ON REGULAR AND IRRREGULAR DYNAMICS OF A NONLINEAR OSCILLATOR WITH SHAPE MEMORY THROUGH A PERTURBATION METHOD

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Abstract. *Shape memory alloys (SMA) are materials that even when submitted to some type of deformation, possess the ability to recuperate its original form through adequate thermal procedures. The main phenomena associated to these alloys are the effect of pseudoelastic and shape memory. Such phenomena happen due to martensite phase transformations that occur in these alloys. In this paper, we consider a single – degree of freedom oscillator, which consists of a mass m connected to a rigid support by a SMA element of length L and cross – sectional area A. A linear viscous damper, associated with a parameter, is also considered. The system is excited harmonically then this problem is well – suitable for the aplication of the method of multiple scales. The constitutive model adopted in the modeling of the system captures the general thermomechanical behavior of SMA´s, allowing the description of various aspects of the dynamical system. Results of numerical simulations in set, with method of the multiples scales, show that this system has a rich behavior with different kinds of response.*

Keywords: *Nonlinear dynamic, shape memory alloys, perturbations techniques, frequency – response.*

1. INTRODUCTION

Shape memory alloy (SMAs) refers to a group of materials, which has the ability to return to a predetermined shape when it is heated. The source of the distinctive mechanical behavior of these materials is a crystalline phase transformation between a high symmetry, highly ordered parent phase (austenite), and a low symmetry, less ordered product phase (martensite).

The SMAs present high reversible strains compared to conventional materials (pseudoelasticity) and permanent deformations that disappear upon an increase in temperature (shape memory).

The observable macroscopic mechanical behavior of SMAs, can be separated into two categories: the shape memory effect and pseudoelastic effect. In the shape memory effect, an SMA material exhibits a large residual strain after the loading and unloading. This strain can be fully recovered upon heating the material. In the pseudoelastic effect, the SMA material achieves a very large stain upon loading that is fully recovered in a hysteresis loop upon unloading

The remarkable properties of SMAs are attracting much technological interest, motivating different applications in several fields of sciences and engineering (Paiva *et al.*2005). They are ideally suited for use as fasteners, seals, connectors and clamps (van Humbeeck, 1999). Self-actuating fasteners, thermally actuator switches are some examples of these applications (Duerig et al., 1999), (Lagoudas et al., 1999). The use of SMAs can help solving many important problems in aerospace technology, in particular those concerning with space savings achieved by selferectable structures and non-explosive release devices (Denoyer et al., 2000), (Pacheco and Savi, 1997). Micromanipulators and robotics actuators have been built employing SMA properties to mimic the smooth motions of human muscles (Garner et al., 2001); (Webb et al., 2000) and (Rogers, 1995). Moreover, SMAs are being used as actuators for vibration and buckling control of flexible structures. In this particular field, SMA wires embedded in composite materials have been used to modify mechanical characteristics of slender structures (Birman, 1997), (Rogers, 1995). The main drawback of SMAs is their slow rate of change (Paiva *et al.*2005).

A number of constitutive models have been developed to describe the thermomechanical behavior of shape memory alloys. The polynomial mathematical model proposed by (Falk, 1980) is based on Devonshire's theory and considers a polynomial-free energy to describe the shape memory and pseudoelasticity effect, (Tanaka, 1986) developed a model based on thermomechanics with an exponential model to describe the martensite volume fraction at different temperatures. (Liang and Rogers, 1990) utilized the constitutive model developed by Tanaka; however a cosine model was used to describe the martensite volume fraction. (Brinson, 1993) modified the models developed by Tanaka, Liang and Rogers and divided the martensite volume fraction into stress-induced and temperature-induced martensite variants. Recent work (Savi and Pacheco, 2002) had studied some characteristic of shape memory oscillators with one and two – degree of freedom, showing the existence of chaos and hyperchaos through of numerical simulations in these systems.

In this paper, we present an analytical solution for the problem of a single degree of freedom pseudoelastic oscillator, where the dynamical response of the system is available. Moreover, we analyze the governing equations of motion using the method of multiple scales to determine the frequency – response of the system to a primary resonance. Results show the effect of nonlinearity, damping and amplitude of the excitation on the response of the system. We note the occurrence of the jump phenomenon to a primary resonance.

