MEASUREMENT OF THE VOLUMETRIC FRACTION AND INTERFACIAL AREA IN DISPERSED OIL-WATER FLOWS BY ACOUSTIC SENSING AND NEURAL NETWORK PROCESSING.

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Abstract. *An innovative method for measuring the volumetric fraction and interfacial area in oil-water flows by acoustic sensing and neural network processing is proposed in this work. Synthetic data, obtained from numerical simulations of the acoustic differential equation, were used to demonstrate the feasibility of the proposed method. The geometrical distribution of the phases within the flow is mapped by the local acoustic propagation velocity, considered in the governing differential equation. This equation is solved numerically by the Finite Difference Method with boundary conditions reproducing the excitation/measurement strategy. A significant number of propagation velocities distributions were considered in the solution of the equation in order to construct a database from which the neural model parameters could be adjusted. The neural model is constructed to map the features extracted from the signals delivered by four acoustic sensors, placed on the external boundary of the sensing domain, into the corresponding volumetric fraction and interfacial area. These features correspond to the time of arrival of the acoustic wave and the corresponding amplitudes and times of the first three peaks. Numerical results show that the neural model can be trained in a reasonable computational time and it is capable of assessing the values of the volumetric fraction and the interfacial area of examples in the test database.*

Keywords: *volumetric fraction, interfacial area, two-phase flows, neural network, acoustic sensing.*

1. INTRODUCTION

The acoustic sensing of industrial processes appears an extremely interesting technique, mainly in applications involving transport and manipulation of multiphase flow. Therefore, physical parameters (flow rate, density, viscosity, volumetric fraction, temperature and others) can be measured and it is possible to obtain more complete information on the phenomenology of the process, normally expressed through images directly mounted or reconstructed from acoustic responses signals (Lynch, 1991). Amongst the main advantages of the acoustic sensing (robustness, immunity to electric noises, low cost and others), the possibility of having a non-intrusive and non-invasive sensing is fundamental for some applications where more direct interventions are not possible. In the case of the acoustic sensing, the property that the ultrasound has to cross different materials allows sensing to be performed by a simple contact with the walls of the tubing.

Thus, it is possible to conclude that the present work has great importance in researches, as one of its objectives is the development of a numerical tool for the acoustic sensing of multiphase flow. More specifically, it aims to determine and quantify how the fields of acoustic sensing are established, in view of their interaction with the flow, as well as with the geometry of the tubing. This is, in fact, a very important problem due to the multiplicity of two-phase interfaces between different acoustic impedances, which it also generates multiple phenomena of reflection, absorption and scattering, so that the structure of the signal obtained as a response to the excitement can be sufficiently complex and its processing extremely delicate.

Such a numerical simulator to study the acoustic propagation in heterogeneous media was developed allowing for the accomplishment of some numerical tests to form a database for the training of artificial neural networks. Specifically, the aim is to implement neural networks that, from the distribution of acoustic variables (arrival times of the acoustic wave, amplitude of the acoustic signals and others), are capable of supplying parameters of the two-phase flow, amongst which the volumetric fraction and the interfacial area are certainly most important.

According to the literature, as well as to the numerical results obtained in this work, the flow of phases with different acoustic properties makes the propagation of acoustic waves extremely complex as well as the interpretation of their response signals. This interpretation can be performed according to two distinct approaches:

(i) through tomographic reconstruction, i.e., calculation of the internal distribution of the acoustic impedance from the measurements of acoustic intensity (Schlaberg, Podd and Hoyle 2000);

(ii) use of artificial neural networks adjusted from a database of training to map the set of possible measurements of acoustic variable on the corresponding topological parameters of the flow.

Both cases are about the solution of an inverse and intrinsically ill-conditioned problem. Therefore, to obtain responses in realistic experimental conditions, concerning the level of noises, for example, the use of techniques of numerical regularization is essential. The tomographic approach, which is generally implemented off-line, allows applying some techniques based on redundant measurements (Rolnik and Seleghim, 2002; Tichonov, 1963) or TSVD (Truncated Singular Value Decomposition - Press et al., 1992). On the other hand, the use of neural networks has the bad conditioning associated with the adjustment of their internal parameters, which can be made a priori, even if it demands similar methods of regularization. Consequently, once trained, the neural network "learns" to regularize the inverse problem and can be implemented in real time.

The main objective of this work is to verify the possibility, through numerical simulations, of developing a methodology for the non-intrusive measurement of the volumetric fraction and the interfacial area of liquid-liquid two-phase flows, using acoustic sensing and solving the associated inverse problem. Therefore, the problem was investigated through the development and training of neural networks for the solution of the inverse problem.

