TWO-DIMENSIONAL MATHEMATICAL MODEL FOR HEAT AND MASS TRANSFER IN THE GRAINS DRYING PROCESS

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Abstract. *Problems of heat and mass transfer inside a granular medium are found in countless situations of technical interest, for example, the grains drying process. In the present work we use a mathematical model in two dimensions that describes the energy and the mass balance for the air and the grains inside a chamber, based on the Navier-Stokes equations. The governing equations of the model are approximated based on the finite difference scheme. The model was already evaluated in previous works for the one-dimensional case for drying air temperature in the range* $55 - 110^0C$ *and drying air velocity in the range* $0.5 - 4.75ms^{-1}$.

Keywords: *Heat and mass transfer, granular medium, fixed bed, two-dimensional, mathematical model*

1. Introduction

The study of heat and mass transfer processes involved in the drying problems are of great interest in the industrial and agricultural sector. To have larger safety in the drying it is important that, during drying, we have control of the air and the grains temperature, of the heat and mass changes between the grains and the air, as well as of the moisture inside the whole dryer [16]. Such needs, associated to the high costs of the prototypes construction based on theoretical models, have increased the development of mathematical models importance, as well as of simulations of the drying conditions and storage, based on experimental data.

When the amount of water contained in the grains is a lot above 10%, it is necessary to extract this excess of water before the grains storage. Figure 1 shows the sketch of a fixed bed dryer inside of which a great mass of humid grains is deposited to begin the drying.

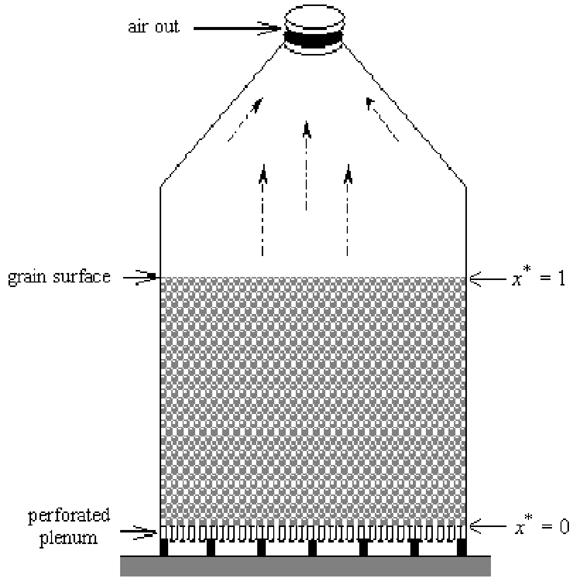

Figure 1. Sketch of a fixed bed dryer

According to Lasseran [7] the artificial grains drying occurs due to the air, usually hot, that is forced through a ventilation system to pass through the granular medium, absorbing the humidity contained in the grains surface and provoking a new humidity gradient, besides the energy gradient (in the case of hot air flow); this provokes a water flow from the center to the grains surface. The water mass that appears in the surface is again removed by the hot air that passes through the grains.

In the experiments realized and presented by Khatchatourian et al. [6] it was verified that immediately after the drying chamber opening the grains surface were apparently without liquid water but, after a short time interval, there appear water drops around the grains. That reinforces the notion that the water leaves the grains in the steam form.

In the literature we can find several models that try to describe the heat and mass transfer inside a granular medium. Courtois et al. [3], in 1991, proposed a model for deep bed based on the mass and energy balance equations for corn grains drying. To describe the moisture variation the grain was divided into three parts: the first formed by the nucleus, the second is the intermediate part and the third the grain outlying part, where the mass transfer of one part to the other occurs by diffusion, with coefficients obtained empirically. For the heat transfer the grain was considered as a uniform layer. The diffusion term was neglected in that model in the energy and the mass equations for the air.

