MODELING RANDOM UNCERTAINTIES USING A PARAMETRIC APPROACH IN A BIOMECHANICAL MODEL OF THE LARYNX FOR MEN AND WOMEN VOICE PRODUCTION

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Abstract. *Almost all biomechanical systems for voice production have been modeled as deterministic. However, models of voice production are better modeled as stochastic processes, taking into account that their parameters are uncertain. The parametric approach requires the adoption of random variables to represent the uncertain parameters of voice production for improving the predictability of the model. For each random variable, a probability density function has to be constructed following a chosen strategy. In this paper, some parameters of a biomechanical model for voice production are considered as uncertain and their density probability functions are modeled using the Maximum Entropy Principle. The construction of the probability distribution is very sensitive to the information used when the Maximum Entropy Principle is applied. The biomechanical model discussed here can represent the voice production for men and women, according to the parameters used. The main objective of this work is to compare the voice production process of men and women taking into account the uncertainties of the process. This comparison is based on a probabilistic analysis of the fundamental frequency of the voice signal obtained for men and women.*

Keywords: *voice production, uncertainties, parametric probabilistic approach, biomechanical models*

1. INTRODUCTION

Voice production (phonation) has been studied by several researchers, and for a number of different reasons such as: to obtain synthesis of voiced sounds (Ishizaka and Flanagan, 1972) (Koizumi et al, 1987) (Cataldo, 2006), to simulate pathological vocal-fold vibrations (Ishizaka, 1976) (Zhang, 2005), and to discuss nonlinearities related to the process (Steinecke, 1995) (Herzel, 1995) (Lucero, 1999). Phonation is one of the laryngeal functions. It results from the vibration of the vocal folds, located in the larynx, which alternately snap together and apart, colliding one with the other, in a periodic (or quasi-periodic) motion.

The laryngeal function is highly similar within the group of voiced sounds produced. Vocal-fold vibration differs little across vowels, and their distinctiveness is determined by the shaping of the *vocal tract*. The vocal folds are set into vibration by the combined effect of the subglottal pressure, the viscoelastic properties of the vocal folds, and the Bernouilli effect, according to the accepted myoelastic theory of voice production proposed by van den Berg (1968) and Titze (1980). The vocal tract acts as a filter which transforms the primary signal (the glottal pulses) into the final voiced sounds.

To characterize the vocal folds vibration, the two-mass model proposed by Ishizaka and Flanagan (1972) has been widely used. In order to model the voice production, besides a vocal-fold model, it is also necessary a model for the vocal tract, usually represented by an acoustic tube. Predictions from the model may be improved with better measures of its parameters. However, since they are related to physiological parameters, accurate measures are, in general, difficult to be done (see, e.g., a discussion of the shape of the vocal tract (Fant, 1960) and technological means to find an approximation for this shape (Titze, 1996) (Takemoto, 2005).

Here, the human voice production system is considered to be non deterministic. The vocal-fold model used has a lot of parameters and, in principle, all of them could be considered as random. However, doing so would be computationally costly and will complicate the interpretation of the physics of the problem. Therefore, we will select only a few important parameters to be random. The choice will be later validated by the obtained results. Besides the choice of the random parameters, the other key point to make a probabilistic model is the association of a probabilistic distribution to these parameters. Of course this could be done through a careful statistics from experimental data. Here this path is not followed and another strategy is used. The probability distributions are characterized using the Maximum Entropy Principle and taking into account some *usable information* on those parameters. The probability distribution constructed is very sensitive to the choice of the usable information. This is shown through an example. This technique is very powerful, since the scarce information used to construct the probability density function is enough to characterize the radiated output pressure and also to measure the robustness of the model.

2. MEAN MODEL

In general, a mathematical model has as objective to predict the output of the real system for a given input. The mean model is the corresponding deterministic mathematical model. The input of the mean model does not exactly represent the input of the real system and, also, uncertainties on the parameters of the mean model have to be taken into account to improve its predictability. The error between the output predicted calculated with the mean model and the response of the real system should be minimized in order to improve its predictability. In general, due to data uncertainties, this error is not sufficiently small.

The mean model adopted here is showed in Fig. 1 (Ishizaka and Flanagan, 1972). Each vocal fold is represented by two (nonlinear) mass-damper-spring systems, coupled through a (linear) spring (k_c) and the vocal tract is represented by a standard two-tube configuration for vowel /a/.

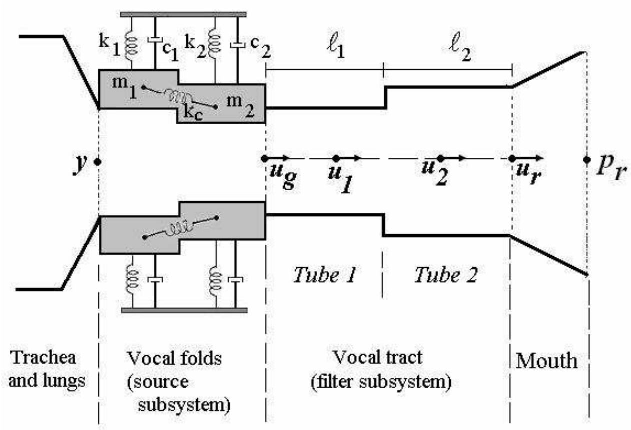

Figure 1. Model used for describing the voice production process.

