Multirate Predictive Control of a 3DOF Helicopter Model

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Abstract. *This paper is concerned with the application of a multirate predictive control strategy to a three-degree-of-freedom (3DOF) helicopter model. This system presents relatively fast dynamics, hence frequent and fast control actions are needed to guarantee the enforcement of constraints and appropriate closed loop performance. Therefore, a small sampling time is required, which may prevent the real-time deployment of the controller because of the computational effort demanded by the solution of a quadratic programming (QP) problem at each sampling time. This problem can be circumvented by adopting different sampling rates for the output and input variables. Such a multirate strategy is usually implemented by including additional equality constraints between the control update instants. In the present paper, blocking and condensing methods are employed to eliminate the need for such additional constraints. The multirate algorithm is applied to the control of a 3DOF helicopter model with two input and three output variables. The problem under consideration consists of carrying out travel manoeuvres with pitch constraints and maintenance of the elevation at a desired operating level. Simulation results show that it is possible to satisfy the operational requirements when output measurements are acquired and predicted at a fast sampling rate and control actions are updated at slower rates.*

Keywords: *predictive control, multirate control, multivariable systems, blocking and condensing.*

1. INTRODUCTION

Model Predictive Control (MPC) is a generic name for a widely used class of control algorithms. The philosophy of such algorithms consists basically of finding the solution to an optimal control problem in real time and implementing the control in a receding horizon manner (Camacho and Bordons, 1999). The main features that have contributed towards the popularity of predictive controllers, according to (Maciejowski, 2002), are the ability to cope with transport delays and large numbers of controlled and manipulated variables, as well as the enforcement of operational constraints, which is of value to reduce the number of emergency stops and system downtime.

Because of the on-line computational effort demanded by predictive controllers, the early applications of MPC were restricted to plants with long time constants. However, the continuing improvement in real-time computational resources and the development of efficient numerical algorithms for MPC problems (Cannon, 2004) is extending the use of MPC to systems with fast dynamics. In this context, MPC may be of interest for the aeronautical sector, as reported in (Wan and Bogdanov, 2001; Manikonda et al., 1999; Jadbabaie and Hauser, 2001). In particular, in order to expand the flight envelope for high-performance aircraft, actuators such as engines, aerodynamic surfaces, lateral thrust and vortex separation devices (Maciejowski, 2002) may have to be employed close to their saturation limits. Therefore, the constraint-handling features of predictive controllers would be of value to ensure a safe operation. In particular, MPC could be employed with models that consider structural modes and aeroelastic phenomena (Smith et al., 2004; Merkel et al., 2004), in order to impose stress restrictions in sections of the aircraft that are prone to structural failure. MPC can also be formulated to deal with multirate (variables updated at different rates) (Ling and Lim, 1996) and multiscale (physical phenomena occurring at different time scales) systems (Stephanopoulos et al., 2000). Typically, both of these characteristics are found in aircraft (Nelson, 1998; Polushin and Marquez, 2004).

In this paper, a multirate state-space MPC formulation is employed for the control of a helicopter model with three degrees of freedom (3DOF). The prediction model is obtained by linearizing a physical model of the system. The effect of changes in the multirate scheme is investigated, as well as the enforcement of output constraints.

1.1 Notation

I_q and $0_{q\times q}$ are a $q \times q$ identity matrix and a $q \times q$ matrix of zeros, respectively. The column vectors of controlled (plant outputs), manipulated (plant inputs), reference and state variables at the k^{th} sampling instant are denoted by $y(k) \in \mathbb{R}^q$, $u(k) \in \mathbb{R}^p$, $r(k) \in \mathbb{R}^q$, $x(k) \in \mathbb{R}^n$, respectively. Increments are denoted by $\Delta y(k) = y(k) - y(k-1)$. The hat $\hat{}$ denotes a predicted value. The i^{th} component of vector y is denoted by y_i.

