Belfiore and Di Benedetto, 2000) The* degrees-of-control K_{ij} *between two links i and j of a kinematic chain is the minimum number of independent actuating pairs needed to determine the relative position between the two links i and j, possibly leaving some other link-relative position undetermined as when K_{ij} is less than the mobility M.*

Based on the definition of degrees-of-control and connectivity, the definition of redundancy may now be introduced.

Definition 4 *The* redundancy R_{ij} *between two links i and j of a kinematic chain is the difference between the number of degrees of control K_{ij} and the connectivity C_{ij} between these links.*

From these definitions the matrices R, C and K do not have to be independently evaluated. It is important to note that the concept of degrees-of-control introduced in (Belfiore and Di Benedetto, 2000) allows the calculation of the redundancy directly from connectivity and degrees-of-control, as stated in the following lemmas (Belfiore and Di Benedetto, 2000)

Lemma 5 *If K_{ij} is greater than λ, then $C_{ij} = \lambda$, otherwise C_{ij} will be equal to K_{ij}.*

Lemma 6 *The redundancy R_{ij} is given as the difference between K_{ij} and C_{ij}: $R = K - C$.*

The above properties are invariantly relative to the permutation of indices: $ij \leftrightarrow ji$; therefore, a convenient way of representing the full set of degrees-of-control, connectivities and redundancies of a kinematic chain is by symmetric matrices.

As an example, the concepts of connectivity, degrees-of-control, and redundancy are applied to a planar, closed-loop kinematic chain with eight links and eight simple 1-DoF kinematic pairs as shown in Figure 1. Let us consider links 1 and 4: their degrees-of-control is $K_{1,4} = 3$, *i.e.* three independent actuators must be used in order to determine the relative position between the two links. The connectivity between the same pair of links is $C_{1,4} = 3$, *i.e.* the two links have full mobility (the relative mobility is equal to the order of the screw system λ where all the joint screws belong). Finally, the redundancy between links 1 and 4 is $R_{1,4} = 0$. If we choose link 1 as the frame and link 4 as the end-effector, the parallel manipulator derived from the kinematic chain has no degree of redundancy. Consider now links 1 and 5 of the kinematic chain in Figure 1. The degrees-of-control between these two links is $K_{1,5} = 4$ and their connectivity is $C_{1,5} = 3$, because it is upper-bounded by the value of λ; therefore, the redundancy is $R_{1,5} = 1$. One conclusion is that choosing link 1 as the frame and link 5 as the end-effector, or vice-versa, we obtain a redundant parallel manipulator from the kinematic chain.

For a better understanding of the importance of the concept of connectivity let us consider Figure 2. Figure 2a represents an open kinematic chain with mobility $M = 8$, but the connectivity between any two links does not exceed 2. Consequently the relative mobility between any two links i and j cannot be greater than 2. Figure 2b represents a closed kinematic chain with mobility $M = 3$, but the connectivity between any two links does not exceed 2. From these two simple examples, it is evident that connectivity, not mobility, determines the ability of an output link to perform a task relative to a frame.

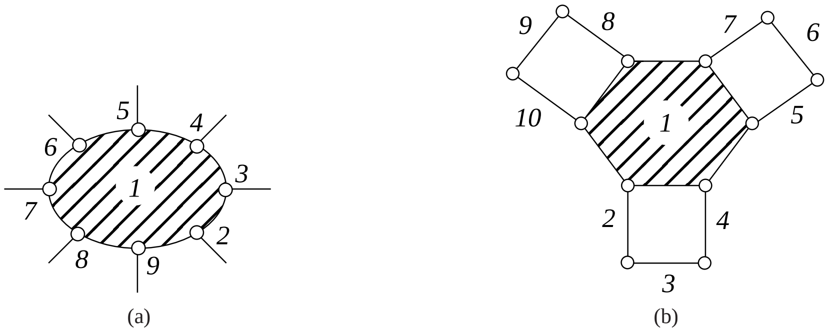


Figure 2: Kinematic chains with maximum connectivity between links of 2 *i.e.* $C_{ij} \leq 2 \;\; \forall i, j$

3. Graph Formulation

In this section, some fundamental concepts of graph theory (Tsai, 2001) are introduced. They are essential for topological analysis and number synthesis of mechanisms. It is important to remember that the topology of a mechanism can be uniquely identified by its graph representation, where links and joints of the mechanism are represented, respectively, by the vertices and edges of the graph.

