MULTI-CRITERIA STRUCTURAL RELIABILITY ANALYSIS BASED ON ANT COLONY OPTIMIZATION

Jhojan Enrique Rojas Flores, jerflores@mecanica.ufu.br
Felipe Antonio Chegury Viana, fchegury@mecanica.ufu.br
Domingos Alves Rade, domingos@ufu.br
Valder Steffen Jr. vsteffen@mecanica.ufu.br
Federal University of Uberlândia, Av. João Naves de Ávila 2121, CP 593, Uberlândia-MG, Brazil, 38400 902

Abstract. *Specific analysis methods have been developed to deal with the statistical nature of some characteristics of the models of a class of mechanical problems, such as geometry, material properties, and loads. In this context, Reliability Analysis intends to find the best compromise between cost and safety and to supply guidelines for carrying out reliable and cost-effective projects, accounting for the statistical variability of the system properties and loads. A set of methods in Reliability Analysis are readily applicable to the cases where the limit state functions are available in explicit analytical form. However, the situation is much more involved when it is necessary to perform the evaluation of implicit limit state functions through numerical models, like those based on the Finite Element Method. In this work, it is presented a Reliability Analysis methodology that couples nature-inspired optimization method, namely Ant Colony Optimization with Finite Element analysis. Numerical applications in static and dynamics considering various limit state functions simultaneously are presented in order to demonstrate the applicability, accuracy and efficiency of the proposed methodology.*

Keywords*: Reliability Analysis, Ant Colony Optimization, Finite Element Analysis.*

1. INTRODUCTION

Traditional structural design procedures utilize exclusively deterministic information of the problem. Values of geometrical features, material properties and loads are assumed, and an analysis is then performed to provide a model of the behavior of the structure. However, fluctuations of loads, variability of material properties and uncertainties regarding the analytical models can contribute to the probability that the structure does not perform as intended. In this context, analysis methods have been developed to deal with the statistical nature of input information. Over the last ten years there has been an increasing trend for analyzing structures using probabilistic information of loads, geometry, material properties, and boundary conditions. As the structures are becoming more complex and the performance requirements are becoming more ambitious, the need for analyzing the influence of uncertainties and computing the probabilities of events has been growing (Rackwitz, 2001).

Reliability Analysis (RA) can be used for the calculation of the probability of failure determined by a limit state function for structural members or structures at any time during their service life. The reliability analysis intends to find the best balance between cost and safety and to supply guidelines for carrying out reliable and cost-effective projects.

Structural time invariant reliability assessment consists in modeling every uncertain design variable as a random variable. After that, a failure criterion is defined by a limit state function or performance function that defines the failure domain in the space defined by those variables. To assess the structure's reliability, it is needed to know the joint probability density function (PDF) of the random variables. The failure probability will then be obtained by integrating the PDF over the failure domain, which is a very difficult task in practical situations, mainly those involving a large number of random variables (Haldar and Mahadevan, 2000).

In general, probabilistic analysis methods use probability functions to represent the design variables. This way, the solutions found through these analyses must also be represented by statistical parameters. Monte Carlo (MC) simulation is often referred to as the “exact” solution. However, for large-scale, high fidelity models, the computational effort renders MC simulation virtually impractical for use. Many other methods, considered to be more efficient, have been devised as alternatives to MC simulation. These methods include the first and second order reliability method: FORM and SORM respectively (Der Kiureghian and De Stefano, 1991); the advanced mean value family of methods: AMV (Wu et al., 1990) and the response surface method: RSM (Faravelli, 1989). These methods replace the original deterministic model with a computationally efficient analytical model in order to speed up the analysis.

Regarding optimization, it is well known that the solution of reliability analysis problems by using classical methods is a difficult task due to the existence of local minima in the design space. This aspect has motivated the authors of this paper to explore a modified approach based on FORM and SORM for the determination of reliability index and design points, based on the so-called Ant Colony Optimization (ACO), which, owing to its random, multi-directional search nature, is believed to be more robust with respect to the presence of local minima . In this paper, it is proposed an algorithm that is able to solve the global search optimization in reliability problems by using ACO. This algorithm is not competing with existing methods, but it is introduced because of its ability to solve global optimization problems efficiently. The analysis methodology integrates a set of reliability analysis tools developed under MATLAB® and finite element analysis using the commercial software ANSYS®. Numerical applications in static and the dynamics are performed in order to check the accuracy and efficiency of the suggested algorithms.