Liu et al., in 1997 [9] [10] [11], presented a stochastic model for the grains drying in a crossed flow to evaluate the moisture, the air temperature distribution and the air flow ratio in a corn sample. In a subsequent work, the same authors introduced an automatic controller at the drying process [8].

Mhimid et al. (2000) [12] analyzed the drying in a deep bed with vertical hot air flow, being the walls of the dryer subject to the Neumann and the Dirichlet conditions. They considered two mathematical models for heat and mass transfer in cylindrical coordinates for local and no local temperature equilibrium.

Khatchatourian et al. [6] adapted the model of Courtois et al. [3] for the soy grains drying process in deep bed, considering the grains to be homogeneous. That model consists of four partial differential equations, where the mass transfer term is obtained starting from the experimental data. They considered, in their model, that the moisture variation ratio along the time is proportional to the mass flow Φ_m, with Φ_m function of M_1, M_2 and M_3 (functions of the temperature obtained from experimental data). In this work they obtaining experimental data for the soy grains drying in deep bed dryers for air temperature till 65^oC. Weber, et all, in 2001 [19] and Borges, in 2002 [2], used the same model and obtained more experimental data for deep bed and for fine layer dryers for higher temperatures (up to 110^oC).

According to Srivastava and John (2002) [18], Boyce proposed a model, wherein a deep bed was modelled as a thin layer using a semi-empirical model. Srivastava and John reported a deep bed model under unsteady conditions for the simulation of air humidity, grains temperature and air temperature inside the bed.

Aguerre and Suarez [1], in 2004, affirmed that the drying of humid solids involve simultaneously heat and mass transfer processes much complicated and so a series of simplifications are usually employed to reduce the complexity of the models that involve those phenomena. In their work, they used a model based on the equation of water diffusion in grains. The variation of the moisture was given by the Fick's law, where the effective mass diffusion coefficient was calculated as a function of the moisture.

Resio et al. [17] presented an expression for the effective diffusion coefficient. According to these authors, this coefficient is a function of the temperature and of the energy activation for the diffusion, that is calculated from the Clausius-Clapeyron equation. The results presented refer to drying temperatures in the range of 40^oC to 70^oC. Gastón et al. [4] proposed an expression to calculate the effective diffusion coefficient as a function of the temperature and of the initial grains moisture for temperatures in the range from 35^oC to 70^oC. Prachayawarakorn et al. [15] discussed the maintenance of the grains quality in the drying at high temperatures and presented an expression for the effective diffusion coefficient as a function just of the temperature.

Petry et al. [13], in a recent work, developed a mathematical model to describe the heat and mass transfer processes in a granular medium based on the Navier-Stokes equations. The source terms were obtained starting from the mass and the energy balance for the air and for the grains. In the source term of the mass conservation equation they considered a mass diffusion coefficient between the grains and the air and had obtained an expression for it as a function of the air speed, the temperature and the difference between the grains moisture and the equilibrium moisture, using experimental data. A numeric scheme based on finite differences was presented together with an analysis of the convergence and stability a in previous work. The model efficiency is also evaluated starting from experimental data for the one-dimensional case [14].

Therefore the objective of this work is apply the model presented in [13] for the two-dimensional case, with the purpose of evaluate the influence of the horizontal grains position in the drying process.

In the sequence, we will present the set of governing equations of our model.

2. Governing equations and initial/boundary conditions

The fixed bed chamber considered in this work, consists of a straight prism of rectangular base inside of which solid spheres of $8mm$ diameter are deposited; hot air passes among the spheres.

The mathematical model considers a set of partial differential equations that describe the air and the temperature distribution, the air humidity and the spheres moisture inside the chamber. The following hypotheses or simplifications are adopted:

- the medium porosity is constant;
- there is no thermal equilibrium between the spheres and the air inside the chamber;
- each sphere has uniform temperature;
- the heat transfer from the air to the solid elements occurs by conduction;
- the water mass transfer from the spheres to the air occurs by diffusion;
- the heat transfer in the air occurs by the conduction and the convection processes;
- the mass transfer in the air is due to the diffusion and the convection processes;
- the chamber walls are isolated.