The networks are trained with synthetic data from numerical simulations. For the accomplishment of such simulations, a numerical tool capable of calculating the acoustic propagation in heterogeneous media was developed. This simulator was used to construct a database constituted of distributions of acoustic contrast (ratio between the acoustic propagation impedances) randomly generated. For each numerical test performed, the volumetric fraction and the interfacial area were obtained, besides four acoustic signals, from which the arrival times of the acoustic pulses, the amplitude of the three highest peaks of each signal, as well as their arrival times. After concluding this stage, the implementation of neural networks and studies was performed to obtain optimized architectures regarding the problem. Next, the training of the best models with examples extracted from the database was realized, in order to determine the parameters, such as volumetric fraction and interfacial area. In particular, it was possible to verify that these models can be trained in a reasonable computational time. The reliable intervals for the neural networks responses were found to be compatible with future practical applications.

2. STATEMENT OF THE PROBLEM

The basic idea is to apply an excitation pulse through a piezoelectric element and to capture the corresponding responses through measuring piezoelectric elements. As previously mentioned, despite the excitation pulse being simple in shape, a Dirac delta, for example, the measured responses usually have a complex shape due to multiple echoes and diffraction associated with the multiphase mixture and pipe walls. Since the speed of sound is much higher than the velocities of the multiphase flow, which is a good assumption in liquid-liquid mixtures, the equation describing the propagation of acoustic waves can be written as (Kinsler and Frey, 1962)

$$\frac{\partial^2 P(t,\overrightarrow{x})}{\partial t^2} - c^2(\overrightarrow{x})\nabla^2 P(t,\overrightarrow{x}) = 0, \quad \text{para } (t,\overrightarrow{x}) \in \mathbb{R}_+ \times \Omega \tag{1}$$

Variables P and c represent the acoustic pressure and the local speed of sound, respectively. It is important to stress that the local speed of sound maps the phase distribution within the measurement volume indicated by Ω. The initial and boundary conditions modeling the excitation/measurement strategy are given by

$$P(0,\overrightarrow{x}) = \begin{cases} 1, & \text{se } \overrightarrow{x} \in L_{exc} \\ 0, & \text{se } \overrightarrow{x} \in \Omega - L_{exc} \end{cases} \tag{2}$$

$$\overrightarrow{n}.\overrightarrow{\nabla} P(t,\overrightarrow{x}) = 0, \quad \text{para } (t,\overrightarrow{x}) \in \mathbb{R}_+^* \times \partial\Omega \tag{3}$$

$$R_k(t) = \oint_{s \in L_k} P(t,\overrightarrow{x})ds, \quad \text{para } L_k \in \partial\Omega \tag{4}$$

L_{exc} in Eq. (2) denotes the excitation element. $R_k(t)$ and L_k in Eq. (4) represent, respectively, the response signal and the $k-th$ measuring element (which may correspond to L_{exc}).

Equations (1) to (4) constitute an ill-posed inverse problem, as P and c are unknown in the measurement volume V and the boundary conditions are known only partially, i.e. local average values on the measuring elements. Given specifics experimental conditions, the number and extension of the measuring elements for instance, the satisfactory reconstruction of the geometric structure of the flow is still an open question. It is more likely that, under strict mathematical conditions, this reconstruction is not possible from the response signals obtained from a practical sensor configuration.

The acoustic pressure signals acquired in this work through numerical simulations have a very complex structure, even in response to an only Dirac-type pulse, applied to the point of intersection of the positive half-axis of the abscissas with the boundary of the domain (circular geometry).

Such complexity is due to the multiple echoes in the walls of the tubing as well as to the reflections and dispersion of the acoustic wave. Another factor that influences the form of the signals is the configuration of the fluid, as the acoustic propagation speed is different in both continuous (water) and dispersed (oil) phases. As the geometric distribution of the phases in the fluid is mapped by the local speed of propagation considered in the differential equation that governs the problem, the acoustic propagation speed was used in water, that is, $c_{\text{água}} = 1,48\frac{Km}{s}$ and in oil, $c_{\text{óleo}} = 1,7\frac{Km}{s}$ (oil of engine SAE 30). In this work four responses signals $R_k, k = 1, ..., 4$, given by Eq. (4) were obtained by numerical tests.

The positions of the measurement elements $L_k \in \partial\Omega, k = 1, ..., 4$ can be seen in Fig. 1. It can be observed that the measurement element L_1 is responsible for the application of the Dirac pulse and also for the attainment of a signal. Elements $L_k, k = 2, 3, 4$ only acquire acoustic signals.