2. RELIABILITY ANALYSIS

Since the design parameters are considered as random variables, the satisfactory performance of a system can not be absolutely guaranteed. Instead, it can be expressed in terms of the probability of a certain failure criterion to be satisfied. In engineering terminology, this probability is called reliability and their counterpart, the failure probability. Thus, reliability is defined as the probability related to a perfect operation of a system (within the bounds specified by the design) during a pre-defined period in normal operation conditions.

Defining the design variables X_i of the structure and a performance function expressed or limit state function as $Z = g(X_i)$ which delimits the surface of failure (defined by the condition $Z = 0$), the safe region ($Z > 0$) and unsafe region ($Z < 0$), of the design space in which the failure occurs. The failure probability is calculated as:

$$P_f = \int_{g(X_i)<0} f_X(X_i)dx \tag{1}$$

where $f_X(X_i)$ is the joint probability density function (PDF) of the design variables.

In practice, it is impossible to obtain the joint PDF in Eq. (1) because of scarcity of statistical data. Even in the case where statistical information is sufficient to determine these functions, it is often impractical to perform numerically the integration indicated in Eq. (1). Moreover, the number of random variables is high; these variables do not appear explicitly in the performance function and there may be correlation among the design variables. These difficulties have motivated the development of various approximate reliability methods (Fiessler et al, 1979).

The main approaches to solve this equation are direct integration of PDF over the failure domain and analytical approximations such as the first and second order reliability methods (FORM and SORM, respectively). These methods use an optimization approach, and are close to the methodology presented in this paper. Because of that, they are briefly reviewed in the following section.

2.1. Reliability index estimation as a general optimization problem

In traditional design optimization, the optimization problem is generally formulated in the physical space of the design variables and consists in minimizing or maximizing an objective function subjected to geometrical, physical or functional constraints in the form:

$$\min f(\{y\}) \tag{2}$$

subjected to $g_k(\{y\}) \le 0$, where $\{y\}$ designates the vector of deterministic design variables.

In reliability analysis, which involves random variables $\{x\}$, the deterministic optimal solution is not considered the exact solution of the optimum design but is one of the most probably design. In this case, the failure surface or limit state function is given by $G(\{x\},\{y\}) = 0$. This surface defines the limit between the safe region $G(\{x\},\{y\}) > 0$ and unsafe region of the design space. The failure occurs when $G(\{x\},\{y\}) < 0$, and the failure probability is calculated as $P_f = prob[G(\{x\},\{y\}) \le 0]$.

The reliability index β is introduced as a measure of the reliability level of the system and is estimated in the so-called reduced coordinate system, where the random variables $\{u\}$ are statistically independent with zero mean and unit standard deviation. Thus a pseudo-probabilistic transformation $\{u\} = T[\{x\},\{y\}]$ must be defined for mapping the original space into the reduced coordinate system (Mohsine, 2006). Considering that the probability density in the reduced space decays exponentially with the distance from the origin of this space, the point with maximum probability of failure (most probable point) on the limit state surface is the point of minimum distance from the origin. The reliability index is thus defined as the minimum distance between the origin of the reduced space and the hypersurface representing the limit state function $H(\{u\},\{y\})$. Hence, it is possible to find the most probable point or design point by solving a constrained optimization problem that is:

$$\beta = \min \sqrt{\sum_{i=1}^{n} u_i^2} \tag{3}$$

subjected to safety constraints:

$$H(\{u\},\{y\}) = 0 \tag{4}$$

By formally introducing a cumulative density function (Φ) of the normal probability distribution function, the first order approximation (tangent plane at the MPP) to P_f can be written as:

$$P_f = \Phi(\beta) \tag{5}$$

This corresponds to the substitution of the hyper surface by the hyper plane passing through the point defined by u_i.

2.2. Reliability assessment by FORM and SORM

FORM and SORM can be considered as gradient-based methods since they require the evaluation of the partial derivatives of the limit state function with respect to the random variables at each iteration step.

