MODAL PARAMETERS IDENTIFICATION USING THE MULTIVARIABLE ARX METHOD

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Abstract. *A considerable amount of researches have focused on the problem of modal parameters estimation. The paper presents, both, the basic formulation and numerical implementation of the time-domain multivariable autoregressive with exogenous input (ARX) model applied to modal analysis identification. The ARX parameter matrices are calculated via least- squares minimizing process. The proposed method is valid for multi input-multi output (MIMO) experimental modal tests. Natural frequencies, damping factors and mode shapes are estimated. In order to present the main characteristics of the method, numerical simulation of a mechanical system is conduced to obtain the input and output sets of data to be used in the test and to compare the difference between the exact and identified system's parameters.*

Keywords: *modal parameters identification, multivariable ARX, signal processing*

1. INTRODUCTION

Parametric system identification is the field of modeling dynamic systems from experimental data (Söderström and Stoica, 1989 and Ljung, 1987). References (Söderström and Stoica, 1989 and Ljung, 1987) approach the subject in a more general form, i.e., a model is fitted to the recorded data by assigning suitable numerical values to its parameters. Such application in modal analysis is of great importance and has an enormous field of interest in both practical and scientific applications see for example references (Maia and Silva, 1997, Gontier et al., 1993 and Juang, 1997). Modal parametric identification deals with the problem of estimating natural frequencies, damping factors and mode shapes of vibratory systems. Most usable experimental data in modal tests are naturally the impulsive responses (IR) or input-output data which are obtained from the action of impact hammer or shakers and accelerometers mounted on the structure to be analyzed.

The mathematical model commonly used for mechanical system is a time invariant matrix second order differential equation. The above references suggest the use of the discrete-time auto-regressive (AR) or auto-regressive with exogenous input (ARX) models to fit, respectively, the provided IRs and input-output data. The principal characteristic of these approaches is the using of least squares (LS) minimization procedure to calculate the coefficient matrices of such models.

Multiple input and multiple output (MIMO) modal testing has many advantages when compared to single input and single output techniques, especially when dealing with larger structures. The force from multiple inputs allows a more uniform distribution of excitation energy throughout the structure, improving the accuracy of identified modal parameters and reducing the testing time. The problem has attracted much attention because of its broad application in many fields.

The present work shows the result of using a multivariable ARX identification process for the determination of modal parameters of vibratory systems valid for the MIMO situation. In order to evaluate the capabilities and limitations of the presented method, simulated data is generated to obtain results of application of the technique.

2. BASIC FORMULATION

2.1. Dynamic System Equations

The motion of a f degrees of freedom linear time invariant mechanical system is dictated by the following second order matrix differential equation,

$$\mathbf{M}\,\ddot{\mathbf{z}}(t)+\mathbf{C}\,\dot{\mathbf{z}}(t)+\mathbf{K}\,\mathbf{z}(t)=\mathbf{f}(t) \tag{1}$$

where **M**, **C** e **K** are the $f \times f$ mass, damping and stiffness matrices, respectively. The $f \times 1$ vectors $\mathbf{z}(t)$ e $\mathbf{f}(t)$ represent, respectively, the generalized displacement and the external forcing which acts on the system. Equation (1) can be expressed in an equivalent continuous time state form (Maia and Silva, 1997 and Gountier et al., 1993) as,

$$\overline{\mathbf{A}}\dot{\mathbf{x}}(t)+\overline{\mathbf{B}}\mathbf{x}(t)=\overline{\mathbf{f}}(t) \tag{2}$$

where $\mathbf{x}(t)=\{\mathbf{z}(t) \quad \dot{\mathbf{z}}(t)\}^T$ is the *n* x 1 state vector, with $n = 2f$, the index *T* denotes vector transposition and

$$\overline{\mathbf{A}}=\begin{bmatrix}\mathbf{C} & \mathbf{M}\\ \mathbf{M} & \mathbf{0}\end{bmatrix},\ \overline{\mathbf{B}}=\begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}\end{bmatrix} \text{ and } \overline{\mathbf{f}}(t)=\begin{Bmatrix}\mathbf{U}_f\,\mathbf{u}(t)\\ \mathbf{0}\end{Bmatrix} \tag{3}$$

are, the first and second, $n\times n$ matrices and the third $n\times 1$ vector. The $m\times 1$ vector $\mathbf{u}(t)$ represents the *m* non null efforts of the input and the $f\times m$ matrix $\mathbf{U}_f$ is an ordering input matrix. The term **0** denotes null vector and null matrix of appropriated dimensions.