Reference and predicted values are stacked in column vectors as

$$R = \begin{bmatrix} r(k+1) \\ \vdots \\ r(k+N) \end{bmatrix}, \hat{Y} = \begin{bmatrix} \hat{y}(k+1|k) \\ \vdots \\ \hat{y}(k+N|k) \end{bmatrix}, \Delta\hat{U} = \begin{bmatrix} \Delta\hat{u}(k+1|k) \\ \vdots \\ \Delta\hat{u}(k+M|k) \end{bmatrix}$$

where N and M are the prediction and control horizons, respectively.

2. MULTIRATE MPC FORMULATION

In a basic state-space (multivariable) MPC formulation, the plant model is employed to calculate output predictions up to N steps in the future, where N is termed "Prediction Horizon". Such predictions are determined on the basis of the state measured at the present time (k^{th} sampling instant), and are also dependent on the control sequence that will be applied. The optimization algorithm is aimed at determining the sequence of future control increments $\Delta\hat{u}(k-1+i|k), i = 1, ..., M$, that minimizes the cost function specified for the problem, subject to constraints on the plant inputs and outputs. The value of M ("Control Horizon") is typically smaller than N, and the optimization assumes that $\Delta\hat{u}(k-1+i|k) = 0$ for $M < i \leq N$. The control is implemented in a receding horizon manner, that is, only the first element of the optimized control sequence is applied to the plant and the optimization is repeated at the next sampling instant, on the basis of fresh measurements.

A commonly used cost function, which penalizes tracking errors at the q plant outputs and control variations at the p plant inputs, is of the type: (Maciejowski, 2002)

$$J(\Delta\hat{U}) = \sum_{j=1}^{q}\sum_{i=1}^{N} \mu_j[\hat{y}_j(k+i|k) - r_j(k+i)]^2 + \sum_{l=1}^{p}\sum_{i=1}^{M} \rho_l[\Delta\hat{u}_l(k-1+i|k)]^2 \tag{1}$$

where $\rho_l > 0$; $l = 1, \ldots, p$ and $\mu_j \geq 0$; $j = 1, \ldots, q$. By defining an output weight matrix W_y as

$$W_y = \begin{bmatrix} \Psi(\mu) & 0_{q\times q} & \cdots & 0_{q\times q} \\ 0_{q\times q} & \Psi(\mu) & \cdots & 0_{q\times q} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{q\times q} & 0_{q\times q} & \cdots & \Psi(\mu) \end{bmatrix} \tag{2}$$

where

$$\Psi(\mu) = \begin{bmatrix} \mu_1 & 0 & \cdots & 0 \\ 0 & \mu_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mu_q \end{bmatrix} \tag{3}$$

and an input weight matrix W_u in a similar manner (using weights ρ_l), the cost function can be rewritten in the form

$$J(\Delta\hat{U}) = (\hat{Y} - R)^{\text{T}} W_y (\hat{Y} - R) + \Delta\hat{U}^{\text{T}} W_u \Delta\hat{U} \tag{4}$$

The relation between $\hat{Y}$ and $\Delta\hat{U}$ can be expressed by a prediction equation based on an incremental state-space model. By assuming a linearized model of the form $x(k+1) = Ax(k) + Bu(k)$, $y(k) = Cx(k)$, the incremental model can be written as $\Delta x(k+1) = A\Delta x(k) + B\Delta u(k)$, $\Delta y(k) = C\Delta x(k)$. Therefore, a prediction equation for Δy can be written as (Maciejowski, 2002)

$$\Delta\hat{Y} = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-1}B & CA^{N-2}B & \cdots & CA^{N-M}B \end{bmatrix} \Delta\hat{U} + \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix} \Delta x(k) = P\Delta\hat{U} + Q\Delta x(k) \tag{5}$$