The dynamics of the system can be written in a simplified form, by Eq.(1) and Eq.(2):

$$\psi_1(\mathbf{w})\dot{u}_g + \psi_2(\mathbf{w})|u_g|u_g + \psi_3(\mathbf{w})u_g + \frac{1}{\tilde{c}_1}\int_0^t (u_g(\tau) - u_1(\tau))d\tau - y = 0 \quad (1)$$

$$[M]\ddot{\mathbf{w}} + [C]\dot{\mathbf{w}} + [K]\mathbf{w} + \mathbf{h}(\mathbf{w}, \dot{\mathbf{w}}, u_g, \dot{u}_g) = 0 \quad (2)$$

where $\mathbf{w}(t) = (x_1(t), x_2(t), u_1(t), u_2(t), u_r(t))^t$. The functions x_1 and x_2 are the displacements of the masses, u_1 and u_2 describe the air volume flow through the (two) tubes that model the vocal tract and u_r is the air volume flow through the mouth. The function p_r gives the output radiated pressure and it is given by $p_r(t) = u_r(t)r_r$, in which $r_r = \frac{128\rho v_c}{9\pi^3 y_2^2}$, ρ is the air density, v_c is the sound velocity, and y_2 is the radius of the second tube. The subglottal pressure is denoted by y and the function u_g describes the glottal signal.

The functions ψ_1, ψ_2, ψ_3, $\mathbf{h}$, and also the matrices $[M]$, $[C]$, $[K]$ are described in the appendix.

Equation (1) is the coupling equation between the vocal tract and the vocal folds and Eq. (2) gives the dynamics of the airflow, from the lungs up to the mouth.

In order to solve the system (Eq. (1) and Eq. (2)); that is, find u_g and $\mathbf{w}$ given y, a *centered finite difference* scheme is used for Eq. (1) and an unconditionnally stable Newmark scheme is used for Eq. (2). This method is proposed because there is a non-linear equation to solve and such an implicit scheme, in general, is more robust to solve stochastic nonlinear elastodynamical systems. The results obtained with this strategy were satisfactory.

For a probabilistic analysis of the fundamental frequency of men and women, taking into account studies discussed by Lucero (1999) and the randomness of some parameters involved in the model considered, it will be necessary to calculate the fundamental frequency, for each realization. Let T be the period of the function u_g and let f_0 be the fundamental frequency of the voice signal. The fundamental frequency of the voice signal is also the fundamental frequency of the glottal signal u_g. Then, $f_0 = 1/T$.

3. STOCHASTIC MODEL

3.1 Stochastic equations

Some parameters will be considered as uncertain and random variables will be associated to these parameters. As the objective is to perform a probabilistic analysis of the fundamental frequency, the main parameters responsible for the changing of the fundamental frequency were chosen. These parameters are the *tension parameter*, *subglottal pressure*, and *neutral glottal area*.

The first parameter, that describes the muscular action to produce the voice, influences several other parameters of the model and it is used an strategy to reduce them to just one dimension. This parameter is called *tension parameter* and it is denoted by q. So, m_1, k_1, m_2, k_2 and k_c are written as $m_1 = \widehat{m}_1/q$, $k_1 = q\,\widehat{k}_1$, $m_2 = \widehat{m}_2/q$, $k_2 = q\,\widehat{k}_2$ and $k_c = q\,\widehat{k}_c$, in which $\widehat{m}_1$, $\widehat{k}_1$, $\widehat{m}_2$, $\widehat{k}_2$ and $\widehat{k}_c$ are fixed values. This is done similarly as Ishizaka and Flanagan (1972). The random variable associated to this parameter will be denoted by Q.

The second parameter is the *subglottal pressure*, denoted by y, and the random variable associated will be denoted by Y.

The third parameter is the *neutral glottal area*, denoted by a_{g0}, and the associated random variable will be denoted by A_{g0}.

The corresponding stochastic equations will be written, from Eq. (1) and Eq. (2), substituting the parameters q, y and a_{g0}, by the corresponding random variables Q, Y and A_{g0}, respectively.

Figure 2 shows a block diagram describing the complete system and the corresponding two subsystems (vocal folds subsystem and vocal tract subsystem), considering that all parameters are deterministic. Figure 3 shows the block diagram of the complete system, but emphasizing the random variables related to the parameters chosen as uncertain.