2. SMA CONSTITUTIVE MODELING

To describe the behavior of the oscillator with shape memory, we adopt in the modeling of the problem the constitutive model proposed by (Falk 1980).This model is based on Devonshire theory and defines a free energy of Helmholtz (ψ) in the polynomial form and is capable to describe the shape memory and pseudoelasticity effects. The polynomial model as more it is known to deals with one – dimensional cases and it does not consider an explicit potential of dissipation, and no internal variable is considered. On this form, the free energy depends only on the observable state variables (temperature and strain), that is, $\psi = \psi(\varepsilon, T)$

The free energy is defined in such way that, for high temperatures ($T > T_A$), the energy possesses only one point of minimum corresponding to the null strain representing the stability of the austenite phase (A); for intermediate temperatures ($T_M < T < T_A$) it presents three points of minimum corresponding to the phases austenitic (A), and two other martensitic phase (M^+ and M^-), which are induced by positive and negative stress fields, respectively; to low temperature ($T < T_M$) there are two points of minimum representation the two variants of martensite (M^+ e M^-), corresponding the null strain.

Therefore, the restrictions below are given by the following equation polynomial;

$$\rho\psi(\varepsilon, T) = \frac{1}{2}q(T - T_M)\varepsilon^2 - \frac{1}{4}b\varepsilon^4 + \frac{1}{6}e\varepsilon^6 \tag{1}$$

where q and b are constants of the material, T_A correspond to the temperature where the austenite phase is stable, T_M correspond to the temperature where the martensitic phase is stable and ρ is the mass density, and the free – energy has only one minimum at zero strain,

$$T_A = T_M + \frac{b^2}{4qe} \tag{2}$$

and the constant e may be expressed in terms of other constants of the material. By definition (Savi and Braga, 1993), the stress – strains relation is given by,

$$\sigma = q(T - T_M)\varepsilon - b\varepsilon^3 + \frac{b^2}{4a(T_A - T_M)}\varepsilon^5 \tag{3}$$

3. THE ANALYZED PROBLEM

We consider the one degree of freedom oscillator which consists of a mass m, connected to a rigid support through of a viscous damping with coefficient c and a shape memory element, where a periodic external force $p(t) = p\cos(\omega t)$, it is applied to the systems, as it shows figure 1.

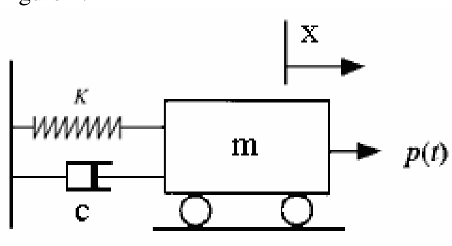


Figure 1

Thus, the equation of motion that governs the vibrating system can be written as,

$$m\frac{d^2x}{dt}+c\frac{dx}{dt}+K(x,T)=p\cos(\omega t) \tag{4}$$

On the other hand, as we saw in the previous section the behavior of the element with shape memory can be described through the constitutive model polynomial. Therefore, the restoring force of the spring is given by,

$$K=K(x,T)=\bar{q}(T-T_M)x-\bar{b}x^3+\bar{e}x^5 \tag{5}$$

where,

$$\bar{q}=\frac{qA}{L};\quad \bar{b}=\frac{bA}{L^3};\quad \bar{e}=\frac{eA}{L^5} \tag{6}$$

it follows that u represents the variable relative to the displacement of the element with shape memory, L is the length and A it is the area of this element.

Then, the governing equation of motion of the oscillator is:

$$m\frac{d^2x}{dt}+c\frac{dx}{dt}+\bar{q}(T-T_M)x-\bar{b}x^3+\bar{e}x^5=p\cos(\omega t) \tag{7}$$

Next, we introducing the following dimensionless variables

$$y=\frac{x}{L}\quad and \quad \tau=\omega_0 t \tag{8}$$

then, Eq. (7) is given by

$$\ddot{y}+2\mu\,\dot{y}+(\theta-1)\,y-\alpha\,y^3+\gamma\,y^5=\delta\cos(\phi\tau) \tag{9}$$

where the dot represents time differentiation and the dimensionless variables are given by,

$$\gamma=\frac{eA}{mL\omega_0^2}\qquad \alpha=\frac{bA}{mL\omega_0^2}\qquad \mu=\frac{c}{2m\omega_0}\qquad \phi=\frac{\omega}{\omega_0}$$

$$\delta=\frac{p}{mL\omega_0^2}\qquad \theta=\frac{T}{T_M}\qquad \omega_0^2=\frac{qAT_M}{mL} \tag{10}$$