It can be easily seen that $\Delta\hat{Y}$ and $\hat{Y}$ can be related as

$$\hat{Y} = \begin{bmatrix} I_q & 0 & \cdots & 0 \\ I_q & I_q & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_q & I_q & I_q & I_q \end{bmatrix} \Delta\hat{Y} + \begin{bmatrix} I_q \\ I_q \\ \vdots \\ I_q \end{bmatrix} y(k) \tag{6}$$

or $\hat{Y} = T_N^{I_q}\Delta\hat{Y} + \Gamma_N^{I_q} y(k)$. By using the identity in (5), it follows that $\hat{Y} = T_N^{I_q} P\Delta\hat{U} + T_N^{I_q} Q\Delta x(k) + \Gamma_N^{I_q} y(k) = G\Delta\hat{U} + F$. Therefore, the cost function (4) can be rewritten as

$$J(\Delta\hat{U}) = (G\Delta\hat{U} + F - R)^{\mathrm{T}} W_y (G\Delta\hat{U} + F - R) + \Delta\hat{U}^{\mathrm{T}} W_u \Delta\hat{U} = \frac{1}{2}\Delta\hat{U}^{\mathrm{T}}\mathcal{G}\Delta\hat{U} + f^{\mathrm{T}}\Delta\hat{U} + \alpha \tag{7}$$

which is quadratic in $\Delta\hat{U}$.

If restrictions on the manipulated and controlled variables of the form $u_{min} \leq \hat{u}(k-1+i|k) \leq u_{max}$, $\Delta u_{min} \leq \Delta\hat{u}(k-1+i|k) \leq \Delta u_{max}$, $i = 1, \ldots, M$ and $y_{min} \leq \hat{y}(k+i|k) \leq y_{max}$, $i = 1, \ldots, N$, are to be satisfied, the minimization of the cost is subject to the following linear constraints on $\Delta\hat{U}$ (Camacho and Bordons, 1999):

$$\begin{bmatrix} I_{pM} \\ -I_{pM} \\ T_M^{I_p} \\ -T_M^{I_p} \\ G \\ -G \end{bmatrix} \Delta\hat{U} \leq \begin{bmatrix} Bl_M[\Delta u_{max}] \\ -Bl_M[\Delta u_{min}] \\ Bl_M[u_{max} - u(k-1)] \\ -Bl_M[u_{min} - u(k-1)] \\ Bl_N[y_{max}] - F \\ -Bl_N[y_{min}] - F \end{bmatrix} \tag{8}$$

where $T_M^{I_p}$ is a lower block-triangular matrix of identities and $Bl_N[\bullet]$ is an operator that stacks N copies of a column vector. The set of constraints (8) can be rewritten as $S\Delta\hat{U} \leq b$. Therefore, the optimization problem to be solved at each sampling period is one of Quadratic Programming (QP) (quadratic cost, linear constraints), for which efficient numerical algorithms are available (Maciejowski, 2002).

When dealing with multirate systems not all outputs and inputs are sampled at the same time. In the worst scenario there can be $q + p$ different sampling periods, one for each output ($T_{y_i}, i = 1, \ldots, q$) and input ($T_{u_j}, j = 1, \ldots, p$). A possible way to deal with this, is to take the discretization period T_s as the greatest common divisor of all the sampling periods (Berg et al., 1988).

2.1 Blocking and Condensing

Blocking and condensing (B & C) has been proposed in MPC context as a technique to improve conditioning of dynamic matrices (matrix G in the formulation above), robustness and computation efficiency (Ricker, 1985; Palavajjhala et al., 1994; Lee et al., 1995). In this work B & C is deployed to cope with multirate systems. It will be shown that this approach allows greater design flexibility and, additionally, avoids the extra equality constraints on control inputs in the QP formulation for multirate MPC, which were proposed in (Ling and Lim, 1996; Ling et al., 2004) and also avoids the unnecessarily large G matrix of the scheme proposed in (Scattolini and Schiavoni, 1995).

B & C can be considered as methods that approximate a high dimensional matrix/vector by lower dimensional ones. Projection matrices $\breve{B}$ (blocking projection matrix) and $\breve{C}$ (condensing projection matrix) are used to obtain a lower dimensional control input vector $\Delta\breve{U} = \breve{B}\Delta U$ and predicted error vector $\breve{E} = \breve{C}(\hat{Y} - R)$, respectively. $\breve{B}$ (or $\breve{C}$) is a matrix derived from an orthogonal one by deleting some of the rows that correspond to the particular linear combinations of the input moves (or output error predictions), which are set to zero (or not considered in the error tracking part of the cost equation) *a priori* and are therefore omitted from the computation.