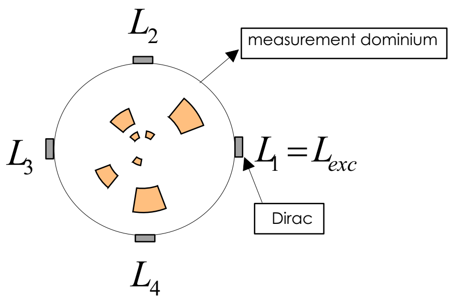


Figure 1. Position of the measurement elements in the circular geometry.

Some characteristics of these signals, such as arrival time, T, of the acoustic wave in each measurement element, amplitude of the three highest peaks of each signal, A_1, A_2 and A_3 , as well as arrival time of the peaks, indicated by T_1, T_2 and T_3 in Fig. 2, were extracted to characterize the complexity of the signal in a sufficient level for the inference of both interfacial area and volumetric fraction.

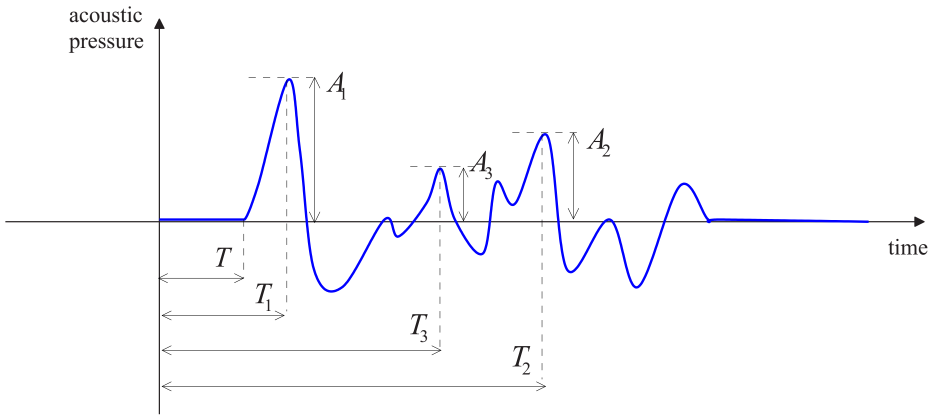


Figure 2. Acoustic signal and respective extracted characteristics.

3. NEURAL NETWORK MODELS

A neural network can be defined as a nonlinear mapping of an input onto an output vector space. This is achieved through layers of activation functions or neurons in which the input coordinates are summed according to specific weights and bias to produce single output or firing values. In this work, a feed forward network was used for which there is no recursiveness, i.e. the input vector of a specific neuron layer is formed only by the firing values of the preceding layer, as shown in Fig. 3.

Formally, if the activation function of i-th neuron in the j-th layer is indicated by $F_{i,j}(.)$, its output $s_{i,j}$ can be calculated from the outputs of the preceding layer $s_{i,j-1}$ and the corresponding bias $b_{i,j}$ and weights $w_{i,k,j-1}$ (the second subscript k indicates the neuron in the $(j-1)$-th layer from which the connection is being established), according to the expression

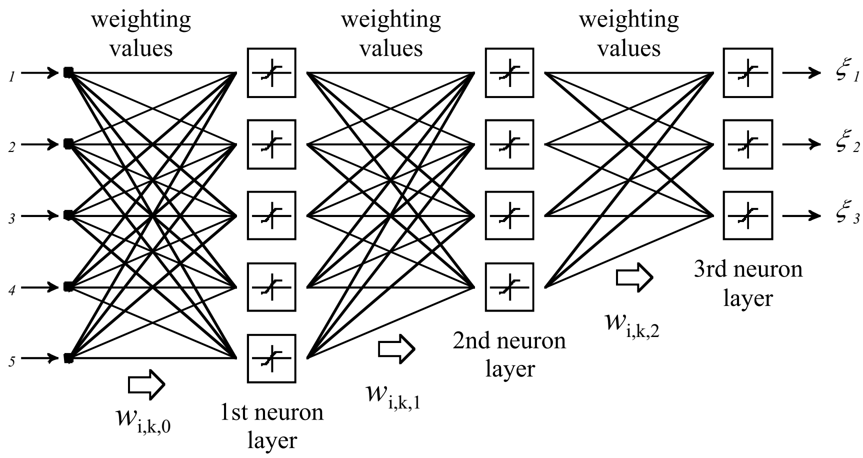


Figure 3. Schematic representation of a feed forward neural network mapping a five-coordinate input vector onto a three-coordinate output vector.