FORM is based on linear (first order) approximation of the limit state surface tangent to the most probable point of the failure surface to the origin of a reduced coordinate system. Thus, the random variables are transformed to reduced variables in a reduced coordinate system. For estimating the reliability index based on FORM one can use the algorithm suggested by Rackwitz and Fiessler (1978) in which the limit state function does not need to be solved because a Newton-Raphson-type recursive algorithm is introduced to find the design point. This algorithm has been widely used in the literature (Haldar and Mahadevan, 2000).

SORM estimates the probability of failure by using a nonlinear approximation of the limit state function by a second order representation. The curvatures of the limit state function are approximated by the second-order derivatives with respect to the original variables. Thus, SORM improves FORM by including additional information about the curvature of the limit state function through of a curvature parameter. SORM was explored by Fiessler et al. (1979) using quadratic approximations. In that work the authors use a simple closed-form solution for the computation of failure probability using a second-order approach given by Breitung (1984) based on the theory of asymptotic approximation.

It is important to notice that the most probable point of FORM and SORM is the same. Additionally, SORM uses as initial value the reliability index value estimated through FORM. Zhao and Ono (1999) and Rojas et al. (2006) give more details of these classical techniques.

2.3. Reliability assessment by ACO

The solution of the optimization problem given by Eq. (2) by using classical gradient-based optimization methods is not a simple task due to the existence of local minima in the design space and the necessity of computation of the gradients (partial derivatives). As a result, accuracy, convergence and computational effort are relevant issues. The existence of multiple MPPs of the limit state functions is likely to introduce additional difficulties. Multiple MPPs are similar to multiple local minima in optimization. The solutions of many problems in structural optimization can be considered to be satisfactory once a local minimum is reached. However, this is an unacceptable procedure in reliability analysis since the local MPP may not represent the worst failure scenario and the actual failure may occur below the predicted level. Hence, only the global MPP represents the actual structural reliability (Wang and Grandhi, 1995).

Another difficulty that must be remembered is that traditional methods FORM and SORM require an initial guess of the solution (reliability index and random variables) and it is not always possible to assure global convergence. These aspects has motivated the authors of this paper to explore an alternative approach for estimation of reliability index, which does not require the computation of gradients of the limit state function and are intrinsically based on multidirectional search. In this work it is used an approach that uses Finite Element analysis to evaluate several implicit limit state functions on RA based on ACO. It is believed that such approach can circumvent some of the difficulties mentioned above, and thus leads to improved results of reliability analysis. It was observed that this methodology is able to handle multiple limit state functions based on numerical models and probabilistic variables related to geometrical, load and material properties parameters (Rojas et al, 2007). In another contribution the authors explore a Heuristic Based Reliability Method (HBRM) which uses of optimization methods such as Genetic Algorithms (GA)

(Michalewicz, 1994; Haupt and Haupt, 2004), Particle Swarm Optimization (PSO) (Kenedy and Eberhart, 1995) and ACO (Venter and Sobieski, 2002). Rojas et al (2007) give more details of HBRM. The following section discusses the main ideas about ACO.

3. ANT COLONY OPTIMIZATION

Even though most of general-purpose optimization software used in industrial applications makes use of gradient-based algorithms, nature-inspired optimization algorithms have experienced ever growing practical applications (Venter and Sobieski, 2002). In spite of the heavy computational effort, when compared to gradient-based techniques, these methods have several advantages, such as the ease to code, the efficiency in making use of parallel computing architectures, the ability to overcome numerical convergence difficulties and the capability of dealing with discrete and continuous variables simultaneously.

ACO is inspired in the behavior of ants and their communication scheme by using pheromone trails (Dorigo, 1992). A moving ant lays some pheromone on the ground, thus marking its path. The collective behavior that emerges from the participating agents is a form of positive feedback in such a way that the more the ants follow a trail, the more attractive that trail will become for being followed.

When searching for food, real ants start moving randomly, and upon finding food they return to their colony while laying down pheromone trails (Socha, 2004). This means that if other ants find such a path, they return and reinforce it. However, over time the pheromone trail starts to evaporate, thus reducing its attraction strength. When a short and a long path are compared, it is easy to see that a short path gets marched over faster and thus the pheromone density remains high. Thus, if one ant finds a short path (from the optimization point of view, it means a good solution) when marching from the colony to a food source, other ants are more likely to follow that path, and positive feedback eventually encourages all the ants in following the same path. The idea behind ACO is to mimic this behavior by using artificial ants. The outline of a basic ACO algorithm is presented in Fig. 1.

