A STUDY OF TRAJECTORIES PASSING NEAR URANUS

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Abstract. The Swing-By maneuver is a technique used in several missions to reduce fuel consumption. To identify one trajectory we use the following variables: 1) J, the Jacobian constant of the spacecraft; 2) The angle y, that is the angle between the line Sun-Uranus and the direction of the periapsis of the trajectory of the spacecraft around Uranus; 3) R_p, the distance from the spacecraft to the center of Uranus in the moment of the closest approach with Uranus (periapsis distance). The Jacobian constant is equivalent to the velocity at periapsis or the hyperbolic excess velocity V_∞, since they can be related by the conservation of energy of the two-body dynamics. For a large number of values of these three variables, the equations of motion are integrated numerically forward and backward in time, until the spacecraft is at a distance where the Uranus's effect can be neglected and the system formed by the Sun and the spacecraft can be considered a two-body system. At these two points, two-body celestial mechanics formulas are valid for the computation of the energy and the angular momentum before and after the close approach. Those quantities are used to identify up to sixteen classes of orbits, according to the changes in the energy and angular momentum caused by the close encounter. In this research, it is especially checked which ones of those orbits have a passage near the Earth in the outbound (starting at the Earth) and in the inbound (starting at Uranus) trajectories. This is very important, because those orbits have potential of practical application. The outbound trajectories are useful to send a spacecraft to Uranus and the inbound trajectories are important because an asteroid passing by Uranus may follow this trajectory to collide with the Earth. The results are shown in plots where one letter describing the effects of the Swing-by is assigned to the respective point in a two-dimensional graph that has in the horizontal axis the angle y (the approach angle) and in the vertical axis the Jacobian constant of the spacecraft. This plot is made for a fixed value of the parameter R_p. . After that, the velocity change required to start or to stop the spacecraft at the Earth and the flight path angle at the meeting point are calculated.

Keywords: swing-by, trajectory, orbits, maneuver.

1. INTRODUCTION

The Swing-By maneuver is a technique used in several missions to reduce fuel consumption like in Weinstein, 1992; Swenson, 1992; Farquhar and Dunham, 1981; Minovich, 1961; Dowling et. al, 1991; Flandro, 1996; Farquhar et. al. 1985; Dunham and Davis, 1985; Prado, 1996, 1997 and 1999; Prado and Broucke, 1995a and 1995b; Broucke and Prado, 1993.

Among the several sets of initial conditions that can be used to identify uniquely one trajectory, the same one used in the paper written by Broucke (1988) is used here.

The Swing-By maneuver was used in several missions, recently, it was used in the Mission New Horizons.

New Horizons is the first mission in NASA's New Frontiers program of medium-class planetary missions, and it will be the first spacecraft to visit Pluto and its moon, Charon. The Swing-By maneuver was used on Feb. 28th 2007, to increase the velocity of the spacecraft, when it reached Jupiter. Due to the Swing-By Maneuver, New Horizons will reach the Pluto System in July 2015, five years earlier than without this maneuver.

The fuel consumption of any mission is an important problem to be discussed, especially because it can make the mission financelery impracticable. The swing-by maneuver is very useful to solve this problem, and besides that, it can decrease the total time of the mission, as seen in the New Horizons Mission.

In this paper, it is specially studied the swing-by maneuver using the planet Uranus, a numerical simulation is made and the results are discussed.

2. DEFINITION OF THE PROBLEM

To solve the problem described above, it is assumed the existence of three bodies: the Sun, the planet Uranus and a third particle of negligible mass (the spacecraft). It is also assumed that the total system (Sun + Uranus + spacecraft) satisfies the hypothesis of the planar restricted circular three-body problem: all the bodies are point masses; the Sun and Uranus are in circular orbits around their mutual center of mass.

With these assumptions, the problem consists in studying the motion of the spacecraft near the close encounter with the planet Uranus. In particular, the energy and the angular momentum of the spacecraft before and after this close encounter are calculated, to detect the changes in the trajectory during the close approach. The orbits are classified in four categories: elliptic direct (negative energy and positive angular momentum), elliptic retrograde (negative energy and angular momentum), hyperbolic direct (positive energy and angular momentum) and hyperbolic retrograde (positive energy and negative angular momentum). The problem now is to identify the category of the orbit of the spacecraft before and after the close encounter with Uranus. Fig. I explains the geometry involved in the close encounter.