After properly designing the blocking ($\breve{B}$) and condensing matrices ($\breve{C}$), the new cost function ($\breve{J}$) and linear restrictions can be written as:

$$\min_{\Delta\breve{U}} \breve{J}(\Delta\breve{U}) = \frac{1}{2}\Delta\breve{U}^{\mathrm{T}}\breve{\mathcal{G}}\Delta\breve{U} + \breve{f}^{\mathrm{T}}\Delta\breve{U} + \breve{\alpha} \qquad \text{subject to} \qquad \breve{S}\Delta\breve{U} \leq \breve{b} \tag{9}$$

where

$$\breve{\mathcal{G}} = 2\breve{B}G^{\mathrm{T}}\breve{C}^{\mathrm{T}}\breve{C}W_y\breve{C}^{\mathrm{T}}\breve{C}G\breve{B}^{\mathrm{T}} + \breve{B}W_u\breve{B}^{\mathrm{T}}, \quad \breve{f} = 2\breve{B}G^{\mathrm{T}}\breve{C}^{\mathrm{T}}\breve{C}W_y\breve{C}^{\mathrm{T}}\breve{C}(F - R),$$

$$\breve{S} = \begin{bmatrix} \breve{B}I_{pM}\breve{B}^{\mathrm{T}} \\ -\breve{B}I_{pM}\breve{B}^{\mathrm{T}} \\ \breve{B}T_M^{I_p}\breve{B}^{\mathrm{T}} \\ -\breve{B}T_M^{I_p}\breve{B}^{\mathrm{T}} \\ \breve{C}G\breve{B}^{\mathrm{T}} \\ -\breve{C}G\breve{B}^{\mathrm{T}} \end{bmatrix}, \ \breve{b} = \begin{bmatrix} Bl_M[\Delta u_{max}] \\ -Bl_M[\Delta u_{min}] \\ Bl_M[u_{max} - u(k-1)] \\ -Bl_M[u_{min} - u(k-1)] \\ \breve{C}(Bl_N[y_{max}] - F) \\ -\breve{C}(Bl_N[y_{min}] - F) \end{bmatrix}$$

Example: Consider a SISO system with $N = 4$, $M = 4$, $T_y = 1$s and $T_u = 1$s (i.e., single rate), so $T_s = 1$s and

$$\begin{bmatrix} \hat{y}(k+1|k) \\ \hat{y}(k+2|k) \\ \hat{y}(k+3|k) \\ \hat{y}(k+4|k) \end{bmatrix} = \begin{bmatrix} CB & 0 & 0 & 0 \\ CAB & CB & 0 & 0 \\ CA^2B & CAB & CB & 0 \\ CA^3B & CA^2B & CAB & CB \end{bmatrix} \begin{bmatrix} \Delta\hat{u}(k|k) \\ \Delta\hat{u}(k+1|k) \\ \Delta\hat{u}(k+2|k) \\ \Delta\hat{u}(k+3|k) \end{bmatrix} + \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ CA^4 \end{bmatrix} \Delta x(k)$$

If it is imposed that control inputs can only be updated every 3 seconds, a simple solution (Ling et al., 2004) is to enforce equalities constraints on the input sequence so that $\Delta\hat{u}(k+1|k) = \Delta\hat{u}(k+2|k) = 0$. However, such constraints may increase the QP computation time prohibitively to applications in systems with fast dynamics. Additionally, if the output needs only be sampled every 2 seconds, then it may not be necessary to take $\hat{y}(k+1|k)$ and $\hat{y}(k+3|k)$ into account.