$$s_{i,j} = F_{i,j}\left(b_{i,j} + \sum_k w_{i,k,j-1} s_{k,j-1}\right) \tag{5}$$

After denoting the network's input and output values, respectively, by η_i, and ξ_i, the mapping relation of one onto another can be calculated by successively applying Eq. 5, which for the example in Fig. 3 results

$$\xi_i = F_{i,3}\left(b_{i,3} + \sum_k^4 w_{i,k,2} F_{k,2}\left(b_{k,2} + \sum_m^5 w_{k,m,1} F_{m,1}\left(b_{m,3} + \sum_n^5 w_{m,n,0}\eta_n\right)\right)\right) \tag{6}$$

Equation (6) makes it clear that the relation between η_i, and ξ_i, is unambiguously defined by choosing the activation functions and by setting the bias and weights. Among many, a very important characteristic of neural networks is the so-called learning potential, i.e. the possibility of adjusting the bias and weights through a convenient training rule to closely reproduce pre-assigned pairs of input/output values. The back-propagation is probably the most used training heuristics and is particularly well adapted to feed forward architectures. It is based on the iterative application of a discrete gradient descent algorithm, computed from the first derivatives of a conveniently defined error function whose arguments are the parameters of the network (weights and bias). In general lines the basic steps of the back-propagation procedure implemented in this work are the following:

(i) Initialize the parameters of the network $b_{i,j}$ and $w_{i,k,j}$ with random numbers;

(ii) From a training data set with pre-assigned input/output pairs take the p-th (η_i^p, δ_i^p) pair, calculate the outputs of the network with the same input and form the pair (η_i^p, ξ_i^p);

(iii) Calculate the error between the desired (δ_i^p) and the obtained (ξ_i^p) output values according to the Euclidean norm:

$$e = \sqrt{\sum_i (\delta_i^p - \xi_i^p)^2} \tag{7}$$

(iv) Calculate the derivatives of error e with respect to $b_{i,j}$ and $w_{i,k,j}$;

(v) Modify the network parameters according to the steepest descent strategy and a specified learning rate γ:

$$b_{i,j} \leftarrow b_{i,j} - \gamma \frac{\partial e}{\partial b_{i,j}} \tag{8}$$

$$w_{i,k,j} \leftarrow w_{i,k,j} - \gamma \frac{\partial e}{\partial w_{i,k,j}} \tag{9}$$

(vi) Iterate from 2 to 5, successively modifying $b_{i,j}$ and $w_{i,k,j}$, until a defined number of learning epochs (cycles) or a convenient stopping criterion has been achieved.

The performance of a neural network is profoundly affected by its internal architecture (number of hidden layers and number of neurons in each one) and the type of interconnections (feed-forward, recursive, winner-take-all, etc.). The exact shape of the activation function has limited effects on the overall performance and is usually set according to the needs of the training heuristics (a sigmoid function in the case of back-propagation method). There is no general mathematical theory but rather a number of empirical rules to be considered when constructing such models.

The neural networks used in this work were implemented in Neural Networks Toolbox of Matlab 6.0 in an AMD Athlon (TM) XP 2600+ computer with 2,08 GHz and 1,5 GB RAM. The architecture of the neural network was defined from preliminary studies in the following way: input layer, two intermediate layers and an output layer. The number of neurons of the intermediate layers was adjusted to enable the nets to learn complex tasks for the gradual extraction of significant information from the inputs (Haykin, 1999; Hagan, Demuth and Beale, 1996). The architecture of the neural network used was 24 - 15 - 11 -2. The activation functions used in neural network are the tangent sigmoid in the intermediate layers and the linear function in the output layer. Each neuron of the output layer is responsible for estimating one of the following parameters of the dispersed two-phase flow: interfacial area and volumetric fraction. The training procedure comprehended the acquisition of acoustic pressure signals of the flow of interest for each numerical simulation performed and the extraction of some particularities that characterize the complexity of the signal in a level sufficient for the inference of the interfacial area and the volumetric fraction to compose the training set.