The dynamic Eq. (2) of the free system,

$$\overline{\mathbf{A}}\dot{\mathbf{x}}(t)+\overline{\mathbf{B}}\mathbf{x}(t)=\mathbf{0} \tag{4}$$

has a solution which is assumed to be of the form $\mathbf{x}(t)=\Psi_j e^{\lambda_j t}$, leading to the standard eigenvalue problem,

$$\left(\lambda_j\overline{\mathbf{A}}+\overline{\mathbf{B}}\right)\Psi_j=0\,. \tag{5}$$

Solutions of such a problem are found to be a set of *n* eigenvalues (or poles) λ_j and *n* associated eigenvectors Ψ_j. If the system is assumed to be underdamped, the eigenvalues appear in complex conjugate pairs such as,

$$\lambda_j,\lambda_{j+1}=-\omega_j\xi_j\pm i\omega_j\sqrt{1-\xi_j^2}\,, \tag{6}$$

where the terms $\omega_j=\left|\lambda_j\right|$, $\xi_j=\mathrm{Re}(\lambda_j)/\omega_j$ denote, respectively, natural frequency and damping factor and $i=\sqrt{-1}$.

Due to the nature of state vector $\mathbf{x}(t)=\{\mathbf{z}(t) \quad \dot{\mathbf{z}}(t)\}^T$, the solution set of the eigenvalue-eigenvector problem is condensed in a $n\times n$ spectral matrix $\overline{\mathbf{\Lambda}}$ and a $n\times n$ modal matrix $\mathbf{\Psi}$ which have the following form,

$$\overline{\mathbf{\Lambda}}=\begin{bmatrix}\mathbf{\Lambda} & \mathbf{0}\\ \mathbf{0} & \mathbf{\Lambda}^*\end{bmatrix} \text{ and } \mathbf{\Psi}=\begin{bmatrix}\mathbf{\Phi} & \mathbf{\Phi}^*\\ \mathbf{\Phi\Lambda} & \mathbf{\Phi}^*\mathbf{\Lambda}^*\end{bmatrix}. \tag{7}$$

The symbol * denotes complex conjugation, and the $f\times f$ matrices, $\mathbf{\Lambda}=diag\{\lambda_1,\cdots,\lambda_f\}$ and $\mathbf{\Phi}=\lfloor\phi_1 \quad \phi_2 \quad \cdots \quad \phi_f\rfloor$ contain, respectively, the poles λ_j and corresponding mode shapes ϕ_j of the system, for *j=1,2,…,f*.

The modal matrix $\mathbf{\Psi}$ has the following orthogonality properties (Maia and Silva, 1997), ,

$$\mathbf{\Psi}^T\overline{\mathbf{A}}\mathbf{\Psi}=\mathbf{I} \text{ and } \mathbf{\Psi}^T\overline{\mathbf{B}}\mathbf{\Psi}=-\overline{\mathbf{\Lambda}} \tag{8}$$

where $\mathbf{I}_n$ is the $n\times n$ identity matrix.

Eq. (2) can be written in a more usual form in the context of system identification as,

$$\dot{\mathbf{x}}(t)=\mathbf{A}\ \mathbf{x}(t)+\mathbf{B}\ \mathbf{u}(t) \tag{9}$$

where $\mathbf{A}=-\overline{\mathbf{A}}^{-1}\overline{\mathbf{B}}$ and $\mathbf{B}=\overline{\mathbf{A}}^{-1}\overline{\mathbf{f}}(t)$ are $n\times n$ matrices. It can be easily shown, using the orthogonality properties of the modal matrix $\mathbf{\Psi}$ described by Eq. (8), that matrix **A** provides a full description of the system , i.e., matrix **A** can be diagonalized by matrix $\mathbf{\Psi}$ as $\mathbf{A}=\mathbf{\Psi}\overline{\mathbf{\Lambda}}\mathbf{\Psi}^{-1}$.