The set of governing equations already in the dimensionless form are given by:

Mass conservation for air:

$$\frac{\partial Y^*}{\partial t^*} = -\overrightarrow{u^*}\overrightarrow{\nabla}Y^* + \frac{D^*}{ReSc}\nabla^2 Y^* + \frac{a^*(1-\phi)}{\phi\rho_g^* Y_0 ReSc} D_s^* \left(\rho_s^* X_0 X^* - \rho_g^* Y_0 Y^*\right) \tag{1}$$

Mass conservation for solid:

$$\frac{\partial X^*}{\partial t^*} = -\frac{a^*}{\rho_s^* X_0 ReSc} D_s^* \left(\rho_s^* X_0 X^* - \rho_g^* Y_0 Y^*\right) \tag{2}$$

Energy equation inside the chamber:

$$\frac{\partial T_g^*}{\partial t^*} = -\overrightarrow{u^*}\overrightarrow{\nabla}T_g^* + \frac{\alpha^*}{RePr}\nabla^2 T_g^* + \frac{a^*(1-\phi)}{\phi\rho_g^* C_{p_g}^* RePr} K_s^* \left(T_s^* - T_g^*\right) \tag{3}$$

Energy equation for the walls:

$$\frac{\partial T_s^*}{\partial t^*} = -\frac{Ec}{ReSc}\frac{a^* L_v^* D_s^* \left(\rho_s^* X_0 X^* - \rho_g^* Y_0 Y^*\right)}{\rho_s^*(X_0 X^* C_{p_w}^* + Cp_s{}^*)} - \frac{1}{RePr}\frac{a^* K_s^* \left(T_s^* - T_g^*\right)}{\rho_s^*(X_0 X^* C_{p_w}^* + Cp_s{}^*)} \tag{4}$$

where Y is the air humidity, X the grains moisture, $\overrightarrow{u}$ the velocity vector, D the water mass diffusion coefficient in air, a the sphere area to volume ratio, ϕ the bed porosity, ρ the density, T the temperature, α the thermal conductivity, C_p the specific heat at constant pressure and K the thermal conductivity. The subscript s, l, g, v and 0 represent the solid, the liquid, the gas, the steam and the initial value, respectively. The symbol * indicates that the variable is in the dimensionless form. $Re = \frac{U_0 L_c}{\nu}$ corresponds to the Reynolds number, $Sc = \frac{\nu}{D_0}$ to the Schmidt number, $Pr = \frac{\nu}{\alpha_0}$ to the Prandtl number and $Ec = \frac{U_0^2}{C_{p0}(T_{air} - T_{amb})}$ to the Eckert number.

The values of mass diffusion coefficients (D and D_s) and the thermal conductivity coefficient K_s were set as constants when writing the equations; however, they are recalculated when performing the iterations, considering $D \propto T_g^{1.5}$ [5]. To obtain the mass diffusion coefficient between the solid and the air we used the expression:

$$D_s = A\sqrt{|\overrightarrow{u}|}D(X - X_e)^n \tag{5}$$

where A and n are defined by experimental data and X_e is the equilibrium moisture. The dependence of the velocity proceeds of the Chilton-Colburn analogy. The set of governing equations used in this work is demonstrated in [13].

To describe the air flow among the grains we considered the:

Continuity equation:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{6}$$

Momentum equations:

$$\frac{\partial u^*}{\partial t^*} = -u^*\frac{\partial u^*}{\partial x^*} - v^*\frac{\partial u^*}{\partial y^*} - \frac{\partial P^*}{\partial x^*} + \frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) \tag{7}$$

$$\frac{\partial v^*}{\partial t^*} = -u^* \frac{\partial v^*}{\partial x^*} - v^* \frac{\partial v^*}{\partial y^*} - \frac{\partial P^*}{\partial y^*} + \frac{1}{Re} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right) \tag{8}$$