The spacecraft leaves the point A, crosses the horizontal axis (the line between the Sun and the planet Uranus), passes by the point P (the periapsis of the trajectory of the spacecraft around Uranus) and goes to the point B. Points A and B are chosen in a such way that the influence of Uranus at those points are neglected and, consequently, the energy is constant after B and before A. Two of the initial conditions are clearly identified in this figure: the periapsis distance R_p (distance measured between the point P and the center of Uranus) and the angle ψ measured from the horizontal axis in the counter-clock-wise direction. The distance R_p is not to scale, to make the figure easier to understand. The third initial condition is the Jacobian constant J of the spacecraft. The second part of the problem is to identify if one particular trajectory passes near the Earth in one or in both directions of time. For that purpose, the numerical integration is extended in each direction of time until one of the following events occur: i) The spacecraft reaches a position inside the Earth's orbit around the Sun. Then it is assumed that the spacecraft crosses the Earth's path in space and, with proper timing conditions, a close encounter with the Earth is possible; ii) The spacecraft goes to far from the Solar System without crossing the Earth's path. Then it is assumed that it does not come back again and a close encounter with the Earth is not possible; iii) The spacecraft remains close to the Solar System, but to much time has been passed without a crossing with the Earth's path. Then it is assumed that a useful close encounter with the Earth is not likely to occur.

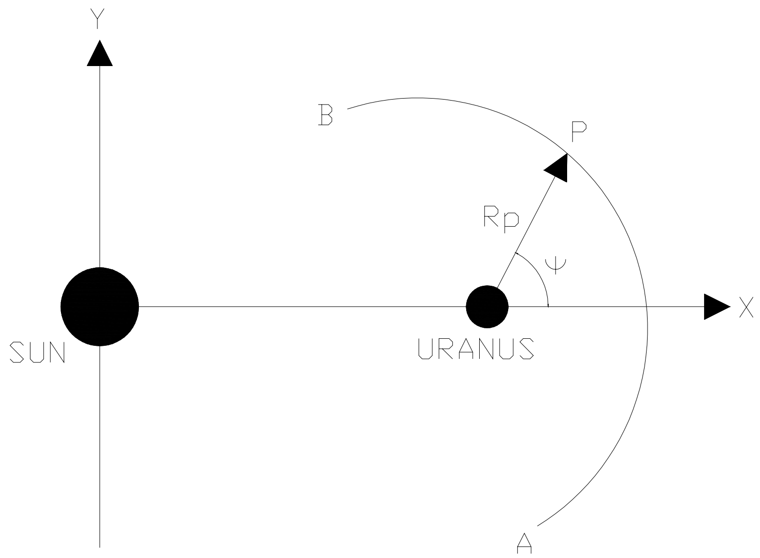

Figure 1. Geometry of the Close Encounter in the Rotating System.

3. MATHEMATICAL MODEL AND ALGORITHM

The equations of motion for the spacecraft are assumed to be the ones valid for the well-known planar restricted circular three-body problem:

$$\ddot{x} - 2\dot{y} = x - \frac{\partial V}{\partial x} = \frac{\partial \Omega}{\partial x} \tag{1}$$

$$\ddot{y} + 2\dot{x} = y - \frac{\partial V}{\partial y} = \frac{\partial \Omega}{\partial y} \tag{2}$$

$$\Omega = \frac{1}{2}\left(x^2 + y^2\right) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} \tag{3}$$

The usual standard canonical system of units is used. It is also necessary to have equations to calculate the energy and the angular momentum of the spacecraft. It can be done with the formulas:

$$E = \frac{(x+\dot{y})^2 + (\dot{x}+y)^2}{2} - \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{4}$$

$$C = x^2 + y^2 + x\dot{y} - y\dot{x} \tag{5}$$

With those equations, it is possible build a numerical algorithm to solve the problem. It has the following steps:
i) Arbitrary values for the three parameters: R_p *J,* ψ are given;
ii) With these values the initial conditions in the rotating system are computed. The initial position is:

$$X_i = R_p \cos(\psi) + (1-\mu) \tag{6}$$

$$Y_i = R_p \sin(\psi) \tag{7}$$

And the initial velocity is:

$$V_{Xi} = -V \sin(\psi) \tag{8}$$

$$V_{Yi} = +V \cos(\psi) \tag{9}$$

Where:

$$V = \sqrt{\dot{x}^2 + \dot{y}^2} \tag{10}$$

iii) With these initial conditions, the equations of motion are integrated forward in time until the distance between the planet Uranus and the spacecraft is bigger than a specified distance limit d_{JS}. At this point the numerical integration is stopped and the energy *(E+)* and the angular momentum *(C+)* after the encounter with Uranus are calculated;

iv) Then the initial conditions are returned to the point P, and the equations of motion are integrated backward in time, until the distance d_{JS} is reached again. Then the energy *(E-)* and the angular momentum *(C-)* before the encounter with Uranus are obtained;

v) With those results, all the information required to calculate the change in energy *(E+ - E-)* and angular momentum *(C+ - C-)* due to the close approach with Uranus are available;

vi) Now, the numerical integration is extended beyond the points A and B and it is verified if the spacecraft has none, one or two possible close encounters with the Earth, by using the conditions described in the previous section;

With this algorithm available, the given initial conditions (values for R_p J, ψ) can be varied in any desired range to study the effects of the close approach with Uranus in the orbit of the spacecraft.

4. RESULTS

The results consist of plots that show the change of the orbit of the spacecraft due to the close encounter with the planet Uranus, for a large range of given initial conditions. First of all it is necessary to c1assify the entire close encounters between Uranus and the spacecraft, according to the change obtained in the orbit of the spacecraft. The letters A to P are used for this classification, according to the rules showed in Tab. 1.

Table 1 – Rules for the assignment of letters to orbits

After: / Before:	Direct Ellipse	Retrograde Ellipse	Direct Hyperbola	Retrograde Hyperbola
Direct Ellipse	A	E	I	M
Retrograde Ellipse	B	F	J	N
Direct Hyperbola	C	G	K	O
Retrograde Hyperbola	D	H	L	P

To indicate which ones of those orbits have possibility of one or two close encounters with the Earth, the following conventions are used: i) Letters in capital case for orbits that do not cross the Earth's path around the Sun and have no possibility of a close encounter with the Earth; ii) Letters in lower case for orbits that cross the Earth's path around the Sun .in only one direction of time. These orbits can be used to send a spacecraft from the Earth to the Uranus or somewhere else using a Swing-by in Uranus; iii) Letters in bold lower case for orbits that cross the Earth's path around the Sun in both directions of time. These orbits can be used to send a spacecraft from the Earth to the Uranus; from Uranus to the Earth; or from the Earth to the Uranus and back to the Earth, without additional maneuvers, if a proper timing condition can be found.

With those rules defined, the results consist of assigning one of those letters to a position in a two-dimensional diagram that has the parameter ψ in the horizontal axis and the parameter J in the vertical axis. There is one plot for each desired value of the periapsis distance. The range for the variables used here is ψ ($180° \leq \psi \leq 360°$) and J ($1.35 \leq J \leq 1.55$). They are very adequate in showing the main characteristic of the plots. The interval $180° \leq \psi \leq 360°$ is used, and not the full range $0° \leq \psi \leq 360°$, because there is a symmetry between the chosen interval and the complementary interval $0° \leq \psi \leq 180$. This symmetry comes from the fact that an orbit with an angle $\psi = \theta$ is different from an orbit with an angle $\psi = \theta + 180°$ only by a time reversal. It means that there is a correspondence between these two intervals. This correspondence is: I↔C, L↔O, B↔E, N↔H, M↔D. The orbits A, F, K and P are unchanged.

To decide the best range of values for the third parameter (periapsis distance) several exploratory simulations have to be made. It was noticed that, for values greater than 50 Uranus's radius, the effects of the Swing-by are very small, with the exception of very few special cases. Then it is decided to make plots for the values: 2.0, 5.0 and 10.0 Uranus's radius. They span a useful range of values and they are able to show very well the evolution of the effects

To have a better understanding of the process, some of the trajectories are plotted in the rotating and fixed frame. The orbits of type F, K, A, a and p are chosen as examples. The numerical values for the limits involved in the results available in this research (in canonical units) are: Distance from Uranus to the points A and B *(d_{JS}): 0.5;* Distance limit for the spacecraft to be considered too far from the Solar System: 2.0; Time limit to stop the numerical integration when searching for a passage close to the Earth: 10.0. Trajectories j and b cross the Earth's orbit making an angle close to 90 degrees, so they are not very useful for practical applications due to the high increment velocity required by the maneuver. They have to be considered just as examples of the trajectories available. The results of this paper makes a survey of a large range of trajectories, including the ones with the lowest possible increments of velocity and flight path angle close to zero.