The observation equation in continuous time is given by,

$$\mathbf{y}(t)=\mathbf{C}\ \mathbf{x}(t) \tag{10}$$

where $\mathbf{y}(t)$ is the $l\times 1$ output measurement vector and **C** is the $l\times n$ output influence matrix.

2.2. Transfer Function in the s-Plane

Pre-multiplying Eq. (2) by $\mathbf{\Psi}^T$ and using the modal transformation $\mathbf{x}(t) = \mathbf{\Psi}\,\boldsymbol{\eta}(t)$, where the n x 1 vector $\boldsymbol{\eta}(t)$ is the so called modal coordinates, leads to,

$$\mathbf{\Psi}^T \overline{\mathbf{A}}\,\mathbf{\Psi}\,\dot{\boldsymbol{\eta}}(t) + \mathbf{\Psi}^T \overline{\mathbf{B}}\,\mathbf{\Psi}\boldsymbol{\eta}(t) = \mathbf{\Psi}^T \overline{\mathbf{f}}(t) \tag{11}$$

Considering the orthogonally properties of modal matrix $\mathbf{\Psi}$ given by Eq. (8) and taking the Laplace transform of two sides of the resulting equation yields,

$$\left(s\,\mathbf{I} - \overline{\mathbf{\Lambda}}\right)\boldsymbol{\eta}(s) = \mathbf{\Psi}^T \overline{\mathbf{F}}(s) \tag{12}$$

where $\overline{\mathbf{F}}(s)$ is the Laplace transform of $\overline{\mathbf{f}}(t)$.

Substituting the value $\boldsymbol{\eta}(s) = \mathbf{\Psi}^{-1}\mathbf{X}(s)$ in Eq. (12), leads to the original coordinate **X**(s) in the s-plane as follows,

$$\mathbf{X}(s) = \mathbf{\Psi}\left(s\,\mathbf{I} - \overline{\mathbf{\Lambda}}\right)^{-1} \mathbf{\Psi}^T \overline{\mathbf{F}}(s) \tag{13}$$

Pre-multiplying Eq. (13) by matrix **C,** and taking the Laplace transform of both sides of Eq. (10), assuming $\mathbf{C} = \left[\mathbf{I}_l\,\mathbf{0}\right]$ and $\mathbf{U}_f = \left[\mathbf{I}_m\,\mathbf{0}\right]^T$ and substituting them in above equation, leads to the input-output relationship in the s-plane,

$$\mathbf{Y}(s) = \left[\mathbf{I}_l\,\mathbf{0}\right]\mathbf{\Psi}\left(s\mathbf{I} - \overline{\mathbf{\Lambda}}\right)^{-1}\mathbf{\Psi}^T \begin{bmatrix}\mathbf{I}_m \\ \mathbf{0}\end{bmatrix}\mathbf{U}(s) = \mathbf{H}(s)\,\mathbf{U}(s) \tag{14}$$

where $\mathbf{H}(s)$ is the $l \times m$ transfer function matrix and $\mathbf{U}(s)$ is the Laplace transform of $\mathbf{u}(t)$.

From the above input-output relationship, the generic element $H_{ij}(s)$ represents the response of the output i to the applied input j. The response can be expanded in the following form,

$$H_{ij}(s) = \sum_{k=1}^{f} \frac{\phi_{ik}\,\phi_{jk}}{s - \lambda_k} + \frac{\phi_{ik}^*\phi_{jk}^*}{s - \lambda_k^*} = \sum_{k=1}^{n} \frac{r_k}{s - \lambda_k} \tag{15}$$

where ϕ_{ij} is an element of matrix $\mathbf{\Phi}$ and $r_k = \phi_{ik}\,\phi_{jk}$ is the residue associated to pole λ_j.

3. LEAST SQUARES MULTIVARIABLE ARX METHOD

The input-output relationship of a linear system may be alternatively described by a finite multivariable ARX difference model (Söderström and Stoica, 1989 and Ljung, 1987) as,

$$\mathbf{y}(k) + \mathbf{A}_1\mathbf{y}(k-1) + \cdots + \mathbf{A}_L\mathbf{y}(k-L) = \mathbf{B}_0\mathbf{u}(k) + \mathbf{B}_1\mathbf{u}(k-1) + \cdots + \mathbf{B}_L\mathbf{u}(k-L) \tag{16}$$

where matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ are ARX coefficient matrices of dimension, respectively, equal to $l \times l$ and $m \times l$. The term L is the order of ARX model and it is assumed to be greater than the order n of the dynamical system which is been identified, i.e., $L > n$.