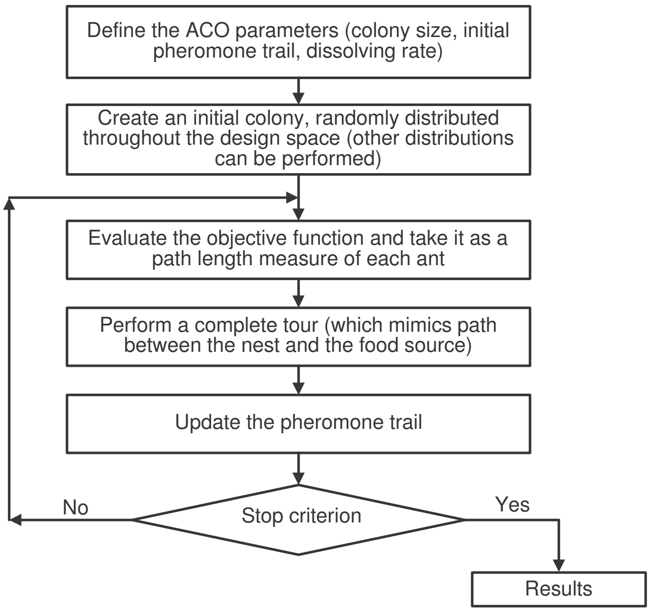

Figure 1. ACO basic algorithm.

The first point that has to be taken into account is how to model the pheromone communication scheme. According to Pourtakdoust and Nobahari (2004), for continuous model implementation, this can be done by using a normal probability distribution function (PDF), as follows:

$$pheromone(x) = e^{-\frac{(x - x_{\min})^2}{2\sigma^2}} \tag{6}$$

where $x_{\min}$ is the best point found within the design space and σ is an index related to the ants aggregation around the current minimum.

In Fig. 1, "To perform a complete tour," means to update the values of each design variable for all ants of the colony. Or, more precisely, it is the process in which, for a given iteration, each ant sets the values for the trial solution based on the probability distribution specified by Eq. (6). Computationally, this can be achieved through a random number generator based on a normal PDF that plays the role of a variable transition (update) rule to choose the next design variable value associated with each ant. From Eq. (6), it can be noticed that each variable uses a different random number generator together with its respective PDF.

Finally, pheromone distribution over the design space is updated by collecting the information acquired throughout the optimization steps. Since the pheromone is modeled by Eq. (6), it is necessary only to update $x_{\min}$ and σ by:

$$\sigma = std(colony) \tag{7}$$

where $std(colony)$ makes use of the colony of ants (candidate solutions) to return a vector containing the standard deviation for each design variable.

Regarding the pheromone scheme, it is possible to see that the accumulation of pheromone increases in the vicinity of the candidate to the optimum. This approach reinforces the probability of the choices that lead to good solutions. However, to avoid premature convergence, negative update procedures are not discarded (Socha, 2004).

In this work, a simple method to perform negative update is used, which consists in dissolving the pheromone. The idea of this scheme is to spread the amount of pheromone by changing the current standard deviation (for each variable) according to the following equation:

$$\sigma_{new} = \gamma\sigma_{old} \tag{8}$$

where $\gamma > 1$ is the dissolving rate.

To initialize the algorithm:

- $x_{\min}$ is randomly chosen within the design space using a uniform PDF;
- σ is taken as being 3 times greater than the length of the search interval

Differently from what occurs with GA and PSO, which have a set of parameters to be defined by the user, ACO has a single special parameter to be chosen, namely the dissolving rate.

A comparison between what happens in nature and the counterparts in the ACO algorithm can be viewed in Table 1:

Table 1. Nature versus ACO.

Nature	**ACO**
Possible paths between the nest and food	Set of possible solution (vector of design variables)
Shortest path	Optimal solution
Pheromone communication in action	Optimization procedure

When solving an optimization problem, one must keep in mind that it will be always necessary to run more than once the optimization procedure. In the case of using classical methods, this is done to avoid local minima by starting from different initial designs. In the case of using nature-inspired methods, one has just to run the algorithm each time with a different seed for the random number generator. At the end, the engineer can compare all results obtained and make decisions about which will be chosen as the final design. Usually, the candidates are either the mean or the best result of the set.