4. ANALYSIS OF THE EQUATIONS OF THE MOTION TROUGH OF THE METHOD OF MULTIPLES SCALES

The underlying idea of the method of multiples scales is to consider the expansion representing the response to be a function of multiple independent variables, or scales, instead of a single variable. The method of multiple scales, though a little more involved, has advantages over another method, for example, it can treat damped systems conveniently (Nayfeh and Mook, 1979).

As the shape memory alloys presents different properties depending on the temperature, in this article we presents the study on the pseudoelastic behavior, considering a higher temperature, where austenitic phase is stable in the alloy ($\theta=2$). Therefore the Eq. (9) becomes

$$\ddot{y}+2\mu\,\dot{y}+y-\alpha\,y^3+\gamma\,y^5=\delta\cos(\phi\tau) \tag{11}$$

Now that we find to the equation of motion in dimensionless form Eq. (11), we use the method of multiples scales (Nayfeh and Mook, 1979) to determine a fourth- order approximate solution for the cases of free and forced vibration.

To analyze this case we need to order the damping, the nonlinearity, and the excitation so that they appear at the same time in the perturbations scheme. Therefore if we let $y=\varepsilon u$, we need to order $\mu\dot{y}$ as $\varepsilon^4\mu\dot{y}$ and δ as $\varepsilon^4\delta$ so that the governing equation becomes

$$\ddot{u} + 2\varepsilon^4 \mu \dot{u} + u - \alpha \varepsilon^2 u^3 + \gamma \varepsilon^4 u^5 = \varepsilon^4 \delta \cos(\phi\tau) \quad (12)$$

In all simulations, it is assumed the following parameters for the shape memory alloy presented in table 1, (Savi and Machado, 2001) and assuming also that the SMA element has the following values: $A = 1.96 \times 10^{-5}\, m^2$, $L = 50 \times 10^{-3}\, m$ and mass unitary.

Table 1: Shape Memory alloy parameters

q (MPa/K)	b (MPa)	e (MPa)	T_M (K)	T_A (K)
260	10^8	3.69×10^{11}	287	314

The numerical simulation results presented were obtained using the Matlab-Simulink™ from Mathworks®.

4.1 System Behavior of the Pseudoelastic Oscillator for Free Vibrations

In the case where the amplitude of the excitation δ is null, we have that the equation of motion is given by

$$\ddot{u} + 2\varepsilon^4 \mu \dot{u} + u - \alpha \varepsilon^2 u^3 + \gamma \varepsilon^4 u^5 = 0 \quad (13)$$

To determine a fourth – order expansion, we need of five scales, $T_0 = t, T_1 = \varepsilon t, T_2 = \varepsilon^2 t, T_3 = \varepsilon^3 t, T_4 = \varepsilon^4 t$ .We seek a solution for Eq. (13) in powers of ε in the form

$$u = u_0(T_0, T_1, T_2, T_3, T_4) + \varepsilon u_1(T_0, T_1, T_2, T_3, T_4) + \varepsilon^2 u_2(T_0, T_1, T_2, T_3, T_4) + \varepsilon^3 u_3(T_0, T_1, T_2, T_3, T_4) + O(\varepsilon^4) \quad (14)$$

By using o method of multiples scales, one obtains the following fourth – order approximation for the solution of Eq. (13):

$$u = a\cos(\tau + \beta) - \varepsilon^2 \frac{\alpha}{32} a^3 \cos(3\tau + 3\beta) + O(\varepsilon^4) \quad (15)$$

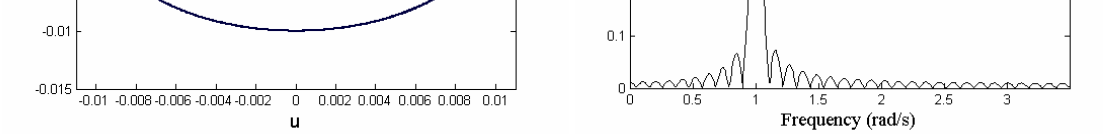
where the amplitude a and phase β are governed by

$$\begin{cases} a' = -\mu a \\ a\beta' = \dfrac{10}{32} a^5 \gamma - \dfrac{15}{256} a^5 \alpha^2 \end{cases} \quad (16)$$

and the prime indicates the derivative with respect to the time scales $\tau = \varepsilon^4 t$.