3.1 Definitions

A *graph* G consists of a set of vertices V connected by a set of edges E. We call a graph with v vertices and e edges a (v,e) graph. Each edge of a graph connects two vertices called *end points*. An edge is specified by its end points; that is, e_{ij} denotes the edge connecting vertices i and j. An edge is said to be *incident* to a vertex if the vertex is an end point of that edge. The two end points of an edge are said to be *adjacent*. Two edges are adjacent if they are incident to a common vertex.

A sequence of alternating vertices and edges, beginning and ending in a vertex, is called a *walk*. A walk is called a *path* if all the vertices, and therefore all the edges, are also distinct. The *length* of a path is defined as the number of edges between the beginning and ending vertices. If each vertex appears once, with the exception of the beginning and ending vertices, which in this case are the same, the path forms a *circuit* or *cycle*.

When a direction is assigned to every edge of a graph, the graph is said to be a *directed graph*; if no direction is assigned, the graph is said to be an *undirected graph*.

A *subgraph* of G is a graph obtained by removing a number of edges and/or vertices from G. The removal of a vertex from G implies the removal of all the edges incident to that vertex, whereas the removal of an edge does not necessarily imply the removal of its endpoints although it may result in one or two isolated vertices.

Two vertices are said to be *connected*, if there exists a path from one vertex to the other. Note that two connected vertices are not necessarily adjacent. A graph G is said to be *connected* if every vertex in G is connected to every other vertex by at least one path.

An undirected graph is said to be *biconnected* (Manber, 1989) if there are at least two vertex disjoint paths from every vertex to every other vertex. A *biconnected component* is defined as a maximal subset of edges such that its induced subgraph is biconnected (namely, there is no subset that contains it and induces a biconnected graph) (Manber, 1989). A connected graph can be partitioned into biconnected components (in (Manber, 1989) an algorithm to find all biconnected components of an undirected subgraph is presented).

If every pair of distinct vertices in a graph is connected by one edge, the graph is called a *complete graph*. A *tree* is a connected graph that contains no circuits.

A *spanning tree* T, is a tree containing all the vertices of a connected graph G. Clearly, T is a subgraph of G. With reference to a spanning tree, the edge set E of G can be decomposed into two disjoint subsets, called *arcs* and *chords*. The arcs of G consist of all the elements of E that form the spanning tree T, whereas the chords consist of all the elements of E that are not in T. The union of the arcs and chords constitutes the edge set E.

In general, the spanning tree of a connected graph is not unique. The addition of a chord to a spanning tree forms, one and precisely one, circuit. A collection of all the circuits with respect to a spanning tree forms a set of *independent loops* or *fundamental circuits*. The fundamental circuits constitute a basis for the circuit space. Any arbitrary circuit of the graph can be expressed as a linear combination of the fundamental circuits using the operation of *mod 2*, *i.e.* , $1 + 1 = 0$.

4. Critical review of connectivity calculation

The importance of the connectivity is emphasised in (Hunt, 1978), (Tischler et al., 2001), (Tischler et al., 1995), (Liberati and Belfiore, 2006), (Belfiore and Di Benedetto, 2000) and others, which drives the efforts to find an algorithm for the numerical calculation of connectivity. In this section, a critical review of the past contributions to the connectivity calculation is presented, and the limits of the various methods are analyzed.