4. NUMERICAL EXAMPLES

The present application is concerned with the use of dynamic vibration absorbers (DVAs) in a dome structure. The objective is to study the design of the DVA in four different scenarios of reliability analysis. These scenarios vary from the case of a single limit state function to the case of multiple limit state functions.

Dynamic vibration absorbers (DVAs) are systems constituted by mass, spring and damping elements, which are coupled to a mechanical system (named primary structure) in order to attenuate the vibrations in a given frequency range. The classical procedure for tuning the DVA, i.e., to define a convenient set of values of the DVA parameters (mass, spring and damping values), is based on the existence of the so-called fixed points of the FRFs (Den Hartog, 1956).

Figure 2 shows details about the finite element (FE) model of a dome structure with a DVA. Figure 2-(a) illustrates the oblique view with the boundary and load conditions. Figure 2-(b) illustrates the target mode shape ($f = 32.2$ Hz). Figure 2-(c) gives details about the cross section of the beams. Finally, Fig. 2-(d) illustrates the scheme of the DVA.

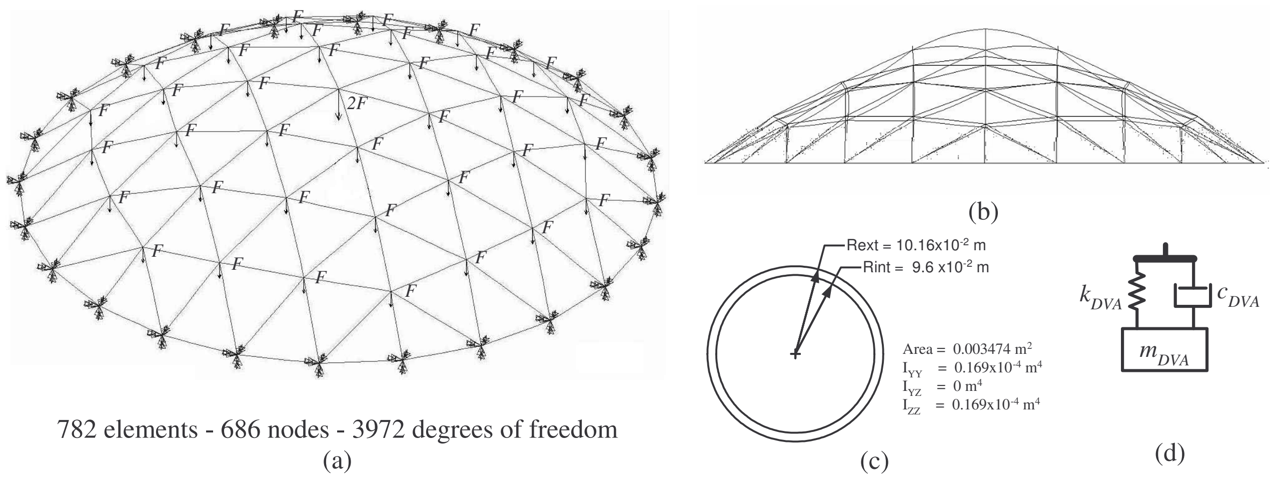

Figure 2. Finite element model of dome.

First of all, the deterministic design of the DVA for the dome structure is treated as an optimization problem in which the goal is to reduce the vibration amplitudes of the first mode. As suggested by Steffen and Rade (2000; 2001) this optimization problem is defined as the minimization of the objective function given by:

$$J\left(m_{DVA}, c_{DVA}, k_{DVA}\right) = \max_{26 \leq f \leq 36} \left\{ \left| H(f) \right| \right\} \tag{9}$$

where $H(f)$ is the amplitude of a given frequency response function of the system with the DVA.

Following this approach, the optimal values for the DVA parameters were found to be $m_{DVA} = 72.96\,\text{kg}$, $k_{DVA} = 2406493.62\,\text{N}/m$ and $c_{DVA} = 6250.24\,\text{N.s/m}$.

Back to the reliability problem, it was considered as random variables: $m_{DVA}, k_{DVA}, c_{DVA}$ and F. The interest in including the dead load F as a random variable is related to the fact that it is expected that the stress-stiffening effect can have some influence on the dynamic behavior of the structural system. This effect has been investigated by Rojas (2004).