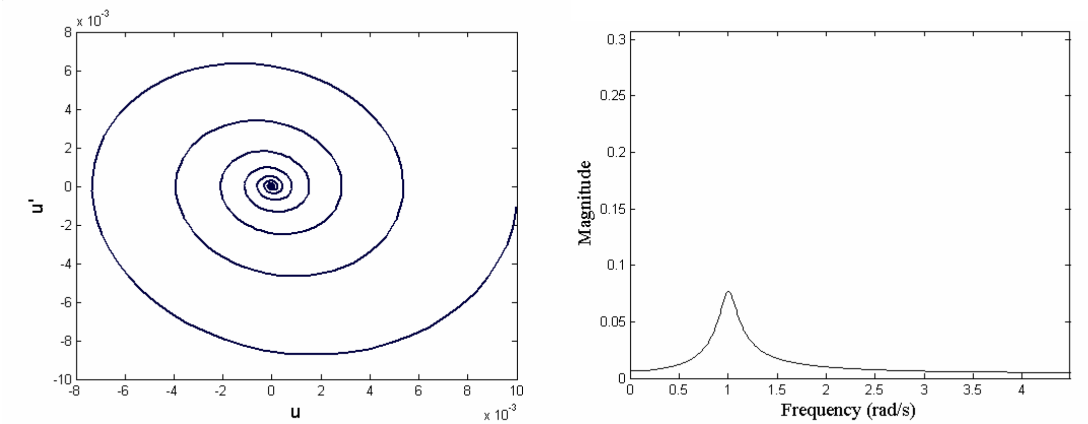
The system above presents only one fixed point. The origin is the only fixed point and there are two possibilities: is a center or a stable spiral, depending if the system to present damping or not.

Below we show to the phase portraits and its respective FFT for both the cases;



(a) (b)

Figure 2: (a) Phase Portrait and (b) FFT spectrum for the case where μ = 0.



(a) (b)

Figure 3: (a) Phase Portrait and (b) FFT spectrum for the case where μ = 0.1.

Figure 4 shows the variation of a and β with T_4 as calculated by numerically integrating system of Eq. (16) for the value of $\mu = 0.1$. We should note that initially a decrease while β increase with T_4, but as T_4 increases a and β tend a constant values. These constant values are usually referred to as stationary values.

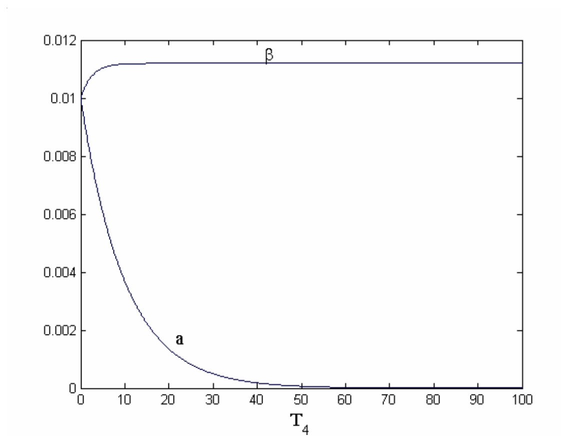


Figure 4: Variation of a and β with T_4 as calculated by numerically integrating system of Eq. (15) for the value of $\mu = 0.1$.

4.2 System Behavior of the Pseudoelastic Oscillator with External Excitation

Now we discuss the dynamic of the pseudoelastic oscillator when the system is harmonically excited. To determine an approximate solution to Eq. (12) free of secular and small – divisor terms, we need to distinguish between secondary and primary resonances.