For an amount of $k = 1,2,\cdots,L+N$ input-output measurements and considering $N > L > n$, Eq. (16) leads to the following matrix equation,

$$\begin{bmatrix} -\mathbf{A}_L & -\mathbf{A}_{L-1} & \cdots & -\mathbf{A}_1 & \mathbf{B}_L & \mathbf{B}_{L-1} & \cdots & \mathbf{B}_0 \end{bmatrix} \begin{bmatrix} \mathbf{y}(1) & \mathbf{y}(2) & \cdots & \mathbf{y}(N) \\ \mathbf{y}(2) & \mathbf{y}(3) & \cdots & \mathbf{y}(N+1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}(L) & \mathbf{y}(L+1) & \cdots & \mathbf{y}(L+N-1) \\ \mathbf{u}(1) & \mathbf{u}(2) & \cdots & \mathbf{u}(N) \\ \mathbf{u}(2) & \mathbf{u}(3) & \cdots & \mathbf{u}(N+1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{u}(L+1) & \mathbf{u}(L+2) & \cdots & \mathbf{u}(L+N) \end{bmatrix} \tag{17}$$

$$= \begin{bmatrix} \mathbf{y}(L+1) & \mathbf{y}(L+2) & \cdots & \mathbf{y}(L+N) \end{bmatrix}$$

or in a more compactly expression as,

$$\mathbf{\Theta}\,\mathbf{X} = \overline{\mathbf{y}} \tag{18}$$

where $\mathbf{\Theta} = \begin{bmatrix} -\mathbf{A}_L & -\mathbf{A}_{L-1} & \cdots & -\mathbf{A}_1 & \mathbf{B}_L & \mathbf{B}_{L-1} & \cdots & \mathbf{B}_0 \end{bmatrix}$ is a $l \times (L\,l + (L+1)m)$ matrix that contains the ARX coefficients matrices , $\mathbf{X}$ is a $(L\,l + (L+1)m) \times N$ matrix that contains the input-output data and $\overline{\mathbf{y}} = \begin{bmatrix} \mathbf{y}(L+1) & \mathbf{y}(L+2) & \cdots & \mathbf{y}(L+N) \end{bmatrix}$ is the l x N matrix that contains only output data.

In general, Eq. (18) represents a over-determined system of algebraic linear equations from which the matrix of ARX coefficients matrices $\mathbf{\Theta}$ are calculated via least-squares minimization as,

$$\mathbf{\Theta} = \overline{\mathbf{y}}\mathbf{X}^T \left(\mathbf{X}\mathbf{X}^T\right)^{-1} \tag{19}$$

In order to estimate the modal parameters of the dynamical system, the z-transform is applied to both sides of Eq. (16), giving the following equation,

$$\left[\mathbf{I}_l + \mathbf{A}_1 z^{-1} + \cdots + \mathbf{A}_L z^{-L}\right]\mathbf{Y}(z) = \left[\mathbf{B}_0 + \mathbf{B}_1 z^{-1} + \cdots + \mathbf{B}_L z^{-L}\right]\mathbf{U}(z) \tag{20}$$

or

$$\left[\mathbf{I}_l + \mathbf{A}_1 z^{-1} + \cdots + \mathbf{A}_L z^{-L}\right]\mathbf{H}(z) = \left[\mathbf{B}_0 + \mathbf{B}_1 z^{-1} + \cdots + \mathbf{B}_L z^{-L}\right] \tag{21}$$

where $\mathbf{I}_l$ is the l x l identity matrix, $\mathbf{Y}(z)$ and $\mathbf{U}(z)$ are, respectively, the z-transform of $\mathbf{y}(k)$ e $\mathbf{u}(k)$ and $\mathbf{H}(z)$ is the $l \times m$ transfer function in the z-domain. Parallel to what was shown in the former section for the s-plane, a generic element of matrix $\mathbf{H}(z)$ can be expressed in a partial fraction expansion form as,