These features of $\Delta\hat{U}$ and $\hat{Y}$ can easily be coped with by the proposed B & C scheme by letting

$$\breve{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; \breve{C} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

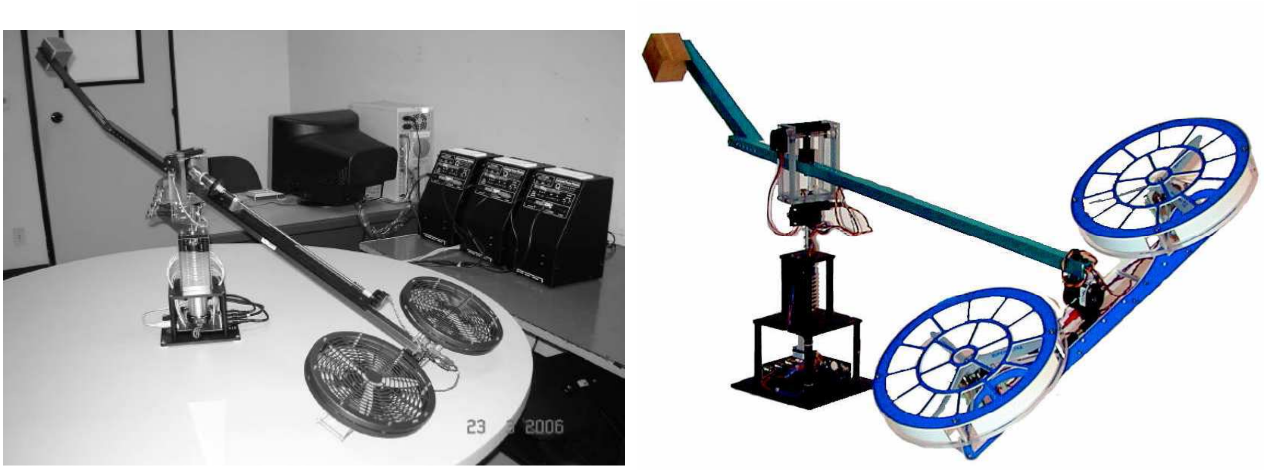
$$\begin{bmatrix} \hat{y}(k+2|k) \\ \hat{y}(k+4|k) \end{bmatrix} = \begin{bmatrix} CAB & 0 \\ CA^3B & CB \end{bmatrix} \begin{bmatrix} \Delta\hat{u}(k|k) \\ \Delta\hat{u}(k+3|k) \end{bmatrix} + \begin{bmatrix} CA^2 \\ CA^4 \end{bmatrix} \Delta x(k)$$

$$\breve{Y} = \breve{C} G \breve{B}^{\mathrm{T}} \Delta\breve{U} + \breve{C} F \Delta x(k)$$

It is clear from this example that this multirate formulation is general with respect to q, p, N and M. The only restriction is that the greatest common divisor between all T_{y_i} and T_{u_j} must be large enough to allow real-time implementation. It was also shown how the addition of the equalities constraints $\Delta\hat{u}(k+1|k) = \Delta\hat{u}(k+2|k) = 0$ to the QP formulation were avoided.

3. SYSTEM DESCRIPTION

In this work, a simulation model for the 3DOF helicopter (Figs. 1a-b) manufactured by Quanser Consulting© was employed for validation of the multirate MPC strategy proposed. This system was previously described elsewhere (Lopes et al., 2006; Lopes, 2007), so just a brief description will now be given.



(a) (b)

Figure 1. 3DOF Helicopter considered in this work.

The dynamics of the helicopter can be described by a nonlinear 6^{th} order model with states corresponding to the angles of pitch (P), travel (T), elevation (E) and their respective derivatives. The travel movement consists of the vertical rotation of the entire system around the vertical axis. The pitch movement corresponds to the change of attitude of the helicopter body. The elevation is defined as the vertical movement of the helicopter body, which corresponds to the rotation of the main sustentation arm around the horizontal axis. The manipulated variables used for control are the armature voltages of the two DC motors connected to the propellers of the helicopter.