For the primary resonance to occur, $\phi \approx 1$ instead of using the frequency of the excitation ϕ as a parameter, we introduce a detuning parameter σ, which quantitatively describes the nearness of ϕ to 1. This has the advantage of helping one to recognize the terms in the equations for u that lead to secular, and nearly secular (small divisor) terms. (Nayfeh and Mook, 1979). Accordingly we write

$$\phi = 1 + \varepsilon^4 \sigma \tag{17}$$

where $\sigma = O(1)$. We use o method of multiples scales and seek a fourth – order uniform expansion of Eq. (12) in the form of the Eq. (14). Substituting the Eq. (14) into Eq. (12) and equating coefficients of like powers of ε, we obtain the following fourth – order approximation for the solution of Eq. (12):

$$u = a\cos(\phi\tau - \psi) - \varepsilon^2 \frac{\alpha}{32} a^3 \cos(3\phi\tau - 3\psi) + 0(\varepsilon^4) \tag{18}$$

where the amplitude a and phase ψ are governed by

$$\begin{cases} a' = -a\mu + \dfrac{\delta}{2}\operatorname{sen}(\psi) \\ a\psi' = a\sigma - \dfrac{10a^5\gamma}{32} + \dfrac{15a^5\alpha^2}{256} + \dfrac{\delta}{2}\cos(\psi) \end{cases} \tag{19}$$

where

$$\psi = \sigma T_4 - \beta \tag{20}$$

Steady – state motion occur when $a' = \psi' = 0$, which corresponds to the singular points of Eq. (19), that is, they correspond to the solution of

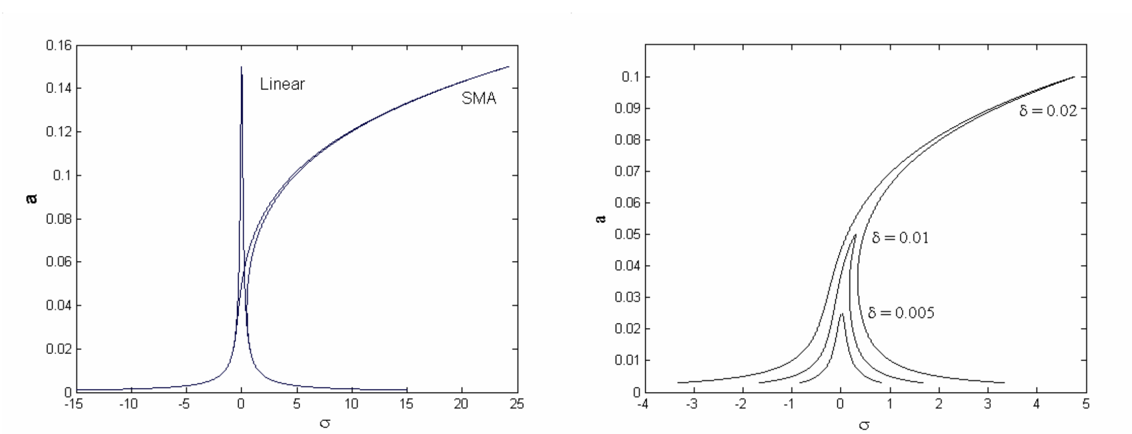
$$\begin{aligned} a\mu &= \frac{\delta}{2}\operatorname{sen}(\psi) \\ -a\sigma + \frac{10a^5\gamma}{32} - \frac{15a^5\alpha^2}{256} &= \frac{\delta}{2}\cos(\psi) \end{aligned} \tag{21}$$

Squaring and adding these equations, we obtain

$$\frac{\delta^2}{4} = a^2\mu^2 + a^2\left(\frac{10a^4\gamma}{32} - \sigma - \frac{15a^4\alpha^2}{256}\right) \tag{22}$$

The Eq. (22) is an implicit equation for the amplitude of the response a as a function of the detuning parameter σ and the amplitude of the excitation δ, it is called the frequency – response equation.