4.1 Contribution of Tischler *et al.*

The concept of variety of a kinematic chain and its first definition is introduced in (Tischler et al., 1995). The relation between variety and connectivity is presented through a series of conjectures, which are referred to in (Tischler et al., 1995) as propositions and corollaries.

Variety is a useful property for determining the relative connectivities within a chain and also for selecting actuated pairs. Variety may also be used to classify kinematic chains according to the constraints required (Tischler et al., 2001). The definition of variety as proposed in (Tischler et al., 1995) is:

Definition 7 *A kinematic chain is* Variety V *if it does not contain any loop, or subset of loops, with a mobility less than $M - V$, but does contain at least one loop, or subset of loops, which has a mobility of $M - V$.*

In (Tischler et al., 1995) the relationship between variety and connectivity is summarised through a series of propositions, originally stated as conjectures, in the absence of counter-examples despite lacking formal proofs.

Conjecture 8 *(Tischler et al., 1995) If a variety V kinematic chain has a mobility less than, or equal to, the order of the screw system,* i.e. *if $M \leq \lambda$, any two links of the chain, separated by at least $M - V$ joints, have a relative connectivity $C \geq M - V$.*

Corollary 9 *If a variety V kinematic chain has a mobility greater than the order of the screw system that generally prevails,* i.e. *if $M > \lambda$, then any two links, separated by at least $\lambda - V$ joints, have relative connectivity $C \geq \lambda - V$.*

Corollary 10 *Two links separated by a minimum of g single-freedom joints, where $g < M - V$ and $g < \lambda - V$, have a relative connectivity $C = g$.*

The formal proof for conjecture 8 is given by the authors in (Martins and Piga Carboni, 2006) an a new algorithm for variety calculation is presented there (Martins and Piga Carboni, 2006).

4.2 Contribution of Shoham and Roth

Another important contribution to the automatic calculation of connectivities in a kinematic chain is found in (Shoham and Roth, 1997). Therein, a correspondence between kinematic chains and graphs is adopted and the connectivity matrix is introduced. A counter-example for the algorithm proposed by Shoham and Roth is presented in (Belfiore and Di Benedetto, 2000).

4.3 Contribution of Belfiore and Di Benedetto

A pure topological treatment of the problem of connectivity calculation is presented in (Belfiore and Di Benedetto, 2000). A new algorithm, referred to as the *topological method*, for the automatic calculation of degrees-of-control, connectivity and redundancy is proposed, derived from the method in (Shoham and Roth, 1997). A counterexample for the algorithm proposed in (Belfiore and Di Benedetto, 2000) is presented in (Liberati and Belfiore, 2006).

4.4 Contribution of Liberati and Belfiore

A new algorithm aiming at correctly detecting partial mobility chains and calculating their mobilities is introduced in (Liberati and Belfiore, 2006). The method is based on the concept of *gradual freezing of the circuits*, introduced in (Mruthyunjaya and Raghavan, 1984) to determine whether a link of a chain is a separation link. A counter-example for this algorithm is presented in (Piga Carboni and Martins, 2007).

4.5 Contribution of Piga Carboni and Martins

New definitions of connectivity, degrees-of-control and variety are introduced in (Martins and Piga Carboni, 2006) and (Piga Carboni and Martins, 2007). These new definitions, not conflicting with the previous ones, have an algorithmically oriented form, and identify a systematic procedure for the calculation of these parameters. A new algorithm is proposed which overcomes the deficiencies of the previous algorithms, and permits the calculation degrees-of-control, connectivity, redundancy and variety of a kinematic chain.

5. Redefined concepts

Of all the definitions presented in section 2, probably the most difficult to calculate is connectivity. The concept of connectivity is relatively easy to understand; however, Definition 2 does not provide a systematic procedure to obtain its value.