The trajectories that have only one encounter with the Earth are studied in more detail, to see if they encounter the Earth before or after the Swing-by with Uranus. The curious result is that only trajectories that encounter with the Earth before the Swing-by with Uranus are found. It means that the only type of trajectory that encounter the Earth after the Swing-by found in this research is the one that has a double-crossing (before and after the Swing-by) with the Earth's path around the Sun.

4.1. The Excess Velocities and the Flight Path Angle

After finding all those orbits, it is interesting to know the magnitudes of the impulses (ΔV) required to start the outbound trajectories at the Earth (to go to Uranus), or to stop the inbound trajectories at the Earth (coming from Uranus). It is assumed that the impulse required is the difference between the inertial velocities of the Earth and the spacecraft. It means that the spacecraft is assumed to be traveling attached to the Earth (they both have the same position and velocity at a given time), but it is free of the attraction of the Earth's gravity field. In other words, the impulse required to escape the Earth is not included in the results shown here. Another quantity calculated is the flight path angle at the Earth, which is defined as the angle between the inertial velocities of the spacecraft and the Earth at the point that their orbits intersect. Fig. 3 (for example) shows the results. All the plots have the angle of approach ψ (in degrees) in the horizontal axis and the Jacobian Constant in the vertical axis. They are: i) The flight path angle (in degrees) for the outbound trajectories; ii) The outcoming excess velocity (in canonical units) to start the outbound trajectories; iii) The flight path angle (in degrees) for the inbound trajectories; iv) The incoming excess velocity (in canonical units) to stop the inbound trajectories; v) The addition of the two excess velocities, also in canonical units. From those figures it is easy to find the regions with minimum excess velocities. It corresponds, as expected, to the regions with flight path angle close to zero.