To evaluate the pressure distribution, we employed the Poisson equation:

$$\nabla^2 P^* = -\frac{\partial}{\partial t^*} \left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} \right) - \frac{\partial}{\partial x^*} \left(\overrightarrow{u^*}.\overrightarrow{\nabla} u^* \right) - \frac{\partial}{\partial y^*} \left(\overrightarrow{u^*}.\overrightarrow{\nabla} v^* \right) + \frac{1}{Re} \left[\frac{\partial}{\partial x^*} \left(\nabla^2 u^* \right) + \frac{\partial}{\partial y^*} \left(\nabla^2 v^* \right) \right] \tag{9}$$

The appropriate implementation of boundary conditions is always important when solving a set of differential equations. Here some assumptions are adopted as following indicated. When starting the simulations, the air and the spheres temperature and humidity, the velocity and the pressure are considered to be:

$T_s^*(x,y,0) = 0, \quad (x,y) \in [0,1]x[0,1]$
$T_g^*(x,y,0) = 0, \quad (x,y) \in (0,1]x[0,1]$
$X^*(x,y,0) = 1, \quad (x,y) \in [0,1]x[0,1]$
$Y^*(x,y,0) = 1, \quad (x,y) \in [0,1]x[0,1]$
$u^*(x,y,0) = 0, \quad (x,y) \in (0,1]x[0,1]$
$v^*(x,y,0) = 0, \quad (x,y) \in [0,1]x[0,1]$
$P^*(x,y,0) = 0, \quad (x,y) \in (0,1]x[0,1]$

For the x-direction boundary conditions we have considered:

$T_g^*(0,y,t) = 1 \quad and \quad \frac{\partial T_g^*}{\partial x^*}(1,y,t) = 0 \;\; for \;\; y \in (0,1) \; and \; t > 0$
$Y^*(0,y,t) = 1 \quad and \quad \frac{\partial Y^*}{\partial x^*}(1,y,t) = 0 \;\; for \;\; y \in (0,1) \; and \; t > 0$
$u^*(0,y,t) = 1 \quad and \quad v^*(0,y,t) = 0 \;\; for \;\; y \in (0,1) \; and \; t > 0$
$P^*(1,y,t) = 0 \quad for \quad y \in (0,1) \; and \; t > 0$

On the chamber walls, that is, for $y^* = 0$ and $y^* = 1$, the variables T_g^*, Y^* and P^* satisfy the Neumann boundary conditions and , $u^* = v^* = 0$. The pressure P^* at $x^* = 0$ is obtained by extrapolation using the following approach: $\Psi_{0,j,k}^{n+1} = 0.75\Psi_{1,j,k}^{n+1} + 0.25\Psi_{2,j,k}^{n+1}$, according to Fig. 2. and the velocity components u^* and v^* at $x^* = 1$ are obtained

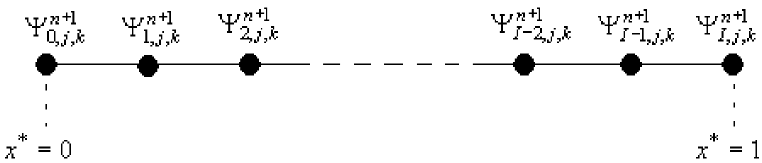

Figure 2. Sketch of extrapolation on $x^* = 0$ and $x^* = 1$

by extrapolation using the expression:
$\Psi_{I,j,k}^{n+1} = 0.75\Psi_{I-1,j,k}^{n+1} + 0.25\Psi_{I-2,j,k}^{n+1}$