The model was linearized around an elevation angle of $E = 27°$ with all other angles and rates equal to zero. The adopted sampling periods (T_s) were 800ms and 100ms and a zero-order-hold is included at the controller output. By defining the state and output vectors as $x = [P, \dot{P}, E, \dot{E}, T, \dot{T}]^{\mathrm{T}}$ and $y = [P, E, T]^{\mathrm{T}}$, respectively, the continuous-time state space model matrices are the following:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1.0389 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1.3421 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 & 0 \\ 2.9626 & -2.9626 \\ 0 & 0 \\ 0.4166 & 0.4166 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad D_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

After model discretization, the eigenvalues of A with $T_s = 100$ms are 1.0 (with multiplicity four) and $0.9948 \pm j0.1017$, which shows that the model is marginally stable. When $T_s = 800$ms the complex conjugate pair of poles move to $0.6856 \pm j0.7280$.

4. SIMULATION SETUP

The simulation scenario under consideration consists of carrying out travel manoeuvres with pitch constraints and maintenance of the elevation at a desired operating level.

The travel manoeuvre consists of a set-point change from 0° to 90° at time instant equal to 5 seconds. To carry out such manoeuvre, the controller has to pitch the helicopter towards the desired direction. By doing so, the helicopter tends to fall below the desired elevation level, wich has to be compensated by the controller.

In this experiment, a maximum overshoot of 10% is set for the travel angle. The pitch angle (P) has no reference trajectory, but upper and lower bounds are imposed such that $|P| \leq 5°$. Pitch angle bounds may have to be enforced in several practical applications, e.g. when the load being carried by the helicopter must rest on a horizontal level, and even to prevent the helicopter from stalling. The elevation angle (E) has a set-point of 27° and upper and lower bounds such that $|E - 27°| \leq 0.8°$. These constraints on the elevation angle are needed, among other reasons, to ensure that the linearized model is valid during the whole manoeuvre. All these constraints and reference trajectories are depicted in Fig. 2 (see next section).

In order to investigate the effects of the multirate strategy proposed, three settings were considered:

- **1st Case**: single rate MPC with sampling time $T_s = 800$ms, prediction horizon $N = 20$, control horizon $M = 5$;
- **2nd Case**: multirate MPC with $N = 160$, predictions made every 100ms in the future for all the variables (P, E and T), $M = 5$ and control inputs updated every 800ms;
- **3rd Case**: multirate MPC with $N = 160$ and predictions made every 100ms in the future for the pitch output, $N = 20$ and predictions made every 800ms in the future for the travel and elevation outputs, $M = 5$ and control inputs updated every 800ms. This difference in the prediction scheme is due to the fact that pitch dynamics are faster than the others.

Note that although the prediction horizon assumes different values in the cases above, the comparison among them is fair in the sense that in all the cases the prediction horizon is fixed in units of time ($N_{\text{time}} = 16$s), the same number of degrees of freedom is employed in the QP solution ($M = 5$) and the inputs are updated at the same rate (every 800ms).

In the 1st case no blocking nor condensing was employed. In the 2nd case, use was made only of blocking. The 3rd case demonstrates the flexibility on how to implement a desired multirate MPC with both blocking and condensing. This flexibility is directly related to the design of the condensing ($\breve{C}$) and blocking ($\breve{B}$) matrices.

Matrices W_y and W_u of the predictive controller were adjusted so that all three cases exhibit similar behavior in the travel and elevation angles. The control weights for the front and back motors were set to $\rho = [15, 15]$ in all the cases under study. The output weights for pitch, elevation and travel were set to $\mu_1 = \mu_3 = [0, 20, 20]$ in the 1st and 3rd case, whereas in the 2nd case $\mu_2 = [0, 1, 1]$. There is no weight for the pitch angle since it has no reference path. Note that μ_2 is considerably smaller than μ_1 and μ_3 because in the 2nd case predictions for the travel and elevation angle are done at every 100ms, resulting in a higher number of tracking error terms included in the cost function (see, e.g., Eq. 1). In contrast, in the other cases the predictions are done only every 800ms. So the intention is to penalize deviations from the reference path in a similar manner in all three cases.

For simplicity, it is assumed that there are sensors available for the measurement of all states.