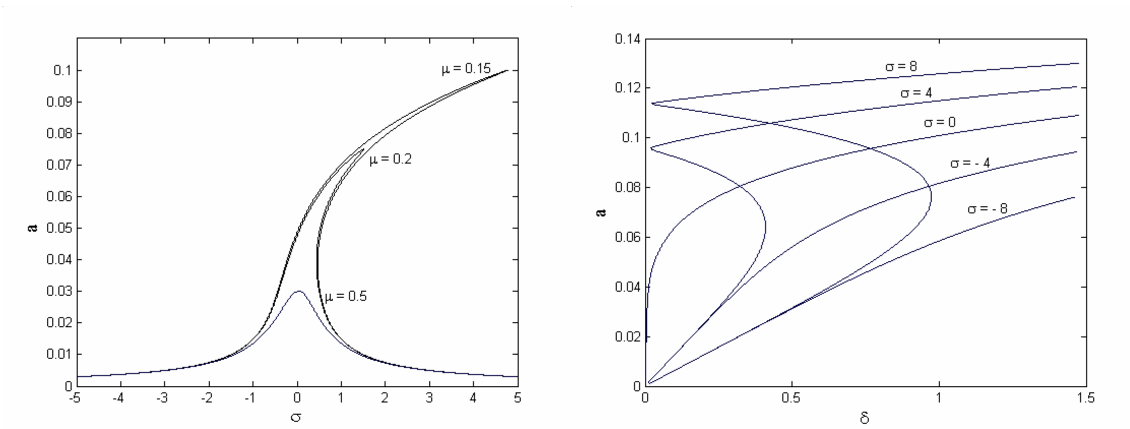
The plot of a as a function of σ for given μ and δ is called a frequency – response curve. Figure 5 (a) shows a comparison of the linear and nonlinear (SMA) response curves. The Eq. (22) indicates that the peak amplitude, which is given by $a_p = \dfrac{\delta}{2\mu}$, is independent of the values of α and γ. Figure 5 (b) shows variation of the frequency – response curves with the amplitude of the excitation.



(a) (b)

Figure 5: Frequency – response curves for primary resonance of the SMA equation: (a) effect of nonlinearity for $\mu = 0.1$, $\delta = 0.03$ and $\varepsilon = 0.001$; (b) effect of amplitude of excitations for $\mu = 0.1$ and $\varepsilon = 0.001$.

Figure 6 (a) show the influence of the damping coefficient μ on the response curves. In the absence of damping, the peak amplitude is infinite. Figure 6 (b) shows the variation of the amplitude of the response with the amplitude of the excitation for several values of σ .



(a) (b)

Figure 6: (a) Effect of damping on the response for $\delta = 0.03$ and $\varepsilon = 0.001$; (b) Amplitude of response as function of amplitude of the excitation for several detunings and for $\mu = 0.1$ and $\varepsilon = 0.001$.

The fixed point of the system of Eq. (19) is (a_0, ψ_0), where the stability is determined by the eigenvalues of the Jacobian matrix

$$J = \begin{bmatrix} -\mu & -a_0\left(\sigma - \frac{10a_0^4\gamma}{32} + \frac{15a_0^4\alpha^2}{256}\right) \\ \frac{1}{a_0}\left(\sigma - \frac{50a_0^4\gamma}{32} + \frac{75a_0^4\alpha^2}{256}\right) & -\mu \end{bmatrix} \tag{23}$$

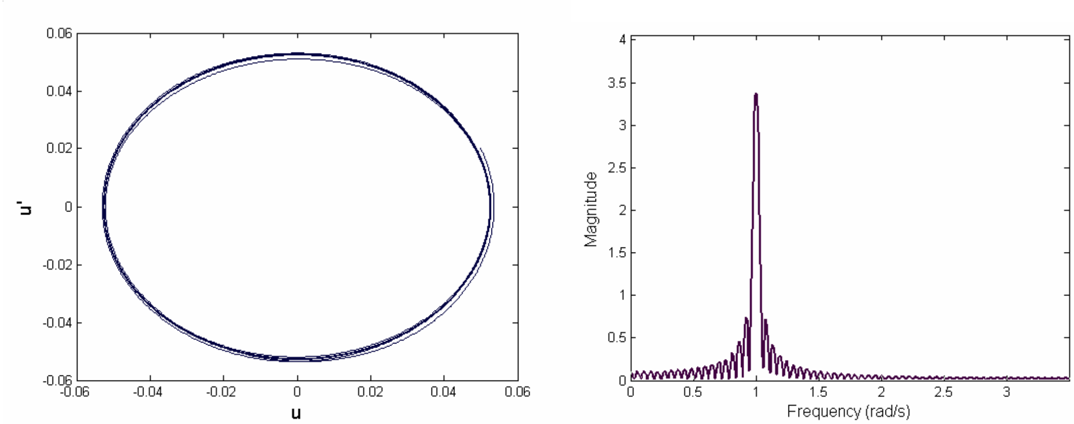
The corresponding eigenvalues λ_i are the roots of