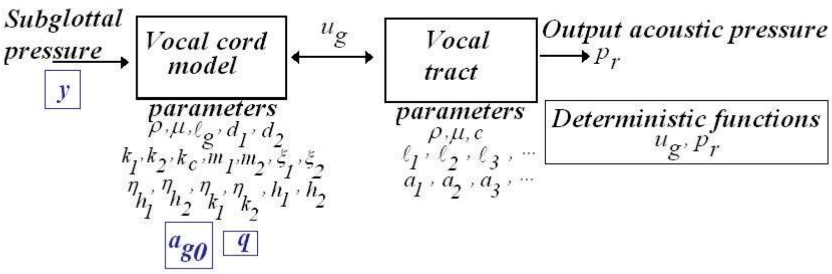

Figure 2. Block diagram of the deterministic complete system.

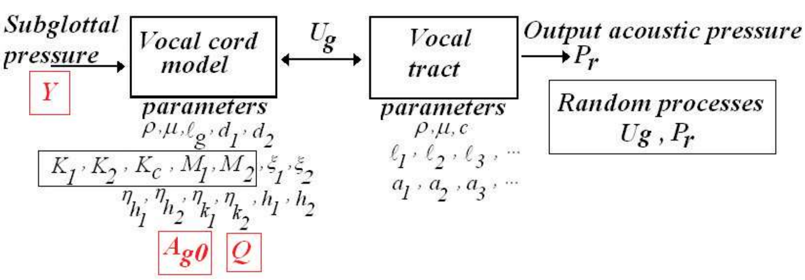

Figure 3. Block diagram of the complete system, emphasizing the three random variables related to the uncertain parameters.

To write the equations of the dynamics of the corresponding stochastic system:

(1) the matrices $[M]$, $[C]$ and $[K]$ will be substituted by random matrices $[\mathbf{M}]$, $[\mathbf{C}]$, and $[\mathbf{K}]$.

(2) the functions $\mathbf{w}$, $\mathbf{h}$, u_g and u_1 will not be deterministic anymore. They are stochastic processes and will be substituted by $\mathbf{W}$, $\mathbf{H}$, U_g and U_1, respectively.

Then, the corresponding stochastic equations are given by Eqs. (3) and (4).

$$\psi_1(\mathbf{W})\dot{U}_g + \psi_2(\mathbf{W})|U_g|U_g + \psi_3(\mathbf{W})U_g + \frac{1}{\tilde{c}_1}\int_0^t (U_g(\tau) - U_1(\tau))d\tau - Y = 0 \tag{3}$$

$$[\mathbf{M}]\ddot{\mathbf{W}} + [\mathbf{C}]\dot{\mathbf{W}} + [\mathbf{K}]\mathbf{W} + \mathbf{H}(\mathbf{W}, \dot{\mathbf{W}}, U_g, \dot{U}_g) = 0\,. \tag{4}$$

The random variable Y appears explicitly in Eqs. 3 and 4. This does not happen with the random variables A_{g0} and Q, which will appear in the entries of the random matrices $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$ and in the definition of the function $\mathbf{H}$.

The next step is, then, to construct the density probability functions of the random variables Q, Y and A_{g0}. The strategy used is based on the Maximum Entropy Principle and relies in some *usable information*, necessary to solve a constrained optimization problem. The resulting density is very sensitive to the information used, as will be shown thorough an example. First, a brief review of the Maximum Entropy Principle will be presented and after the probabilistic models are constructed.

3.2 The Maximum Entropy Principle

This principle consists in maximizing the entropy subjected to constraints defined by the usable information. In the context of information theory, Shannon (1948) introduced an entropy as the measure of uncertainty for probability distributions. In the context of Statistical Mechanics, Jaynes (1957a, 1957b) used this measure to define the Maximum Entropy Principle for the construction of a probability distribution.

This Principle permits to construct the probability density function p_X of a random variable X from a set of information, called *usable information*.

If X is a continuum random variable, the Entropy $S(p_X)$ of its probability density function p_X is defined by Eq. (5).

$$S(p_X) = -\int_{\mathbb{R}} p_X(x)\,\ell n(p_X(x))\,dx\,. \tag{5}$$

According to the Maximum Entropy Principle, the probability density function p_X to be constructed is the one with the largest uncertainty, measured by the entropy, among the sets of all of the probability density functions that verify the constraints defined by the usable information.

Let p_X be the probability density function of the random variable X with values in $\mathbb{R}$. We suppose that p_X is unknown, but its support $s \subset \mathbb{R}$ is known and also the m real numbers $\phi_1, \ldots, \phi_m$ such that

$$E\{g_j(X)\} = \int_{\mathbb{R}} g_j(x)p_X(x)dx = \phi_j \in \mathbb{R}\ ,\ j = 1, \ldots, m \tag{6}$$

with g_j a real measurable function.

The m equations given by Eq. (6) define what we call the usable information to construct the probability density function p_X, knowing the functions $g_1, \ldots, g_m$ and the corresponding values of $\phi_1, \ldots, \phi_m$.