The need for a constructive method to obtain the connectivity prompted the authors to redefine connectivity and degrees-of-control in an algorithmically orientated form. These new definitions, listed below, are inspired by previous publications on connectivity, specifically (Shoham and Roth, 1997), (Belfiore and Di Benedetto, 2000), and (Liberati and Belfiore, 2006).

The new definitions are not in conflict with the previous definitions found in the literature, such as Definitions 2-3. Instead, alternatives way of defining the degrees-of-control and the connectivity are presented, which identify a systematic procedure for the calculation of these parameters.

Definition 11 *In a kinematic chain represented by a graph G, the connectivity between two links i and j is*

$$C_{ij} = \min : \{D_{\min}[i,j], M, M'_{\min}, \lambda\} \tag{2}$$

where $D_{\min}[i,j]$ is the minimum distance between vertices i and j of G, M is the mobility of the kinematic chain considered, $M'_{\min}$ is the minimum mobility closed-loop biconnected subchain of G containing vertices i and j, and λ is the order of the screw system.

Definition 12 *In a kinematic chain represented by a graph G, the degrees-of-control between two links i and j is*

$$K_{ij} = \min : \{D_{\min}[i,j], M, M'_{\min}\} \tag{3}$$

The definition of *redundancy* is based on the concepts of degrees-of-control and connectivity, as previously introduced in (Belfiore and Di Benedetto, 2000).

Definition 13 *In a kinematic chain represented by a graph G, the redundancy between two links i and j is the difference between K_{ij} and C_{ij}*

$$R_{ij} = K_{ij} - C_{ij} \tag{4}$$

A direct consequence of Definitions 11 and 12 is

$$C_{ij} = \min : \{K_{ij}, \lambda\} \tag{5}$$

Following Definition 11, 12 and 13, a new methodology for connectivity, degrees-of-control and redundancy calculation of a kinematic chain is proposed. The algorithm, based on the complete correspondence between kinematic chains and graphs, may be divided into three main parts.

In the first part, a graph representation of the kinematic chain is adopted and the incidence and adjacency matrix of the graph are built. The mobility and the number of fundamental circuits of the graph are evaluated. The minimum distance matrix D_{min} between each pair of links is calculated.

As stated in Definition 11, connectivity and redundancy values depend also on the mobility of the biconnected subchains of the kinematic chain examined. Hence, the second part of the algorithm is the enumeration of all possible closed-loop connected subchains.

In the last part, each biconnected subchain (more precisely, each biconnected subgraph) is analyzed, and the mobility evaluated. Each biconnected subchain is checked for properness and the algorithm stops if an improper subchain (see Section 5.1) is found. An improper kinematic chain is a kinematic chain where at least one biconnected subchain has mobility $M' \leq 0$.

Otherwise, based on Definition 11, 12 and 13, connectivity and redundancy are finally calculated.

5.1 Improper kinematic chains

An improper kinematic chain is a kinematic chain where at least one biconnected subchain has mobility $M^* \leq 0$. As an example of an improper kinematic chain, consider the kinematic chain in Figure 3a and its corresponding graph in Figure 3b. For the subchain formed by links 1-2-3-4-5-6-7-8-9, the mobility is $M^* = 0$ and the links act as a rigid body. A further inspection permits the identification of this subchain as a Baranov chain, or Baranov truss (Manolescu, 1979).

Improper chains are of no interest in pure kinematic analysis; therefore, the algorithm stops displaying an output message identifying the biconnected subchain $G^* \subset G$ with mobility $M^* \leq 0$.