Table 2 summarizes the design parameters and their statistical moments.

Table 2. Random variables and statistic parameters of DVA.

Random variable	**Distribution**	**Mean**	**Standard deviation**
m_{DVA} (Kg)	Normal	72.96	7.296
k_{DVA} (N/m)	Normal	2406493.62	240649.362
c_{DVA} (N.s/m)	Normal	6250.24	625.024
F (N)	Normal	40000	4000

4.1. First scenario

The first scenario is dedicated to the study of reliability analysis based on a single state limit function. Under this circumstance, it is possible to check the possibility of using ACO approach to generate an initial guess to FORM and SORM. The limit state function is defined as:

$$G_1\left(F, m_{DVA}, c_{DVA}, k_{DVA}\right) = 1 - \frac{\left| f_3 - f_3^{dome} \right|}{\Delta f} \tag{10}$$

where:

- f_3 is the third natural frequency of the whole structure (dome + DVA),

- f_3^{dome} is the third natural frequency of the dome, and
- $\Delta f = 5$ Hz is the frequency band of interesting.

It is easy to see that this function measures the tuning of the DVA for the 3*th* mode.

Table 3 and others shows the results obtained from 20 runs of ACO. Through previous analyses it was possible define the number of runs of ACO aiming the equilibrium between time computational effort and a relative results convergence. It was observed that better results are obtained for more runs and computational effort. Moreover, statistical parameters of the results are considering acceptable for 20 runs of ACO. In these tables: Min. β is the less reliable design, Max. β is the most reliable design, μ_i is the average of the 20 runs, σ_i is the standard deviation of the 20 runs, and δ_i is the coefficient of variation of the 20 runs.

It can be observed that the standard deviations for each design variable are always small when compared to the mean values. It means that the ACO was capable to reach solutions that are close in the design space.

Table 3. Results of the first scenario.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	$P_f\,(\%)$	$R_{level}\,(\%)$
Min. β	39681.78	71.78	2337996.69	5995.30	0.5292	29.83	70.17
Max. β	46205.01	75.07	2324807.72	6511.53	1.6674	4.77	95.23
μ_i	40558.45	72.10	2435668.64	6295.67	0.9532	17.82	82.18
σ_i	2269.95	3.49	95362.35	320.81			
δ_i	0.06	0.05	0.04	0.05			

The best result was obtained in the Experiment #14, with a reliability level of 95.23%, which is considered satisfactory as a final design. The worst result was obtained in the Experiment #8 with a reliability level of 70.17%, which is not a good final design. Following the proposed strategy, the values of the design variables obtained in the Experiment #8 were used to feed a cascade-type approach with FORM and SORM. Table 4 shows the results. It can be seen that for both FORM and SORM the results are the same and there is a significant improvement when compared with the initial guess of the Experiment #8.

Table 4. Results of FORM and SORM.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	$P_f\,(\%)$	$R_{level}\,(\%)$
FORM	39681.78	85.01	1585039.72	5995.30	3.7109	0.0103	99.9897
SORM	39681.78	85.01	1585039.72	5995.30	3.7109	0.0103	99.9897

4.2 Second Scenario

Here it is added a second limit state function. According to what was discussed in the previous sections, the reliability problem is treated as a constrained optimization problem. It is important to notice that FORM and SORM are not capable to deal with this type of problem.

The second limit state function is written as:

$$G_2\left(F, m_{DVA}, c_{DVA}, k_{DVA}\right) = 1 - \frac{\delta_y}{\delta_y^{\lim}} \tag{11}$$

where:

- δ_y is the displacement in the vertical direction of the DVA attachment point.
- $\delta_y^{\lim}$ is the limit assumed for the displacement in the vertical direction of the DVA attachment point.

According to the literature (ABNT/CB 02, 2007), this value is given by $\delta_y^{\lim}(cm) = L/200$, where L is diameter of the dome. Differently from G_1, this function tries to ensure that the maximum displacement be lesser than the pre-defined limit.

Table 5 shows the results obtained from 20 runs of ACO. Similarly to the previous case, the standard deviations for each design variable are small when compared to the mean values.

The best result was obtained in the Experiment #15, with a reliability level of 90.65%, which is not a satisfactory result but can be considered as a final design. The worst result was obtained in the Experiment #17 with a reliability level of 68.15%, which is not a good final design.