Let $\mathcal{C}$ be the space of the functions p_X, from $\mathbb{R}$ to $\mathbb{R}_+$, with the same support $s \subset \mathbb{R}$, verifying the Eqs. 7 and 8.

$$\int_{\mathbb{R}} p_X(x)dx = 1 \tag{7}$$

$$\int_{\mathbb{R}} g_j(x)p_X(x)dx = \phi_j \tag{8}$$

To construct p_X, we will maximize the entropy $S(p_X)$ under the $m + 1$ constraints defined by Eqs. (7) and (8); that is, we will solve the optimization problem given by Eq. (9).

$$\max_{p_X \in \mathcal{C}} S(p_X)\,. \tag{9}$$

We introduce $1+m$ Lagrange's multipliers $(\lambda_0 - 1) \in \mathbb{R}$, $\lambda_1 \in \mathbb{R}, \ldots, \lambda_m \in \mathbb{R}$, associated with the $1+m$ constraints. The corresponding Lagrangian $\mathcal{L}$ is given by Eq. (10):

$$\mathcal{L}(p_X) = S(p_X) - (\lambda_0 - 1)\left\{\int_{\mathbb{R}} p_X(x)dx - 1\right\} - \sum_{j=1}^{m}\left(\lambda_j \int_{\mathbb{R}} g_j(x)p_X(x)dx - \phi_j\right). \quad (10)$$

From the Calculus of Variations, we obtain that p_X is given by

$$p_X(x) = \mathbf{1_B}(x) exp\left(-\lambda_0 - \sum_{j=1}^{m}\lambda_j\right) g_j(x) \quad (11)$$

in which $\mathbf{1_B}(x) = 1$ if $x \in \mathbf{B}$ and 0 if $x \notin \mathbf{B}$. It can be also proved that this is the only extreme and it is a maximum.

We have then to calculate the $1+m$ multipliers $\lambda_0, \lambda_1, \ldots, \lambda_m \in \mathbb{R}$ using the $1+m$ equations given by Eqs. (7) and (8).

Let $\boldsymbol{\lambda}$, $\boldsymbol{\phi}$ and $\mathbf{g}$ be vectors given by $\boldsymbol{\lambda} = (\lambda_0, \lambda_1, \ldots, \lambda_m)$, $\lambda_j \in \mathbb{R}$, $\phi = (1, \phi_1, \ldots, \phi_m)$ and $\mathbf{g} = (1, g_1, \ldots, g_m)$.

Then, we must find $\boldsymbol{\lambda}$ that minimizes the function Δ defined by

$$\Delta(\boldsymbol{\lambda}) = \lambda_0 + \lambda_1\phi_1 + \ldots + \lambda_m\phi_m + \int_s exp(-\lambda_0 - \lambda_1 g_1(x) - \ldots - \lambda_m g_m(x))dx\,. \quad (12)$$

It can be proved that Δ is a strictly convex function and consequently there is only one extreme. It can be then proved that it is a minimum. Consequently, there is only one $\boldsymbol{\lambda}$ that minimizes Δ. In general, the usable information used refers to the range of the density probability functions, the moments one expects are finite, the possibility to solve the inverse problem, etc. Also one tries to solve the optimization problem analytically, but of course there is no need of an analytical solution and one could as well work numerically.

In the following, we will apply these results to find density probability functions corresponding to the chosen uncertain parameters.

3.3 Probability model of the uncertain parameters

In order to construct a coherent probability model, only the available information on the parameters is used and the Maximum Entropy Principle is applied.

With the nature of the available information used for the probabilistic models, the application of the Maximum Entropy Principle yields independent probability density functions for A_{g0}, Y and Q (Soize, 2000, 2001).

3.3.1 Tension parameter

The tension parameter is modeled by the random variable Q. The usable information are: (i) Its support is $]0, +\infty[$, (ii) its mean value is $E\{Q\} = \underline{Y}_s$, (iii) $E\{1/Q^2\} < +\infty$. Information (iii) is due to $M_1 = \widehat{m}_1/Q$ is a second-order random variable. Then, it is necessary that $E\{M_1^2\} < +\infty$ yielding $E\{1/Q^2\} < +\infty$.

Applying the Maximum Entropy Principle, the probability density function is given by:

$$p_Q(q) = \mathbf{1}_{]0,+\infty[}(q)\frac{1}{\underline{Q}}\left(\frac{1}{\delta_Q^2}\right)^{\frac{1}{\delta_Q^2}} \frac{1}{\Gamma\left(1/\delta_Q^2\right)}\left(\frac{q}{\underline{Q}}\right)^{\frac{1}{\delta_Q^2}-1} exp\left(-\frac{q}{\delta_Q^2\underline{Q}}\right) \ , \quad (13)$$

where the positive parameter $\delta_Q = \sigma_Q/\underline{Q}$ is the dispersion coefficient, satisfying $\delta_Q < 1/\sqrt{2}$, and σ_Q is the standard deviation of Q.