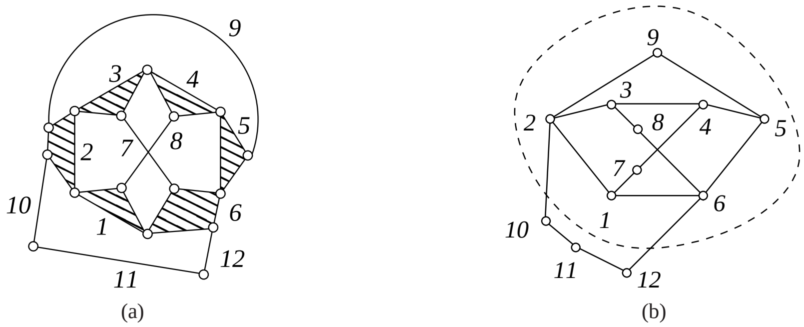


Figure 3: Improper planar kinematic chain with $M = 1$ and partial mobility ($V = 1$) because it contains a Baranov subchain G^* 1-2-3-4-5-6-7-8-9
(evidenced in dashed line)

6. Proposed algorithm for variety and connectivity calculation

Based on the previous definitions and theorems, let us examine the steps of the algorithm proposed. Let a kinematic chain with g joints and l edges be represented by its graph G.

1. Calculate the mobility M of the kinematic chain $M = g - \lambda\nu$ where λ is the order of the screw system and ν is the number of independent circuits; ν is obtained using Euler's equation or by inspection.

2. Build the minimum distance matrix $D_{\min}$, whose element $D_{\min}[r, s]$ is the minimum distance between vertices r and s.

3. Build the incidence matrix A_a of the graph G.

4. Build the adjacency matrix A_j of the graph G.

5. Enumerate all the circuits of the graph G: (a simple method is suggested in (Seshu and Reed, 1961)), *i.e.* considering the vector space of the circuits of the graph generated by the basis B_f of fundamental circuits. Alternative algorithms are proposed in (Liu and Wang, 2006), (Johnson, 1975), (Gibbs, 1969), (Honkanen, 1978) which permit a faster execution, because the sets of disjoint circuits are not generated.

6. A matrix B is generated, in which the columns are the edges of the graph, and the rows are all the circuits of graph G.

7. Enumerate all the biconnected subgraphs of graph G (every biconnected subgraph corresponds to a closed-loop subchain). We can consider the linear combinations of the rows of matrix B, using Boolean algebra. In this way, a large number of subgraphs are considered (*i.e.* non biconnected subgraphs are included). In fact, for the connectivity determination we need to consider biconnected subgraphs only; therefore, a biconnectivity test is useful to discard non biconnected subgraphs.

8. Copy graph G into graph G'.
9. Iterate steps 9.1-9.7 for each subgraph G_k of graph G:
   - 9.1 identify the vertices which belong to the subgraph represented by the row of matrix B examined (use the incidence matrix A_a)
   - 9.2 Calculate the mobility of the subgraph M_k.
   - 9.3 If $M_k \leq 0$ then exit the algorithm because an improper subchain exist.
   - 9.4 If $M_k \leq M$ continue from the following step, if $M_k > M$ consider a new subgraph
   - 9.5 Build the subgraph G_k, composed of the edges and vertices identified.
   - 9.6 Build the complete graph KG_k of G_k.
   - 9.7 For every edge $t - h$ of KG_k perform the following steps:
     - 9.7.1 Find every pair of vertices r and s of G that corresponds to the end of the edge $t - h$ of KG_k
     - 9.7.2 If $M_k < D_{\min}[r, s]$ then add to G' a virtual edge of weight equal to M_k.
10. Calculate a new matrix $D'_{\min}$ of the minimum distance between the vertices of graph G'.
11. Build the connectivity matrix C and redundancy matrix R in such a way that the element $C[i, j] = D'_{\min}[i, j]$ and $R[i, j] = 0$ if $D'_{\min}[i, j] \leq \lambda$. Otherwise, *i.e.* if $D'_{\min}[i, j] > \lambda$, $C[i, j] = \lambda$ and $R[i, j] = D'_{\min}[i, j] - \lambda$.

6.1 Application

Let us apply the algorithm to the kinematic chain of Figure 4a. It is a planar kinematic chain with $\lambda = 3$ and $M = 2$. Considering the corresponding graph G in Figure 4b, let us apply Steps 2 and 3 of the algorithm.