It was observed in Table 2 that the standard deviations of parameters are assumed as 10% of mean values. It is known that the probability of failure increases with standard deviation. Therefore, a pre-analysis of probabilistic parameters may be able to define minor values of standard deviations if compared with the adopted values.

Table 5: Results of the second scenario.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	$P_f(\%)$	$R_{level}(\%)$
Min. β	38970.60	72.60	2490490.97	6362.26	0.4719	31.85	68.15
Max. β	36157.08	78.93	2481119.65	6393.75	1.3197	9.35	90.65
μ_i	39953.01	72.66	2413646.73	6369.26	0.9970	16.39	83.61
σ_i	2469.73	3.82	119889.67	235.92			
δ_i	0.06	0.05	0.05	0.04			

4.3 Third Scenario

In this scenario, a third function is added to the constrained optimization problem:

$$G_3(F, m_{DVA}, c_{DVA}, k_{DVA}) = 1 - \frac{F_{\max}}{F_y^{\lim}} \tag{12}$$

where:

- $F_{\max}$ is the maximum reaction force on the dome structure, and
- $F_y^{\lim}$ is the assumed limit for the maximum reaction force. According to the literature (Haldar and Mahadevan, 2000), this value is given by $F_y^{\lim} = f_{yk} A$, where $f_{yk} = 250$ MPa and A is cross section area.

This function tries to ensure that the maximum reaction force be lesser than the pre-defined limit.

Table 6 shows the results obtained from 20 runs of ACO. Once again, the standard deviations for each design variable are small when compared to the corresponding mean values.

Table 6. Results of the third scenario.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	$P_f(\%)$	$R_{level}(\%)$
Min. β	40322.51	70.42	2372503.89	6434.26	0.4838	31.43	68.57
Max. β	34221.59	73.34	2382159.46	6616.36	1.5630	5.90	94.10
μ_i	40242.74	74.10	2403407.09	6127.73	0.98	17.57	82.43
σ_i	2504.88	3.81	70244.92	334.29			
δ_i	0.06	0.05	0.03	0.05			

The Experiment #7 was the best result obtained, with a reliability level of 94.10%, which is a satisfactory result considered as a final design. The worst result was obtained in the Experiment #18 with a reliability level of 68.57%, which is not a good final design.

4.4 Fourth Scenario

Here, a fourth function is added to the constrained optimization problem:

$$G_4(F, m_{DVA}, c_{DVA}, k_{DVA}) = 1 - \frac{F}{F^{crit}}, \tag{13}$$

where F^{crit} is the buckling load of the dome structure.

This function is particularly interesting since F, which is not a DVA parameter, is explicitly used in the formulation.

Table 7 shows the results obtained from 20 runs of ACO. Again, the standard deviations for each design variable are small when compared with the mean values.

Table 7. Results of the fourth scenario.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	$P_f(\%)$	$R_{level}(\%)$
Min. β	38925.15	72.18	2387927.58	6139.25	0.3482	36.39	63.61
Max. β	38114.12	69.99	2644614.33	6883.82	1.5474	6.09	93.91
μ_i	40233.96	71.33	2404621.49	6197.09	1.1405	13.60	86.40
σ_i	2299.97	3.25	156146.44	412.96			
δ_i	0.06	0.05	0.06	0.07			

In the Experiment #1 was obtained the best result, with a reliability level of 93.91%, which is a satisfactory result considered as a final design. The worst result was obtained in the Experiment #10 with a reliability level of 63.61%, which is not a good final design.

5. CONCLUSIONS

In this work it is proposed a new reliability analysis methodology which integrates the nature-inspired optimization method Ant Colony Optimization and was used to estimate the design point and reliability level to dynamic and static parameters using different limit state functions simultaneously. There were four probabilistic variables, three of them related to the design of a DVA and one related to the load condition. In the applications it was used the reliability analysis tools of HBRM integrated with finite element analysis. In most of the cases, the best results in different scenarios can be considered as final design, mean and standard deviation of the results was considered satisfactory. Taking the performance of HBRM into account, it can be concluded that the presented methodology is able to handle limit state functions based on numerical models and probabilistic variables related to both physical or geometrical parameters, as well as loads. The results obtained encourage the authors to improve this methodology for use in complex problems.

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