3.3.2 Subglottal pressure

The subglottal pressure is modeled by the random variable Y. The usable information are: (i) Its support is $]0, +\infty[$, (ii) its mean value is $E\{Y\} = \underline{Y}$, (iii) The second-order moment of its inverse is finite $E\{1/Y^2\} < +\infty$. Information (3) is used because 0, and values near to it, should be repulsive values for Y, since there is a minimum pressure for causing phonation. The probability density function, applying the Maximum Entropy Principle, will be given by:

$$p_Y(y) = \mathbf{1}_{]0,+\infty[}(y)\frac{1}{\underline{Y}}\left(\frac{1}{\delta_Y^2}\right)^{\frac{1}{\delta_Y^2}} \frac{1}{\Gamma\left(1/\delta_Y^2\right)}\left(\frac{y}{\underline{Y}}\right)^{\frac{1}{\delta_Y^2}-1} exp\left(-\frac{y}{\delta_Y^2\underline{Y}}\right) \ , \quad (14)$$

where the positive parameter $\delta_Y = \sigma_Y/\underline{Y}$ is the dispersion coefficient, satisfying $\delta_Y < 1/\sqrt{2}$, and σ_Y is the standard deviation of Y.

3.3.3 Neutral glottal area

We will construct two models for the neutral glottal area. Both models are reasonable and the decision which one is best to describe the voice production process must be taken from experimental results.

***model I*: There is no information about the dispersion of A_{g0} :** The neutral glottal area is modeled by the random variable A_{g0}. The usable information are: (i) Its support is $]0,+\infty[$ and (ii) its mean value is $E\{A_{g0}\} = \underline{A}_{g0}$. Applying the Maximum Entropy Principle, the probability density function yields:

$$p_{A_{g0}} = \mathbf{1}_{]0,+\infty[}a_{g0}\frac{1}{\underline{A}_{g0}}e^{-a_{g0}/\underline{A}_{g0}}\,. \tag{15}$$

In the *model I*, the information used is such that the random variable is not of second-order. However, it is reasonable to think that A_{g0} is a second-order random variable. It is, then, interesting to see the consequences of the imposition that A_{g0} is a second-order random variable. This information will be added and a new probability density function will be constructed. It will be calle *model II*.

***model II*: Adding a new usable information to A_{g0}: it is a second-order random variable**

Now, the usable information for constructing the probability density function of A_{g0} are: (i) Its support is $]0,+\infty[$, (ii) its mean value is $E\{A_{g0}\} = \underline{A}_{g0}$ and (iii) it is a second-order random variable; it means, $E\{A_{g0}^2\} < +\infty$.

In this case, it will not be obtained a known expression of the probability density function as before (exponential and gamma). The probability density function of A_{g0} will be given by Eq.(11) ($m = 2$), rewritten by Eq.(16)

$$p_{A_{g0}}(a_{g0}) = \mathbf{1}_{]0,+\infty[}e^{-\lambda_0-\lambda_1 x-\lambda_2 x^2} \tag{16}$$

where λ_0, λ_1 and λ_2 are the values that minimize the function Δ, given by Eq.(17):

$$\Delta = \lambda_0 + \lambda_1\underline{A}_{g0} + \lambda_2 c \tag{17}$$

with $E\{X^2\} = c\,,\; c < +\infty$.

It will be created a coefficient of dispersion $\delta_{A_{g0}} = \frac{\sigma_{A_{g0}}}{\underline{A}_{g0}}$, where $\sigma_{A_{g0}}$ is the standard deviation of A_{g0}. It can be proved that $c = \underline{A}_{g0}{}^2\left(1+\delta_{A_{g0}}^2\right)$.

Figure 4 shows the probability density functions obtained for the two models.

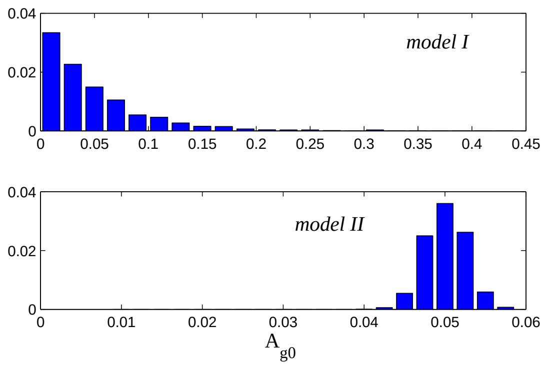

Figure 4. Probability density functions constructed for *model I* (top) and for *model II* (bottom).

4. SIMULATION

The corresponding stochastic solver is based on a Monte Carlo numerical simulation. Realizations of the random variables Q, Y and A_{g0} are constructed according to the their probability density functions. For each independent realization $A_{g0}(\theta)$, $Y(\theta)$ and $Q(\theta)$, a realization of the random fundamental frequency $F_0(\theta)$ is calculated in the same way as described for the mean model.

The convergence analysis with respect to n is carried out studying the convergence of the estimated second-order moment of F_0 defined by

$$\text{Conv}(n) = \frac{1}{n}\sum_{j=1}^{n} F_0(\theta_j)^2. \tag{18}$$

This convergence analysis is performed for different values of δ_Y, δ_Q and $\delta_{A_{g0}}$ and for $n \geq 500$, the convergence is always reached. Then, $n = 500$ was taken for all further estimations.