4.2. Results for R_p=1R_u

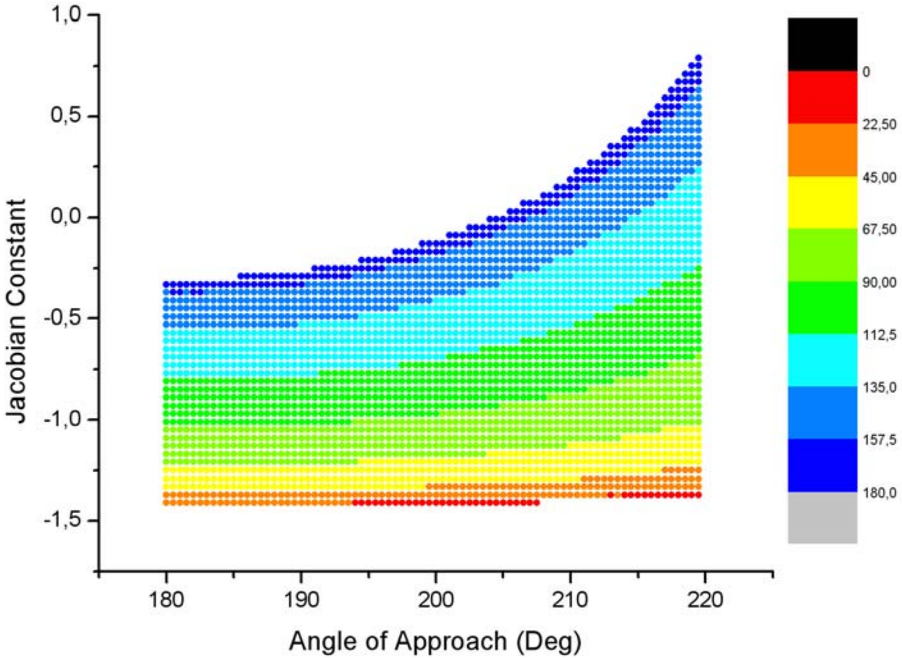

Figure 3. Flight Path Angle for Outbound Trajectories

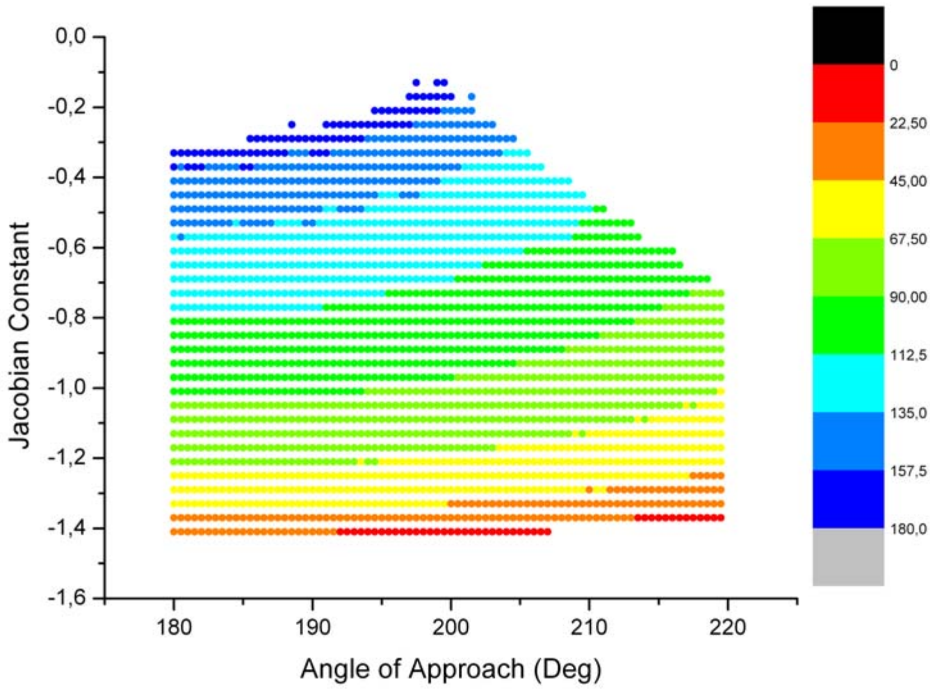

Figure 4. Flight Path Angle for Inbound Trajectories

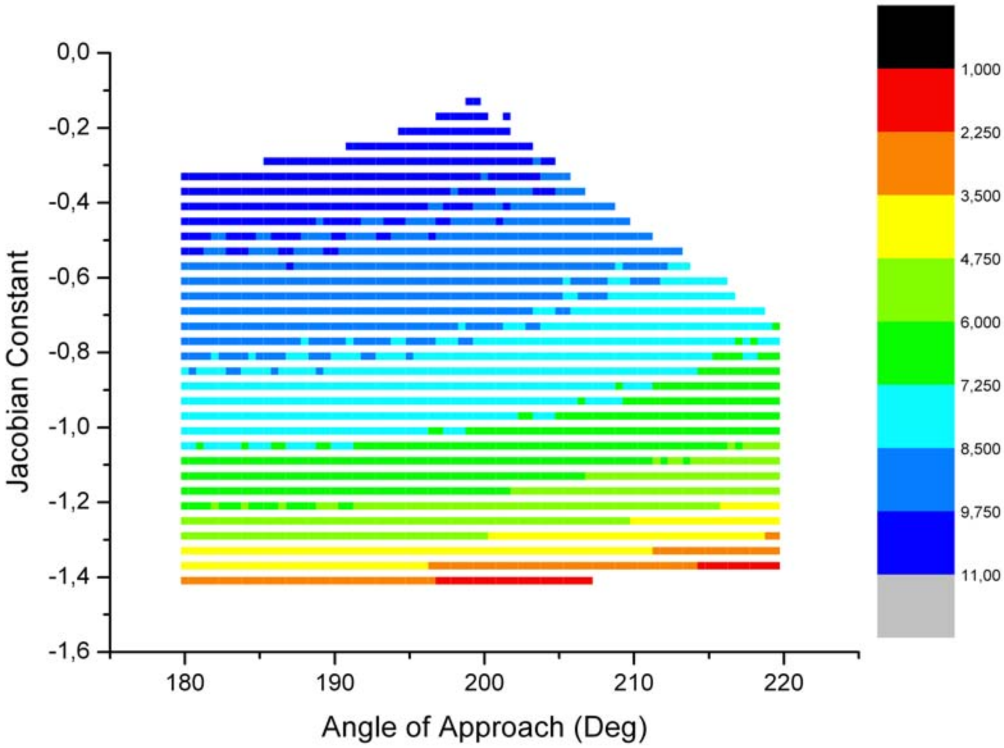

Figure 5. Total Velocity Variation (ΔV_{total})