$$H_{ij}(z) = \sum_{k=1}^{lL} \frac{r_k}{1 - z_k z^{-1}} \tag{22}$$

where z_j is one of a totality of lL z-poles of the multivariable ARX model described by Eq. (16). When a n order dynamical system is identified using the above ARX model, n of those z_j poles are directly associated with the dynamics of the system, whereas $lL - n$ are purely computational ones do not bringing any major information about the system behavior. The s-poles λ_j are used to estimate the modal parameters of the system and they are related to corresponding z-poles z_j as,

$$z_j = e^{\lambda_j \Delta t} \tag{23}$$

where Δt is the sampling time.

Alternatively, Eq. (21) can be written in terms of a companion matrix as,

$$\left\{\begin{bmatrix} z\mathbf{I}_l & \mathbf{0}_l & \cdots & \mathbf{0}_l & \mathbf{0}_l \\ \mathbf{0}_l & z\mathbf{I}_l & \cdots & \mathbf{0}_l & \mathbf{0}_l \\ \mathbf{0}_l & \mathbf{0}_l & \cdots & \mathbf{0}_l & \mathbf{0}_l \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0}_l & \mathbf{0}_l & \cdots & \mathbf{0}_l & z\mathbf{I}_l \end{bmatrix} - \begin{bmatrix} -\mathbf{A}_1 & -\mathbf{A}_2 & \cdots & -\mathbf{A}_{L-1} & -\mathbf{A}_L \\ \mathbf{I}_l & \mathbf{0}_l & \cdots & \mathbf{0}_l & \mathbf{0}_l \\ \mathbf{0}_l & \mathbf{I}_l & \cdots & \mathbf{0}_l & \mathbf{0}_l \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0}_l & \mathbf{0}_l & \cdots & \mathbf{I}_l & \mathbf{0}_l \end{bmatrix}\right\} \begin{Bmatrix} z^{L-1}\mathbf{I}_l \\ z^{L-2}\mathbf{I}_l \\ z^{L-3}\mathbf{I}_l \\ \vdots \\ \mathbf{I}_l \end{Bmatrix} \mathbf{H}(z) = \begin{Bmatrix} z^L\mathbf{B}_0 + \cdots + z\mathbf{B}_L \\ \mathbf{0}_{l\times m} \\ \mathbf{0}_{l\times m} \\ \vdots \\ \mathbf{0}_{l\times m} \end{Bmatrix} \tag{24}$$

where $\mathbf{0}_l$ e $\mathbf{0}_{l\times m}$ represent, respectively, the l x l and l x m null matrices. Equation (24) can be written in a more compactly form as,

$$\left\lfloor z\mathbf{I}_{l\,L} - \tilde{\mathbf{A}} \right\rfloor \tilde{\mathbf{I}}(z)\ \mathbf{H}(z) = \tilde{\mathbf{B}}(z) \tag{25}$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} -\mathbf{A}_1 & -\mathbf{A}_2 & \cdots & -\mathbf{A}_{L-1} & -\mathbf{A}_L \\ \mathbf{I}_l & \mathbf{0}_l & \cdots & \mathbf{0}_l & \mathbf{0}_l \\ \mathbf{0}_l & \mathbf{I}_l & \cdots & \mathbf{0}_l & \mathbf{0}_l \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0}_l & \mathbf{0}_l & \cdots & \mathbf{I}_l & \mathbf{0}_l \end{bmatrix} \tag{26}$$

is the lL x lL companion matrix and

$$\tilde{\mathbf{I}}(z) = \begin{Bmatrix} z^{L-1}\mathbf{I}_l \\ z^{L-2}\mathbf{I}_l \\ z^{L-3}\mathbf{I}_l \\ \vdots \\ \mathbf{I}_l \end{Bmatrix} \text{ and } \tilde{\mathbf{B}}(z) = \begin{Bmatrix} z^L\mathbf{B}_0 + \cdots + \mathbf{B}_L \\ \mathbf{0}_{l\times m} \\ \mathbf{0}_{l\times m} \\ \vdots \\ \mathbf{0}_{l\times m} \end{Bmatrix} \tag{27}$$

are, respectively, lL x l and lL x m matrices.