The confidence region associated with a probability level P_c is constructed using quantiles (Serfling, 1980). Let $F_{F_0}(f_0) = P\{F_0 \leq f_0\}$ be the cumulative distribution function of random variable F_0. For $0 < p < 1$, the p^{th} quantile of F_{F_0} is defined as $\zeta(p) = \text{Inf}\{f : F_{F_0}(f) \geq p\}$. Then, the upper envelope, f^+, and the lower envelope, f^-, of the confidence interval are defined by $f^+ = \zeta((1 + P_c)/2)$ and $f^- = \zeta((1 - P_c)/2)$. Let $f_1 = F_0(\theta_1), \ldots, f_n = F_0(\theta_n)$ be n independent realizations of random variable F_0. Let $\widetilde{f}_1 < \ldots < \tilde{f}_n$ be the order statistics associated with $f_1, \ldots, f_n$. Therefore, we have the following estimation: $f^+ = \widetilde{f}_{j^+}$ with $j^+ = \text{fix}(n(1 + P_c)/2)$ and $f^- = \widetilde{f}_{j^-}$ with $j^- = \text{fix}(n(1 - P_c)/2)$ in which fix(z) is the integer part of the real number z.

The values used for simulations to reproduce the signals of voice produced by men and women are the same ones discussed by Lucero (2005) and reproduced below.

For male configuration: $\widehat{m}_1 = 0.125\,\text{g}$, $\widehat{m}_2 = 0.125\,\text{g}$, $\widehat{k}_c = 25\,\text{N/m}$, $\widehat{k}_1 = 80\,\text{N/m}$, $\widehat{k}_2 = 8\,\text{N/m}$, $\xi_1 = 0.1$, $\xi_2 = 0.6$, $\ell_g = 1.4\text{cm}$, $d_1 = 0.25\,\text{cm}$, $d_2 = 0.05\,\text{cm}$ and for the vocal tract model (Ishizaka and Flanagan, 1972) (Goldstein, 1980): $S_1 = 1\,\text{cm2}$, $S_2 = 7\,\text{cm2}$, $L_1 = 8.9\,\text{cm}$, $L_2 = 8.1\,\text{cm}$.

For female configuration: $\widehat{m}_1 = 0.0456\,\text{g}$, $\widehat{m}_2 = 0.0091\,\text{g}$, $\widehat{k}_c = 17.85\,\text{N/m}$, $\widehat{k}_1 = 57.14\,\text{N/m}$, $\widehat{k}_2 = 5.71\,\text{N/m}$, $\xi_1 = 0.1$, $\xi_2 = 0.6$, $\ell_g = 1\text{cm}$, $d_1 = 0.179\,\text{cm}$, $d_2 = 0.036\,\text{cm}$ and for the vocal tract model (Ishizaka and Flanagan, 1972) (Goldstein, 1980): $S_1 = 0.688\,\text{cm2}$, $S_2 = 4.816\,\text{cm2}$, $L_1 = 6.3\,\text{cm}$, $L_2 = 7.8\,\text{cm}$.

The mean values considered for the random variables are: $\underline{Y} = 800\,\text{Pa}$, $\underline{Q} = 1$ and $\underline{A}_{g0} = 0.05\,\text{cm}^2$.

The estimation of the probability density function p_{F_0} of random variable F_0 is constructed as follows. Let M be the number of intervals. Let $I_j = [\nu_j, \nu_j + \Delta\nu[$ for $j = 1, \ldots, M$ with $\nu_1 = \widetilde{f}_1$ and $\Delta\nu = (\widetilde{f}_n - \widetilde{f}_1)/M$. An estimation $\widehat{p}_{F_0}$ of the probability density function of F_0 is given by

$$\widehat{p}_{F_0}(f_0) = \sum_{j=1}^{M} \mathbf{1}_{I_j}(f_0)\frac{N_j}{n\Delta\nu}\,. \tag{19}$$

Figure 5 shows the probability density function taking into account 500 realizations of the random variables Q, Y and *model I* for A_{g0}. It was used the value 0.05 for the coefficient of dispersion δ_Q and also for δ_Y. When sounds are not produced, the fundamental frequency is set to 0, and these values are not shown in the plots.

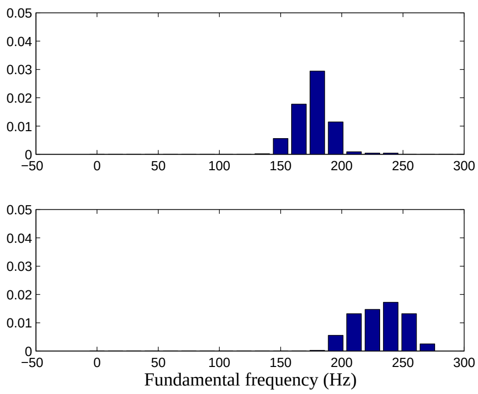

Figure 5. Probability density functions of the fundamental frequency for men (top) and women (bottom).