4. NUMERICAL SIMULATIONS

Considering the geometry of the problem, mainly the layout of the measurement elements, it is justifiable to disregard the axial variations of form to obtain a bidimensional equation. Therefore, making the terms of Eq. 1 explicit for a bidimensional problem, where , one obtains

$$\frac{\partial^2 P}{\partial t^2} - c_0^2 c^2(x,y)\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) = 0, \quad \text{em } \Omega \tag{10}$$

where P is the acoustic pressure, $c(x,y)$ is the local speed acoustic propagation, t is the time, c_0 is the reference speed, Ω is the domain of the problem and (x,y) represents a point in the interior of Ω. In the bidimensional formulation, the interfacial area and the volumetric fraction are calculated, respectively, by

$$\Gamma(t) = \frac{\sum P_i(t)}{A} \tag{11}$$

$$\alpha(t) = \frac{\sum A_i(t)}{A} \tag{12}$$

where P_i and A_i are the instantaneous perimeter and the area of all the circumscribed drops in the transversal section of the tubing, respectively, and A is the total area of the transversal section.

4.1 Discretization of the differential equation in polar coordinates

To perform numerical simulations in circular section pipes of radius R, eq. 10 was written in polar coordinates, r and θ, where $x = r\cos\theta$ and $y = r\sin\theta$ obtaining

$$\frac{\partial^2 P}{\partial t^2} - c_0^2 c^2(r,\theta)\left[\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{1}{r^2}\frac{\partial^2 P}{\partial \theta^2}\right] = 0, \tag{13}$$

Equation 13 was non-dimensional and discretized using the Finite Differences Method, obtaining

$$\frac{1}{\Delta t^2}\left(P_{i,j}^k - 2P_{i,j}^{k-1} + P_{i,j}^{k-2}\right) - \frac{c_{i,j}^2}{\Delta r^2}\left(P_{i+1,j}^k - 2P_{i,j}^k + P_{i-1,j}^k\right) - \frac{c_{i,j}^2}{r_{i,j}}\frac{1}{2\Delta r}\left(P_{i+1,j}^k - P_{i-1,j}^k\right) - \frac{c_{i,j}^2}{r_{i,j}^2}\frac{1}{\Delta\theta^2}\left(P_{i,j+1}^k - 2P_{i,j}^k + P_{i,j-1}^k\right) = 0 \tag{14}$$

where i and j indicate the coordinates in the mesh in the r and θ directions, respectively, $\Delta t = 10^{-7}$, $\Delta r = \dfrac{R}{N}$ and $\Delta\theta = \dfrac{2\pi}{M}$, where N and M are the numbers of points in $[0, R]$ and $[0, 2\pi]$ intervals, in the r and θ directions, respectively.

Equation 13 presents a problem: it is not defined for $r = 0$. There exist at least three different ways to overcome this situation, according to Davis (1979):

(i) avoiding the fall of nodes of the mesh onto the point where $r = 0$;

(ii) using L'Hôpital's rule to eliminate the indefinition;

(iii) using cartesian coordinates in the central point.

The third alternative was used in this work, and its principle is the discretization of Eq. 13 in the central point using cartesian coordinates, i.e., Eq. 10 in $r = 0$ was applied to the numerical model of the circular pipe, while Eq. 14 was utilized in the other points of the mesh, generating a linear system whose unknowns are the nodal values of P's.

Concerning the application of this strategy, special care must be taken in the partitioning of the interval $[0, 2\pi]$. The number of points in this interval must be defined in such a way that two segments of straight line are contained in the cartesian axis. It can be observed in Fig 4. that the mesh defined in the circular domain possesses two orthogonal segments $\overline{AB}$ and $\overline{CD}$ contained in a cartesian axis. ($\overline{AB}$ in axis x and $\overline{CD}$ in axis y).

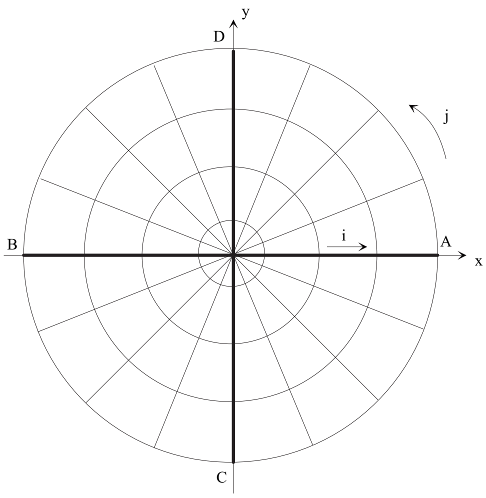


Figure 4. Mesh in polar coordinates.

4.2 Storage and solution of linear systems

For the storage of the matrices of linear systems, the idea presented by Press et al. (1992) in the book Numerical Recipes was used. It consists of a method of storage indexed by lines, where only nonzero elements of the matrix are stored. Therefore, assuming a dimension matrix $N \times N$, two vectors are used:

(i) a vector of real elements (with simple or double precision), responsible for storing the nonzero values of the matrix;

(ii) a vector of entire elements, responsible for the storage of the indices of the nonzero values.