The simulation starts with the elevation angle and the input variables set to its steady state values. The study was carried out in Matlab/Simulink 6.5 R13 under Windows XP. A Pentium IV 3.0 GHz computer with 1.0 GB of RAM was employed.

5. RESULTS AND DISCUSSION

Figure 2 shows the pitch, travel and elevation angles obtained during the specified travel manoeuvre. It is clear from this figure that the travel and elevation trajectories are similar for all three cases. These trajectories exhibit no constraint violation and accomplish the specified goals (see Section 4). The pitch angle for the 2nd and 3rd cases remains within the constraint boundaries, as desired. In the 1st case, however, there are very pronounced violations on the pitch constraints. When a closer look is taken on these violations (Fig. 2b), it is possible to note that they occur between the sampling instants and, therefore, between the predictions made in the QP solution. So the controller never became aware of these violations and, in fact, at the sampling instants $t = [6.4, 7.2, 8, 8.8]$s the pitch lies on the lower constraint. An analogous behavior is observed on the violation of the pitch upper constraint.

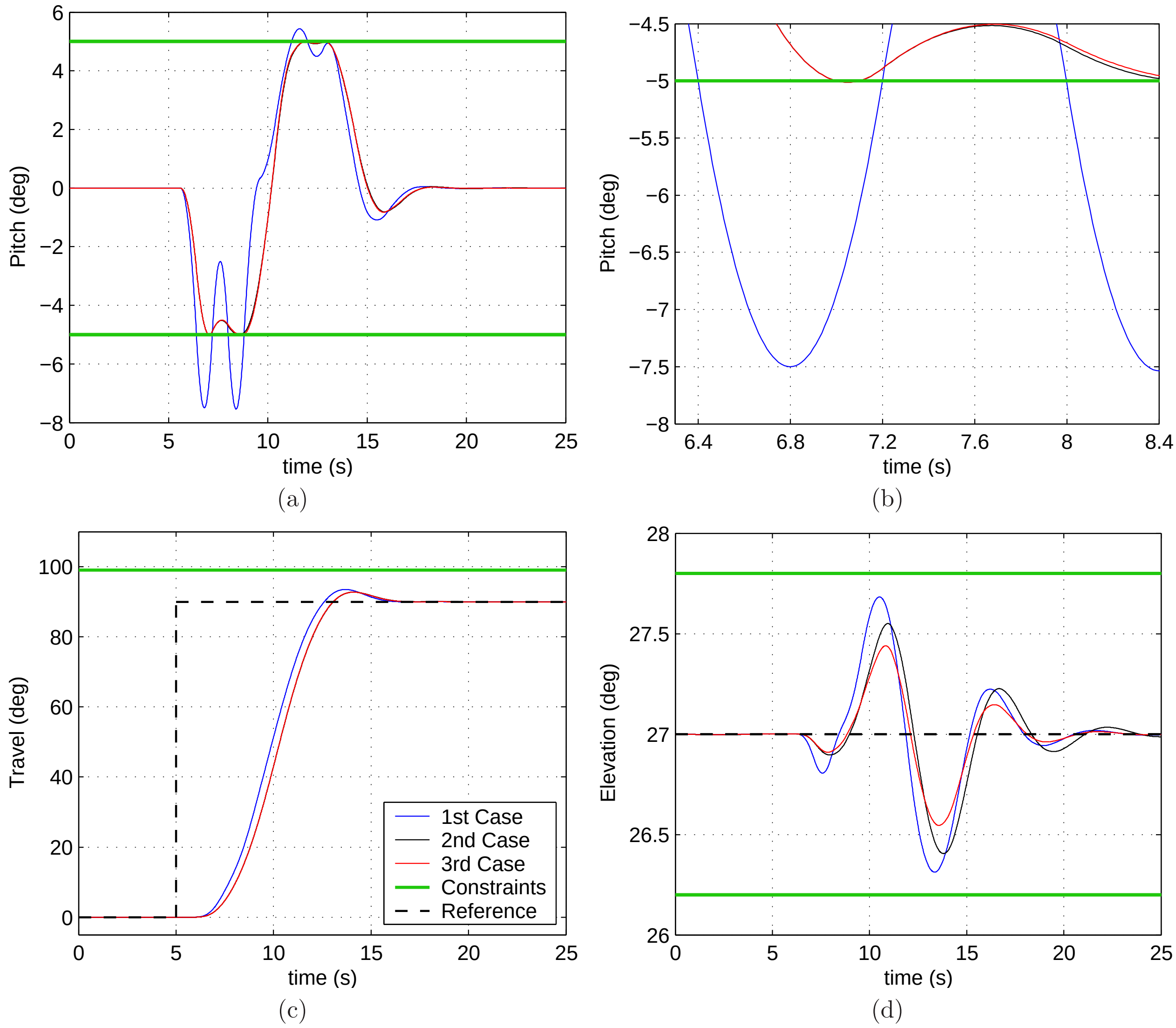


Figure 2. (a-b) Pitch, (c) Travel and (d) Elevation angles obtained during the desired travel manoeuvre.

Another important aspect to consider is the time spent to obtain the QP solution. If such time is large with respect to the control updating rate, then real-time implementation becomes somewhat deteriorated because of the non-constant input delay and may eventually lead to infeasibility of the QP solution, specially when the constraints are active. Figure 3 elucidates this aspect by showing the computation time in the QP solution. It is clear from this figure that it is not possible to arbitrarily increase the input control updating rate because eventually the computation of the QP solution will take longer than the control updating period. In this case the controller would not update the control action, which could lead to constraint violation and even to closed-loop instability.