It can be noted that the shape of the probability density functions are completely different, although the coefficients of the dispersion of the parameters have the same value.

In addition to this, confidence intervals of the fundamental frequency were constructed and showed in Fig. 6 for different levels of dispersion of Q. Only this coefficient of dispersion of Q was varied from $\delta_Q = 0.01$ up to $\delta_Q = 0.4$.

The coefficient of dispersion of Y was considered fixed at 0.01. As the model used for A_{g0} was *model I*, there is no coefficient of dispersion associated to it.

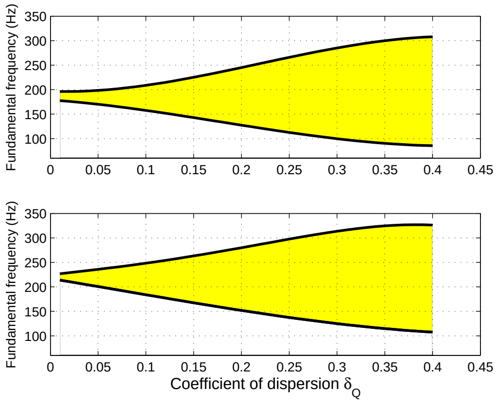

Figure 6. Confidence interval for men (top) and women (bottom) considering subglottal pressure deterministic and δ_Q varying.

It is important to say that the values used for constructing the two plots are discrete. However, to construct the confidence interval, the values were fitted (polynomial fitness).

It can be noted that the confidence intervals have the same shape (for men and women), although the probability density functions have presented different shapes.

The results obtained for the probability density functions did not seem good when *model I* is used. The *model II* for A_{g0} is then applied and the probability density function of A_{g0} constructed.

Figure 7 shows the density probability functions taking into account 500 realizations of the random variables Q, Y and A_{g0}. The same values for the coefficients of dispersion were taken into account: $\delta_{A_{g0}} = \delta_Q = \delta_Y = 0.05$.

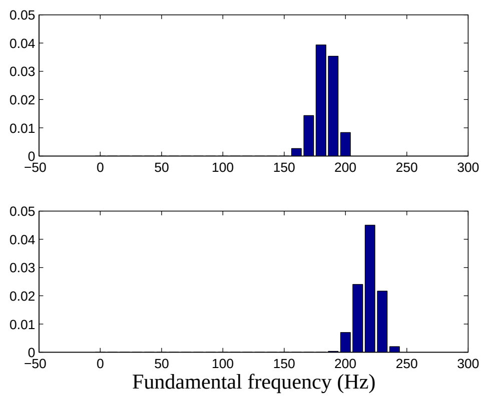

Figure 7. Probability density functions of the fundamental frequency with *model II* for men (top) and women (bottom)
.

Now, with the additional requirement that A_{g0} is of second order, it can be seen that the shapes of the probability density functions are similar. One sees that the results are very sensitive to the information used to derive the probability distribution. Hence, one must be careful not to add information that are not really sure. In a future work, that is being prepared, it will be shown how to update the results obtained from the available information and the Maximum Entropy Principle using information from experiments and techniques from Bayesian Statistics.

5. CONCLUSIONS

A parametric probabilistic approach is proposed to take into account uncertainties present on a biomechanical model for the production of voiced sounds. This approach allows a comparison between the systems of voice production of men and women. The novelty of this work is mainly to consider the voice production as a stochastic process and to use a strategy to construct probability density functions to the random variables associated to the uncertain parameters. The strategy used is based on the Maximum Entropy Principle, which is especially powerful because experimental data sets available are not sufficiently large. The results obtained with the first information about A_{g0} (*model I*) did not seem satisfactory, and new information was added to fit a better model. This study shows a way to discuss uncertaintin models of voice production and mainly to show that the behavior of the same model, when men and women are considered, should be analyzed in different ways.

6. APPENDIX

$$\psi_1(\mathbf{w}) = (\frac{\rho d_1}{a_{g_0}+2\ell_g x_1} + \frac{\rho d_2}{a_{g_0}+2\ell_g x_2} + \tilde{\ell}_1)$$

$$\psi_2(\mathbf{w}) = (\frac{0.19\rho}{a_{g_0}+2\ell_g x_1} + 2\ell_g x_1) + \frac{\rho}{(a_{g_0}+2\ell_g x_2)^2}\left[0.5 - \frac{a_{g_0}+2\ell_g x_2}{a_1}\left(1 - \frac{a_{g_0}+2\ell_g x_2}{a_1}\right)\right]$$

$$\psi_3(\mathbf{w}) = (12\mu\ell_g \frac{d_1}{(a_{g_0}+2\ell_g x_1)^3} + 12\ell_g^2 \frac{d_2}{(a_{g_0}+2\ell_g x_2)^3} + r_1)$$