The main advantage of this technique, beyond optimizing the use of the memory, is that the stored data can be accessed faster, considerably reducing the time spent on the resolution of the linear system. Concerning the assembly of the storage vectors, the algorithm suggested by Press et al. (1992) is based on the sweepings of all original array elements to identify those different from zero, requiring therefore, an excessively long time. In the present work, the subroutine developed for the storage of the nonzero elements of the matrix was based on the idea of Press et al. (1992) and also on the knowledge of the structure of the matrix. Thus, the two vectors of storage were built directly, significantly reducing the time of calculations.

The linear systems were solved by the Method of Pre-Conditioned Bi-Conjugated Gradients (PBCG), which is a generalization of the Method of the Conjugated Gradient, with the advantage of solving any linear system since the matrix is not singular. In the implementation, a ready routine of PBCG of the computational package Numerical recipes in Fortran (Press et al., 1992) was used.

5. RESULTS AND DISCUSSION

The results of the training of the neural networks showed that they are capable of reproducing the input/output relation of the data of the training set, for both volumetric fraction and interfacial area. To evaluate the generalization capacity of RNAs, the characteristic data of numerical tests with new distributions of local speed of acoustic propagation were presented to the neural networks. A good correlation among the input and output data could be observed for the cases of volumetric fraction and interfacial area, as seen in the following sub-section.

5000 numerical simulations were performed (4000 used in the training of neural networks and 1000 to test the network's generalization capacity) to compose the database from which the characteristics of the signals supplied to the RNA were extracted. The training set was composed of 4000 examples, where each one represented a numerical test performed. To evaluate the generalization ability of the RNAs, the characteristic data of 1000 new numerical tests with new distributions of local speed of acoustic propagation were presented to the neural networks. The learning rate used in the neural network was 0,1 and the training time was 15 hours, approximately. The network was trained during 200000 epochs, since after that the variation of the error stabilized, as shown in Fig. 5. Although the error of training of the neural network was not so small (of the order of 10^{-1}), it could provide good test results, which will be further shown.

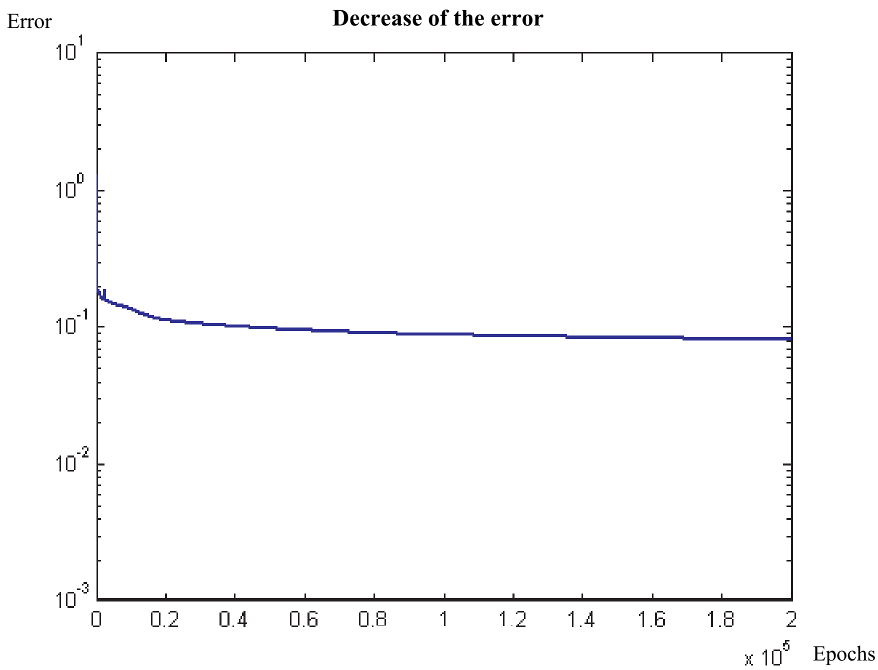
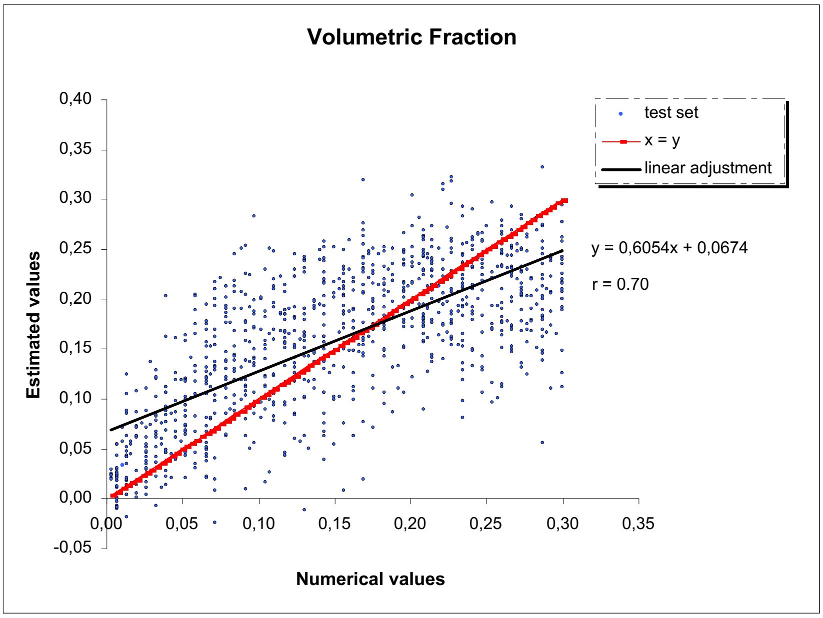