Equation (19) gives the matrix of ARX coefficients matrices which is used to build the companion matrix $\tilde{\mathbf{A}}$. The eigenvalue problem of companion matrix $\tilde{\mathbf{A}}$ can be written from Eq. (25),

$$\left\lfloor z_i\mathbf{I}_{mL} - \tilde{\mathbf{A}} \right\rfloor \tilde{\phi}_i = \mathbf{0} \tag{28}$$

Which leads to the calculation of the z-poles z_j, where $lL - n$ of then (computational poles) may be separated from the identification process.

The n s-poles λ_j associated with the corresponding n z-poles z_j of the system are easily calculated using Eq. (23). Then, the natural frequencies ω_j and modal damping ξ_j are estimated from λ_j according to Eq. (6). Finally, the f associated eigenvectors $\tilde{\phi}_j$ of companion matrix $\tilde{\mathbf{A}}$ can be used to estimate the mode shapes ϕ_j using the following relation, (Maia and Silva, 1997),

$$\tilde{\phi}_j = \mathbf{I}(z)\phi_j \tag{29}$$

where $\tilde{\phi}_j$ is a lL x 1 vector and ϕ_j is a l x 1 vector.

4. RESULTS

This section demonstrates the performance of the presented multivariable ARX algorithm. A simple seven degrees of freedom model, shown in Fig. (1), consisting of seven masses connected in series by seven springs and dampers is used to generate the simulated data sets. Adoption of the model's parameters as $m_1 = \cdots m_7 = 0.1\,Kg$, $c = 2\,Ns/m$ and $k = 250\;N/m$, leads to the exact values of natural frequencies and damping factors shown in Table 1. The experience consists of exciting simultaneously blocks 1 and 5 by means of two different white noise input signals of zero mean and same amplitude and taking numerically all the seven corresponding system's blocks displacements, thus producing a set of 2-input and 7-outputs data measurements. Additive noise with Gaussian distribution, with zero mean and adjusted variance as to produce a RMS noise to signal ratio of the order of 0.4 per cent is added to seven output signals.

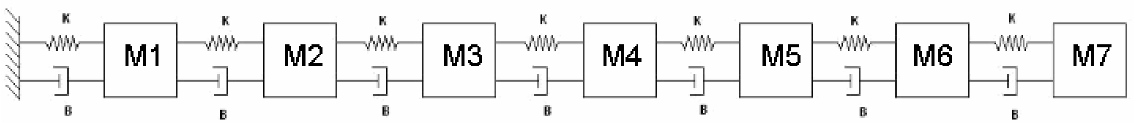

Figure 1. Seven degrees of freedom model

The discretization interval Δt used is 0.027 seconds. The parameters used in the identification process are $m = 2$ inputs, $l = 7$ outputs, $L = 4$ and $N = 246$ for a total number of samples $L + N = 250$, resulting in a 38×246 matrix $\mathbf{X}$ and a 28×28 companion matrix $\tilde{\mathbf{A}}$. The identified natural frequencies and damping factors are derived from the eigenvalues of $\tilde{\mathbf{A}}$. Also, in accordance to Eq.(29), the modal shapes are estimated from the eigenvectors of $\tilde{\mathbf{A}}$.

In order to obtain unbiased model parameters estimation, a model is fitted whose order is larger than the number of modes that are actually present. The separation between the system and spurious modes is based on the repetition of system's modes for different values of *L*. The identified natural frequencies and damping ratios are shown in Tab. 1 and the corresponding estimated modal shapes are shown in Fig. 2.

Table 1. Exact and identified modal parameters

Mode	Exact natural frequency $f_{n\,j}$ (Hz)	Identified natural frequency $f_{n\,j}$ (Hz)	Exact damping factor ξ_j	Identified damping factor ξ_j
1	1.6636	1.6635	0.0418	0.0418
2	4.9182	4.9188	0.1236	0.1236
3	7.9577	7.9669	0.2000	0.1981
4	10.6495	10.7504	0.2677	0.2709
5	12.8759	13.0265	0.3236	0.3442
6	14.5395	14.6161	0.3654	0.1686
7	15.5677	15.5182	0.3913	0.4369

It can be seen from Table 1 a very good correlation between the identified and exact parameters, even in the presence of noise added to the data, displaying the accuracy of the method for the given simulation. The identified mode shapes of the system displayed in figure 2 are also in very good agreement with those exacted from the original system. The high discrepancy in the damping factor estimation for the sixth mode is attributed to the presence of noise added to the data.