3. Numerical Results

The finite difference Gauss-Seidel approach is employed to obtain the numerical results. First order time and a second order space approximations were used, as follows:

$$\frac{\partial \Psi}{\partial t^*} \approx \frac{\Psi_{i,j}^{n+1} - \Psi_{i,j}^n}{\Delta t^*} \tag{10}$$

$$\frac{\partial \Psi}{\partial x^*} \approx \frac{\Psi_{i+1,j}^n - \Psi_{i-1,j}^n}{2\Delta x^*}, \quad \frac{\partial \Psi}{\partial y^*} \approx \frac{\Psi_{i,j+1}^n - \Psi_{i,j-1}^n}{2\Delta y^*} \tag{11}$$

$$\frac{\partial^2 \Psi}{\partial x^{*2}} \approx \frac{\Psi_{i+1,j}^n - 2\Psi_{i,j}^n + \Psi_{i-1,j}^n}{(\Delta x^*)^2}, \quad \frac{\partial^2 \Psi}{\partial y^{*2}} \approx \frac{\Psi_{i,j+1}^n - 2\Psi_{i,j}^n + \Psi_{i,j-1}^n}{(\Delta y^*)^2} \tag{12}$$

To obtain the numerical results we considered the drying chamber with $32cm$ of height and $12cm$ of width. The space domain was divided in a grid, as shown in Fig. 3.

The terms A and n in the equation (5), which represents the mass diffusion coefficient between the air and the grains, were obtained starting from soy grains drying experimental data, being $A = 2.764\text{x}10^{-11}$ and $n = 1.05$. The numerical simulations presented in this work consider $T_{air} = 90^0C$, $T_{amb} = 25^0C$, $X_0 = 0.32$, $U_0 = 1.0ms^{-1}$ and the air relative humidity of 80%, what corresponds to the initial air humidity of approximately $Y_0 = 1.4\text{x}10^{-2}$.

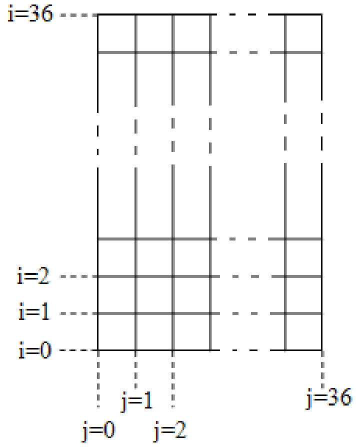

Figure 3. Computational grid

Figure 4 shows the numerical simulations for the dimensionless grains temperature $T^* = \frac{T - T_{amb}}{T_{air} - T_{amb}}$ for different times: $t = 0.5h$ (a), $t = 1h$ (b), $t = 1.5h$ (c) and $t = 2h$ (d). For each time, we showed the variation of the grains temperature in relation to horizontal grains position (value of j) for different values of the vertical position x^* ($x^* = 0$, for $i = 0$, $x^* = 1/4$, for $i = 9$, $x^* = 1/2$, for $i = 18$, $x^* = 3/4$, for $i = 27$ and $x^* = 1$, for $i = 36$).