Figure 5. Decrease of the error during the training of the neural network to estimate the volumetric fraction and the interfacial area for the data obtained for circular geometry.

Figure 6 shows the comparison among the values estimated for the neural network and the numerical values for the volumetric fraction and the interfacial area, respectively, for the data extracted from the signals obtained in the circular geometry, referring to the examples contained in the test set. In the graph, the straight line $x = y$ represents the ideal result and the blue points are the values estimated through the neural network.

The straight lines represented in the graphs by the black line are the ones that better represent the correlation among the values estimated by the RNA and numerical values. The equation for the case of the volumetric fraction is $y = 0,6054x + 0,0674$ and for the interfacial area it is $y = 0,5539x + 0,3688$ (Yong, 1962; Doebelin, 1990). The correlation coefficient among the values of the volumetric fraction was 0,7, while for the interfacial area it was 0,65. It is possible to observe that the data are correlated and the neural network is capable of providing good results.

The standard deviation of the volumetric fraction values is 0,06, while for the interfacial area it is 0,33. Therefore, it is possible to plot the straight lines $y = 0,6054x + (0,0674 \pm 3 \times 0,06)$ that form an envelope around the points referring to the values of the volumetric fraction and the straight lines $y = 0,5539x + (0,3688 \pm 3 \times 0,33)$ around the values of the interfacial area, as it can be seen in the graphs of Fig. 7, which show the reliable interval of the results obtained by the RNA.



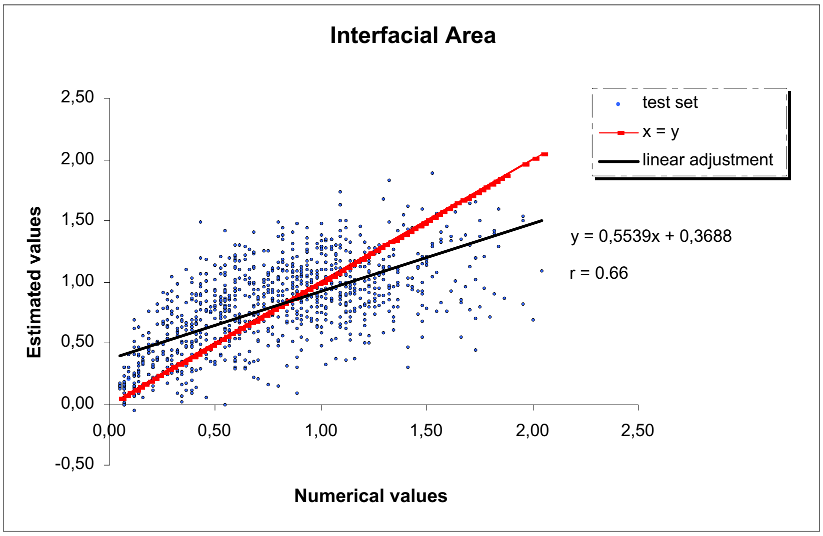
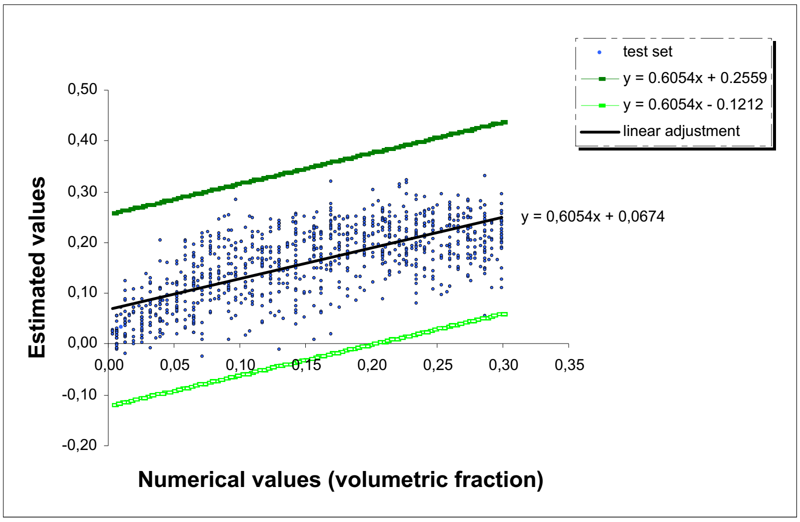