$$\lambda^2 + 2\lambda\mu + \mu^2 + \left(\sigma - \frac{10a_0^4\gamma}{32} + \frac{15a_0^4\alpha^2}{256}\right)\left(\sigma - \frac{50a_0^4\gamma}{32} + \frac{75a_0^4\alpha^2}{256}\right) = 0 \tag{24}$$

From the Eq. (24), we find that the sum of the eigenvalues is -2μ . This sum is negative because $\mu > 0$. Consequently at least one of the two eigenvalues will always have a negative real part. This fact eliminates the possibility of a pair of purely imaginary eigenvalues and, hence, a Hopf Bifurcation. However, static bifurcation can occur. To this end, we find that one of the eigenvalues is zero when

$$\mu^2 + \left(\sigma - \frac{10a_0^4\gamma}{32} + \frac{15a_0^4\alpha^2}{256}\right)\left(\sigma - \frac{50a_0^4\gamma}{32} + \frac{75a_0^4\alpha^2}{256}\right) = 0 \tag{25}$$

Figure 7 (a) shows the phase portrait, the stability of an equilibrium solution is ascertained by investigating the eigenvalues of the Jacobian matrix. In this case the response is an orbit periodic and stable. In figure 7 (b) shows the associated FFT.



(a) (b)

Figure 7: (a) Phase portrait and (b) FFT for the parameters, $\mu = 0.1$, $\delta = 0.03, \varepsilon = 0.001$ and $\sigma = 0.1$.

4.3 Jump Phenomena

The multivaluedeness of the response curves due to the nonlinearity has significance from the physical point of view because it leads to jump phenomena (Nayfeh and Mook, 1979). For fixed $\mu > 0$, $\sigma > 0$ we show the variation de *a* with δ in Figure 8. In this bifurcation diagram, the solid and broken lines correspond to the stable and unstable fixed point of the Eq. (19). As the control parameter δ is gradually increased from a zero value, *a* follows the curve AFB until the critical value $\delta = \delta_2$ is reached. As δ is increased further, a jump from point B to point C takes place with an increase of *a*, after which *a* increases slowly with δ . At $\delta = \delta_2$, a saddle – node bifurcation occurs, and locally, there are no other solutions for $\delta > \delta_2$.

If the process is reverse, *a* decreases slowly as δ from point D to point E. As δ is decreased further, a jump from point E to point F takes place, with an accompanying decrease in *a*, after which *a* decreases slowly with decreasing δ . At the critical point $\delta = \delta_1$, a saddle – node bifurcation occurs, and, locally, there are no other solutions for $\delta < \delta_1$. We note that points B and E are points of vertical tangencies (Nayfeh and Balachandran, 1995). To this end, we find from the Eq. (22) that

$$\frac{d\delta^2}{da^2} = 4\left[\mu^2 + \left(\sigma - \frac{50a^4\gamma}{32} + \frac{75a^4\alpha^2}{256}\right)\left(\sigma - \frac{10a^4\gamma}{32} + \frac{15a^4\alpha^2}{256}\right)\right] \tag{26}$$

which is zero by virtue of the Eq. (25). Because saddle – node bifurcation points are locations of vertical tangencies, they are called tangent bifurcations.