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\ell}_1+\tilde{\ell}_2 & 0 & 0 \\ 0 & 0 & 0 & \tilde{\ell}_2+\tilde{\ell}_r & -\tilde{\ell}_r \\ 0 & 0 & 0 & -\tilde{\ell}_r & \tilde{\ell}_r \end{bmatrix}, \quad [C] = \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0 \\ 0 & 0 & r_1+r_2 & 0 & 0 \\ 0 & 0 & 0 & r_2 & 0 \\ 0 & 0 & 0 & 0 & r_r \end{bmatrix},$$

$$[K] = \begin{bmatrix} k_1+k_c & -k_c & 0 & 0 & 0 \\ -k_c & k_2+k_c & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\tilde{c}_1}+\frac{1}{\tilde{c}_2} & -\frac{1}{\tilde{c}_2} & 0 \\ 0 & 0 & -\frac{1}{\tilde{c}_2} & \frac{1}{\tilde{c}_2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{h}(\mathbf{w},\dot{\mathbf{w}},u_g,\dot{u}_g) = \begin{bmatrix} s_1(x_1)+t_1(x_1)\dot{x}_1 - f_1(x_1,u_g,\dot{u}_g) \\ s_2(x_2)+t_2(x_2)\dot{x}_2 - f_2(x_1,x_2,u_g,\dot{u}_g) \\ -\frac{1}{\tilde{c}_1}u_g \\ 0 \\ 0 \end{bmatrix},$$

where
$\tilde{\ell}_n = \frac{\rho\ell_n}{2\pi y_n^2}$, $\tilde{\ell}_r = \frac{8\rho}{3\pi^2 y_n}$, $r_n = \frac{2}{y_n}\sqrt{\rho\mu\frac{\omega}{2}}$, $\omega = \sqrt{\frac{k_1}{m_1}}$, $a_n = \pi y_n^2$, $\tilde{c}_n = \frac{\ell_n \pi y_n^2}{\rho v_c^2}$, ℓ_n is the length of the n th tube, y_n is the radius of the n th tube, and μ is the shear viscosity coefficient.

$$s_\alpha(w_\alpha) = \begin{cases} k_\alpha \eta_{k_\alpha} x_\alpha^3 \ , \ x_\alpha > -\frac{a_{g_0}}{2\ell_g} \\ k_\alpha \eta_{k_\alpha} x_\alpha^3 + 3k_\alpha \left\{\left(w_\alpha + \frac{a_{g_0}}{2\ell_g}\right) + \eta_{h_\alpha}\left(w_\alpha + \frac{a_{g_0}}{2\ell_g}\right)^3\right\} \ , \ x_\alpha \le -\frac{a_{g_0}}{2\ell_g} \end{cases}, \alpha = 1, 2.$$

$$t_\alpha(x_\alpha) = \begin{cases} 0 \, , \, x_\alpha > -\frac{a_{g_0}}{2\ell_g} \\ 2\xi\sqrt{m_1 k_1} \, , \, x_\alpha \le -\frac{a_{g_0}}{2\ell_g} \end{cases}, \alpha = 1, 2.$$

$$f_1(x_1,u_g,\dot{u}_g) = \begin{cases} \ell_g d_1 p_{m_1}(x_1,u_g,\dot{u}_g) \, , \, x_1 > -\frac{a_{g_0}}{2\ell_g} \\ 0 \, , \, otherwise \end{cases}$$

$$f_2(x_1,x_2,u_g,\dot{u}_g) = \begin{cases} \ell_g d_2 p_{m_2}(w_1,w_2,u_g,\dot{u}_g) \, , \, x_1 > -\frac{a_{g_0}}{2\ell_g} \text{ and } x_2 > -\frac{a_{g_0}}{2\ell_g} \\ \ell_g d_2 p_s \, , \, x_1 > -\frac{a_{g_0}}{2\ell_g} \text{ and } x_2 \le -\frac{a_{g_0}}{2\ell_g} \\ 0 \ , \ otherwise \end{cases}$$

$$p_{m_1}(x_1,u_g,\dot{u}_g) = p_s - 1.37\frac{\varrho}{2}\left(\frac{u_g}{a_{g_0}+2\ell_g x_1}\right)^2 - \frac{1}{2}\left(12\mu\ell_g \frac{d_1}{(a_{g_0}+2\ell_g x_1)^3} + \frac{\rho d_1}{a_{g_0}+2\ell_g x_1}\right)\dot{u}_g$$

$$p_{m_2}(x_1,x_2,u_g,\dot{u}_g) = p_{m_1} - *$$

$$* = \frac{1}{2}\left\{(12\mu\ell_g \frac{d_1}{(a_{g_0}+2\ell_g x_1)^3} + 12\ell_g^2 \frac{d_2}{(a_{g_0}+2\ell_g x_2)^3})u_g + (\frac{\rho d_1}{a_{g_0}+2\ell_g x_1} + \frac{\rho d_2}{a_{g_0}+2\ell_g x_2})\dot{u}_g\right\} - \frac{\varrho}{2}u_g^2\left(\frac{1}{(a_{g_0}+2\ell_g x_2)^2} - \frac{1}{(a_{g_0}+2\ell_g x_1)^2}\right)$$

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