Figure 6. Comparison between the values estimated by the neural network and the numerical values for the volumetric fraction and the interfacial area, respectively.



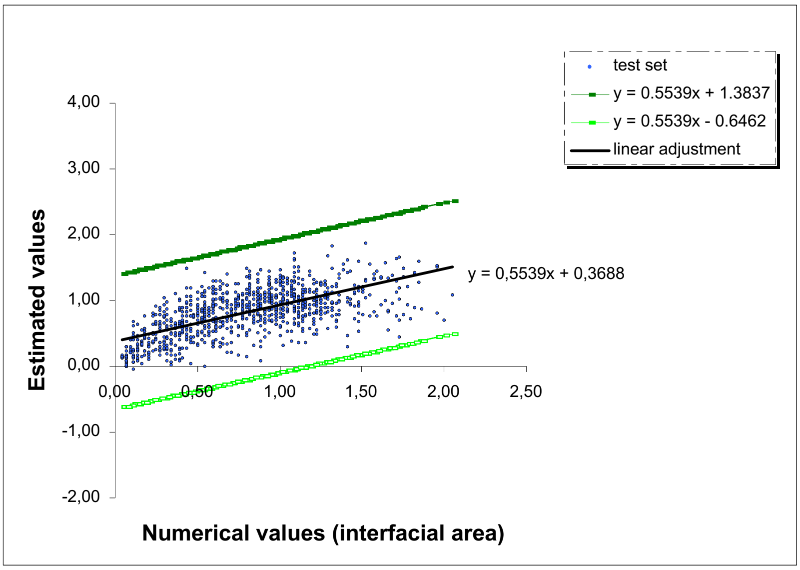


Figure 7. Reliable interval of the results supplied by the RNA for the values of the volumetric fraction and the interfacial area, respectively.

6. CONCLUSIONS

A new methodology for the non-intrusive measurement of the volumetric fraction and the interfacial area of two-phase flow was proposed in this work, using acoustic sensoring and solving the inverse problem associated through the development and training of artificial neural networks. The mathematical model was constructed from the acoustic propagation equations. The geometric distribution of the phases inside the flow was mapped by the local speed of acoustic propagation, considered in the differential equation that governs the problem. This equation was numerically solved by the Finite Differences Method with the boundary conditions reproducing the strategy of pulse/echo.

The neural networks were trained with data derived from numerical simulations. For the realization of such simulations, a numerical tool was developed to calculate the acoustic propagation in heterogeneous ways. A significant number of distributions (random) of acoustic propagation speeds were considered in the solution of the differential equation to construct a database, from which the parameters of the neural network (weights and bias) could be adjusted. More specifically, the neural models were constructed to map characteristics extracted from signals obtained from four acoustic sensors, located in the external boundary of the sensoring domain, estimating the corresponding volumetric fraction and interfacial area. These characteristics correspond to the amplitude and the arrival times of the three highest peaks of the acoustic wave.

After concluding this stage, the implementation of artificial neural networks and preliminary studies were carried out aiming to obtain architectures optimized regarding the problem. After that, the training of the best models with examples extracted from the database was performed to determine the volumetric fraction and the interfacial area, which are important parameters of the two-phase flow.

The results of the neural networks were quite satisfactory, as they showed to be capable of estimating the values of the volumetric fraction and the interfacial area for examples that were presented to them in the training. It was also verified that these models are trained in a reasonable computational time.

7. ACKNOWLEDGEMENTS

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