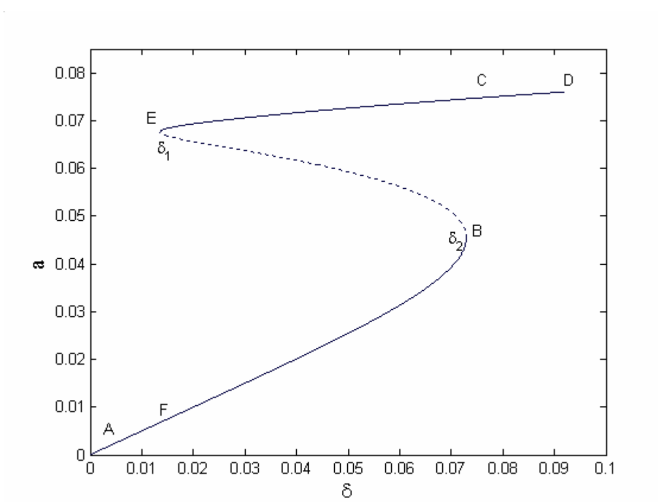


Figure 8: Bifurcation diagram constructed by using δ as a control parameter

Figure 9 show a representative response curve. The bending of the response is due to the nonlinearity and it responsible for a jump phenomenon. Next, we considerer the bifurcation diagrams when σ is used as a control parameter for $\delta > 0$ and $\mu > 0$. Again, the solid and broken lines correspond to the stable and unstable fixed point of the Eq. (19).

Already in this case when the value of σ is increases until the point D, it describes to the curve DCE, where in the point E happens a jump from point E to point B, with a decrease in *a*. At $\sigma = \sigma_2$, a saddle – node bifurcation occurs, and locally, there are no other solutions for $\sigma > \sigma_2$.

On the other hand if the value of σ now corresponding to point A, σ decreased and *a* increases through point B until point F, a jump from point F to point C takes place with an increase in *a*, after which *a* decreases with decreasing σ. At the critical point $\sigma = \sigma_1$, a saddle – node bifurcation occurs, and, locally, there are no other solutions for $\sigma < \sigma_1$.

To verify if these bifurcations points are points of vertical tangencies, we determine from Eq. (22) that

$$\frac{d\sigma}{da^2} = \frac{5a^2\gamma}{8} - \frac{15a^2\alpha^2}{128} + \frac{\mu^2 + \left(\frac{10a^4\gamma}{32} - \sigma - \frac{15a^4\alpha^2}{256}\right)^2}{\left(\frac{10a^4\gamma}{32} - \sigma - \frac{15a^4\alpha^2}{256}\right)} \left(\frac{1}{2a^2}\right) \tag{27}$$

Because the Eq. (25) is satisfied at the bifurcation points, we substitute for μ^2 from Eq. (25) into Eq. (27). After simplifying, we find that

$$\frac{d\sigma}{da^2} = 0 \tag{28}$$

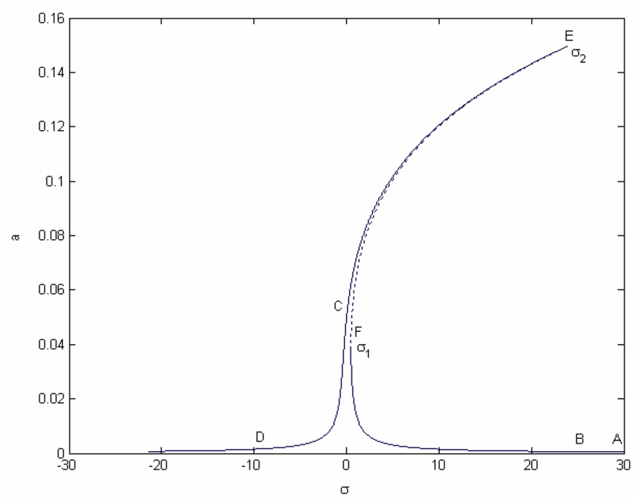


Figure 9: Bifurcation diagram constructed by using σ as a control parameter.

5. CONCLUSION

In this paper, we present a study of the dynamic behavior of a shape memory oscillator, where it was focused on its property of pseudoelasticity. The oscillator is studied using a constitutive model polynomial to describe the restitution force. We use the method of multiple scales to find the equation of motion of the oscillator analytically and thus we determine the frequency – response curves for the primary resonance.

The equilibrium solutions of the modulation equations in the case of primary resonance show that the frequency – response curve exhibits a hardening – spring behavior.

Results of the numerical simulations obtained from the analytical equations they show important dynamic characteristics of the system such as damping, nonlinearity and the amplitude of the excitations effects, presenting still a periodic behavior for these situations.

The jump phenomena it was obtained and analyzed.

6. ACKNOWLEDGEMENTS

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