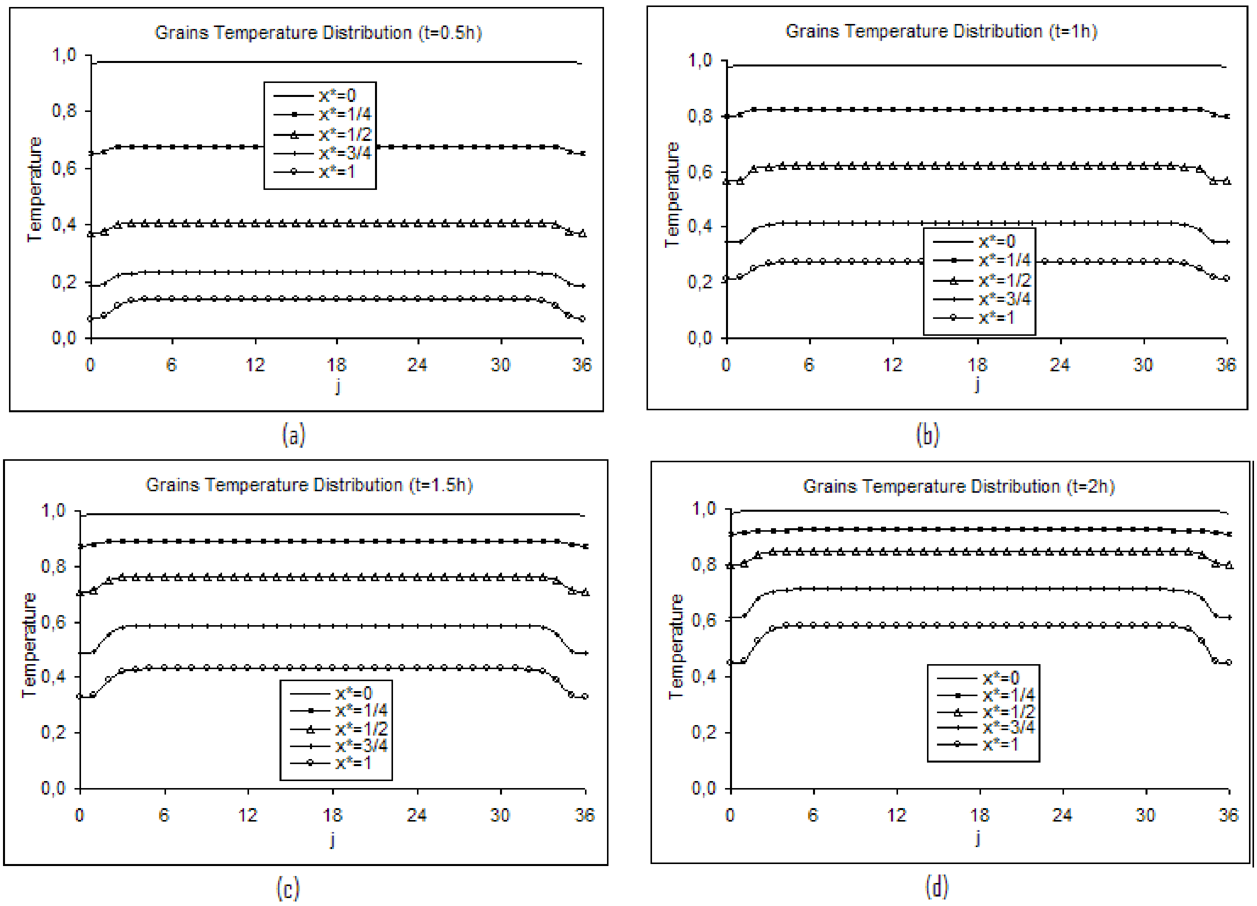

Figure 4. Grains temperature distribution for $t = 0.5h$ (a), $t = 1h$ (b), $t = 1.5h$ (c) and $t = 2h$ (d)

Notice that the temperature rises in all positions of the drying chamber along the time. Also observe that in the chamber entrance (on $x* = 0$) the temperature rises practically in an uniform way. At the ends of the chamber (close of x*=1) it is observed that in the proximities of the walls, the temperature stays lower than in the central part, approaching the parabolic form.

A numerical simulation for the dimensionless grains moisture $X^* = \frac{X}{X_0}$ for the times: $t = 0.5h$ (a), $t = 1h$ (b), $t = 1.5h$ (c) and $t = 2h$ (d) at the some positions that the temperature is shown in Fig. 5.

We observe that the grains moisture decreases along the time in the whole drying chamber. To exemplify what happens with the temperature curves, again the process is slower close to the walls. It is believed that in the real dryers case, where the dimensions are quite larger then considered here, that effect can be more significant, resulting a parabolic profile in the grains temperature and the moisture curves.