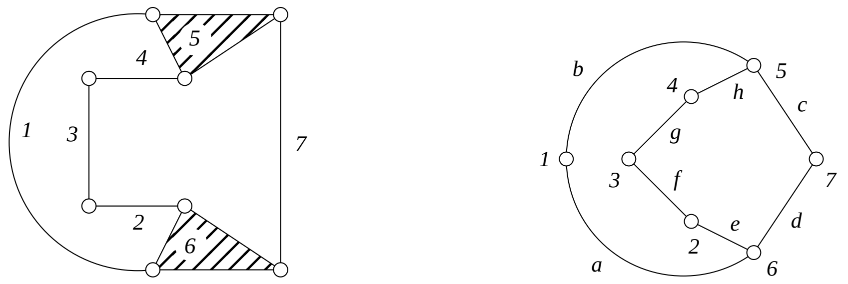


Figure 4: Planar kinematic chain with $M = 2$ and $\lambda = 3$ and its corresponding graph

The minimum distance matrix $D_{\min}$ and the incidence matrix A_j are:

$$D_{\min} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 2 & 3 & 2 & 1 & 1 & 2 \\ 2 & 0 & 1 & 2 & 3 & 1 & 2 \\ 3 & 1 & 0 & 1 & 2 & 2 & 3 \\ 2 & 2 & 1 & 0 & 1 & 3 & 2 \\ 1 & 3 & 2 & 1 & 0 & 2 & 1 \\ 1 & 1 & 2 & 3 & 2 & 0 & 1 \\ 2 & 2 & 3 & 2 & 1 & 1 & 0 \end{bmatrix} \end{array} \qquad (6)$$

and

$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad (7)$$

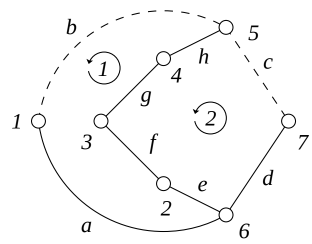


Figure 5: Minimum spanning tree of Graph G with a set of fundamental circuits

Considering (Step 5) the minimum spanning tree of Figure 5, two fundamental circuits are found in graph G. The matrix B_f of the fundamental circuits is obtained applying the method proposed in (Seshu and Reed, 1961).

$$B_f = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \quad (8)$$

Now matrix B (Step 6) is obtained, where each row represents a circuit of the graph.

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad (9)$$

Consider all linear combinations of the rows of matrix B using Boolean algebra. A new matrix B_s is obtained, where the string at the beginning of each row indicates the linear combination of the three rows of matrix B.

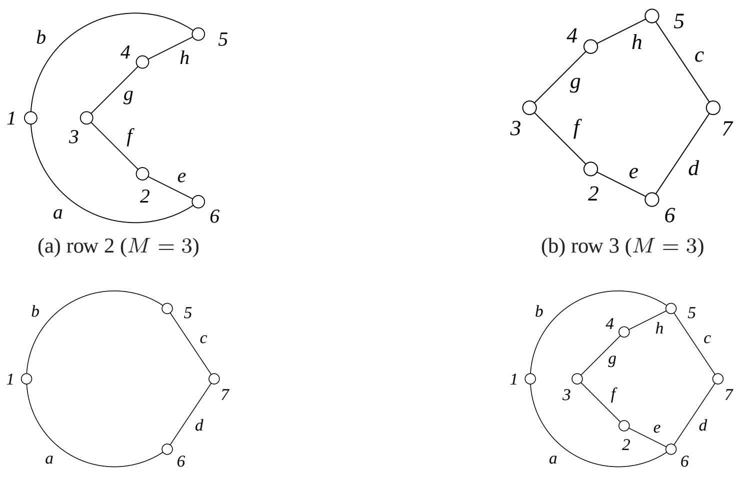
$$B_s = \begin{array}{c} \\ 0+0+0 \\ 1+0+0 \\ 0+1+0 \\ 0+0+1 \\ 1+1+0 \\ 1+0+1 \\ 0+1+1 \\ 1+1+1 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \quad (10)$$