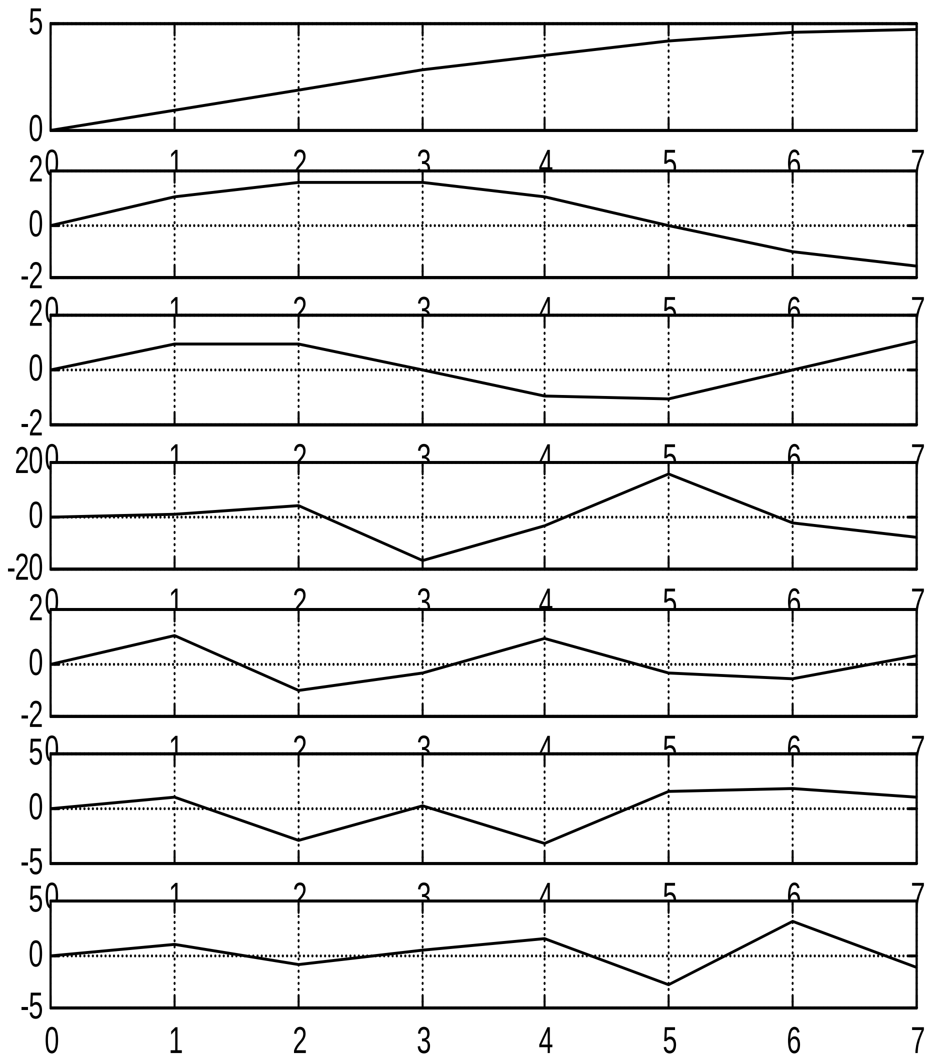

Figure 2. Identified Mode Shapes of the System

5. CONCLUSION

The ability to deal effectively with multivariable system is one of the most important characteristics of system identification. The multivariate ARX method has been proved to be an useful tool for estimation of modal parameters for MIMO tests. This work shows the identification of accurate natural frequencies, damping factors and mode shapes obtained by the present method using simulated data. The main difficulty of such a method is the distinction between the system and spurious modes.

6. ACKNOWLEDGEMENTS

The work presented here was supported through the Scientific Initiation Program of the Universidade Federal do Paraná, Bolsa UFPR/TC Edital IC 2005-2006.

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