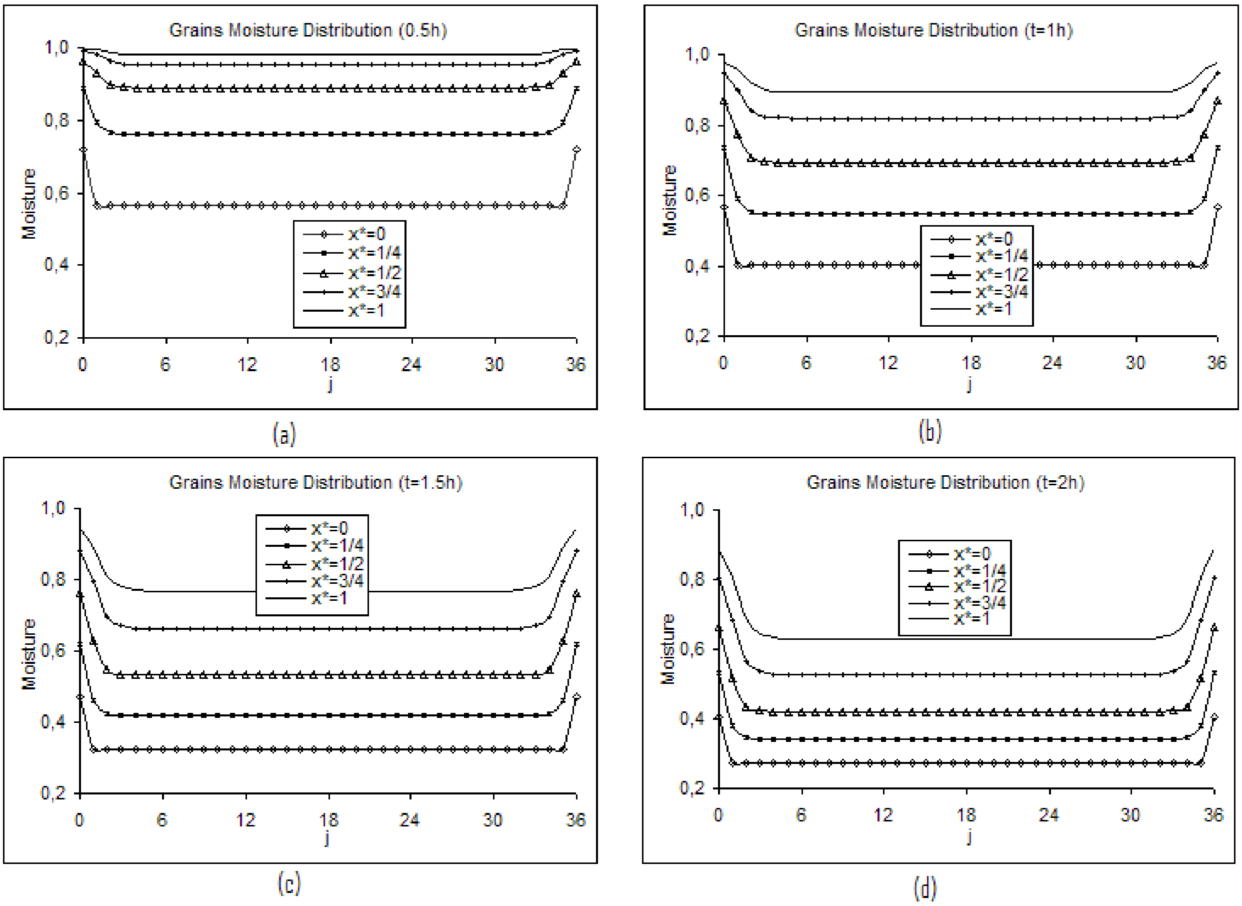

Figure 5. Grains moisture distribution for $t = 0.5h$ (a), $t = 1h$ (b), $t = 1.5h$ (c) and $t = 2h$ (d)

4. Conclusions

We have used a mathematical model and a numerical scheme to solve the heat and mass transfer problem in the grains drying process for the case in two dimensions.

Obtained results indicate the importance of considering more than a dimension in the dryers simulation. It is believed that in the real dryers case, that effect can be more significant.

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