The rows of matrix B_s represent all possible subgraphs of graph G; in fact, some subgraphs are repeated, *e.g.* the last four rows. It should be noticed that keeping repeated subgraphs does not affect the algorithm, but diminishes considerably the efficiency due to extra and unnecessary calculation. For the connectivity calculation, only biconnected subgraphs should be examined; hence a biconnectivity test needs to be applied to the subgraphs to be examined. In matrix B_s all independent biconnected subgraphs (Step 7) are represented by rows 2, 3, 4 and 5, as sketched in Figure 6.

Considering the incidence matrix A_a of graph G (which relates vertices to edges), as in Equation 7, for each set of edges of a subgraph G_k (a row of matrix B_s), it is possible to identify the set of vertices which belong to the same subgraph G_k. It is now possible to calculate the mobility of the subgraph using the mobility equation.

Consider now a copy G' of graph G. Applying to each subgraph of graph G the Step 9.1 - 9.7, we add virtual edges to graph G' where necessary. A useful representation for graph G' is given by the adjacency matrix A_j; a virtual edge of weight W between vertices i and j of graph G' may be added simply by setting $A_j[i,j] = W$. The adjacency matrix A_j of graph G' with all virtual edges added is:

$$A_j = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & 2 & 0 \\ 1 & 2 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \quad (11)$$



(a) row 2 ($M = 3$) (b) row 3 ($M = 3$)

(c) row 4 ($M = 1$) (d) row 5 ($M = 2$)

Figure 6: All different subgraphs of G as identified by lines 2, 3, 4 and 5 of matrix B_s

As we found that the biconnected subgraph in Figure 6c, corresponding to row 4 of matrix B_s, has mobility $M = 1$.

Finally (Steps 10 - 11), the minimum distance matrix $D'_{\min}$ of graph G' can be calculated, and the connectivity matrix of the kinematic chain in Figure 4a is evaluated as:

$$C = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 2 & 1 & 1 & 1 \\ 2 & 0 & 1 & 2 & 2 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 & 2 & 2 \\ 2 & 2 & 1 & 0 & 1 & 2 & 2 \\ 1 & 2 & 2 & 1 & 0 & 1 & 1 \\ 1 & 1 & 2 & 2 & 1 & 0 & 1 \\ 1 & 2 & 2 & 2 & 1 & 1 & 0 \end{bmatrix} \end{array} \qquad (12)$$

and the redundancy matrix R is null. In short, we have a kinematic chain with no redundancy at all, and connectivity shown in Equation (12).

7. Conclusion

A new definition of *connectivity* and *degrees-of-control* has been introduced. These new definitions, not conflicting with the previous ones found in literature, are built in an algorithmically orientated form, and identify a systematic procedure for the calculation of these parameters.

Based on these definitions, a new methodology for the calculation of connectivity and redundancy is proposed. The new algorithm may be applied to kinematic chains with full mobility (variety $V = 0$) and partial mobility ($V \neq 0$). The full set of connectivities, degrees of control and redundancies is calculated. The algorithm may be easily extended to partial mobility kinematic chains (chains with cut edges or cut vertices) applying the algorithm to the biconnected components.

Considering the mobility of all possible subchains, the connectivity is correctly evaluated in the kinematic chains where previous algorithms had some flaws.

The algorithm here proposed is a valid solution for kinematic chains with a small number of independent loops, otherwise the number of subchains may increase dramatically, and the computational time required to perform the analysis may be excessively long. However, to the authors' knowledge, this is the first algorithm that accurately calculates connectivity and redundancy in all cases, without exception.

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9. Responsibility notice

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