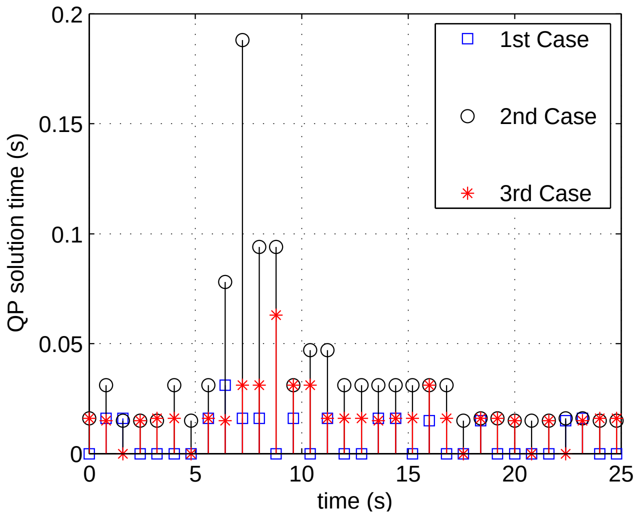


Figure 3. Computation time during the QP solution (small values were rounded down to zero).

The performance, in terms of the variables trajectories, obtained in the 2nd and 3rd case are very alike. However the 3rd case is less expensive in computational terms because not all outputs are predicted every 100ms, but only the pitch angle. This characteristic also improves the performance in real-time applications because a smaller delay is inserted in the control loop.

6. CONCLUSIONS

This work investigated some aspects of a multirate MPC strategy applied to the control of a 3DOF helicopter model. This strategy consists of blocking and condensing control input and output prediction sequences, respectively. Such operations are general with respect to the number of input and output variables as well as their respective sampling periods.

Three settings of multirate MPC were compared in a simulation scenario. The results showed that the enforcement of output and input constraints may require a proper choice of output and input sampling times. More specifically, it was shown that faster dynamics need smaller sampling periods so as to satisfy the specified goals, whereas slower dynamics can be sampled sparsely without degradation of overall performance which reduces the on-line computational effort.

Future works could analyze the robustness and stability of the employed multirate strategy. The design of a multirate state estimator and subsequent real-time implementation on the helicopter itself would also be interesting follow-ups to the present work.

7. ACKNOWLEDGEMENTS

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9. Responsibility notice

The authors are the only responsible for the printed material included in this paper