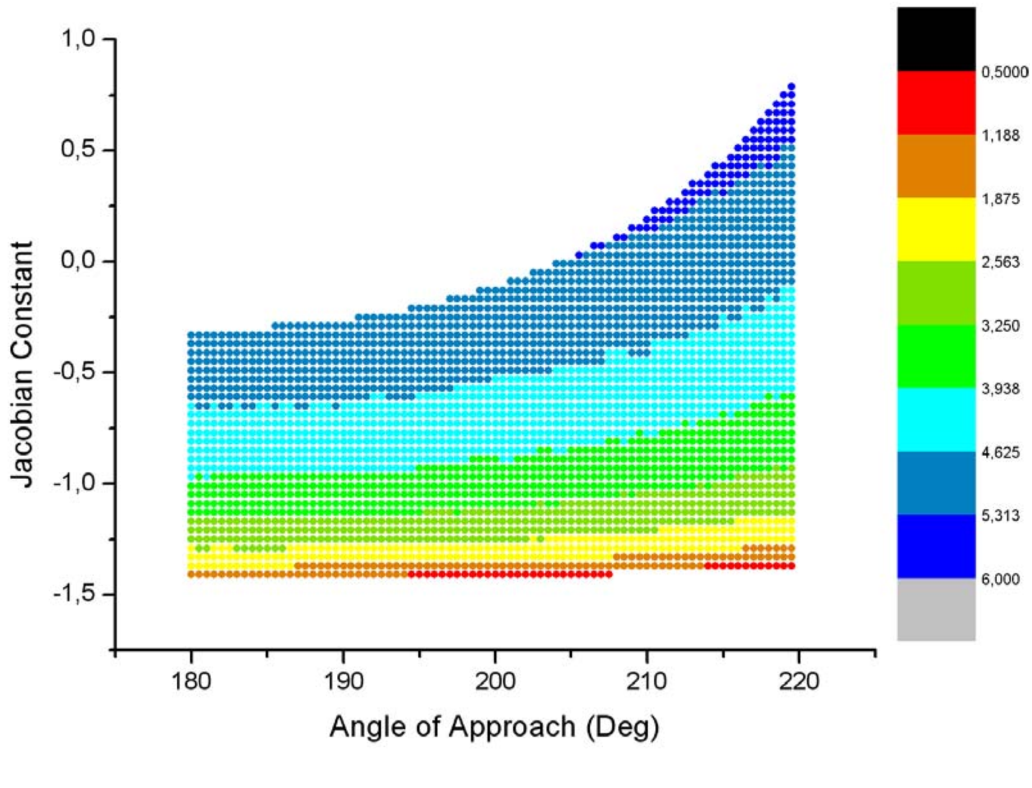

Figure 6. Variation of the Initial Velocity

4.3. Results for R_p=2R_u

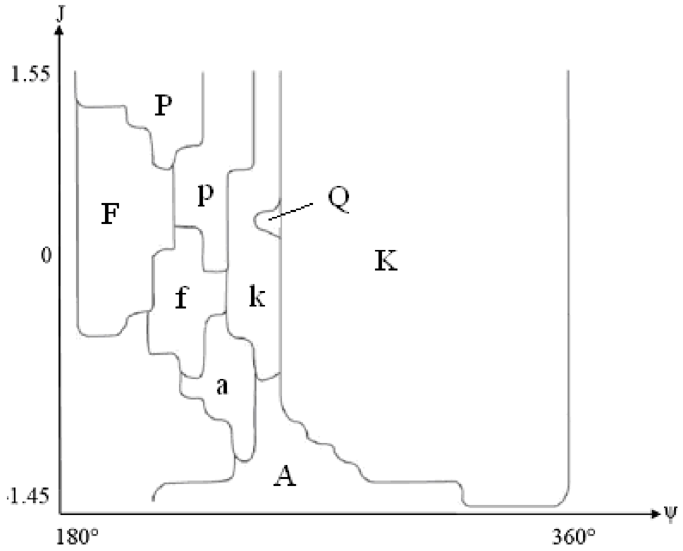

Figure 7. R_p=2R_u

4.4. Results for R_p=5R_u

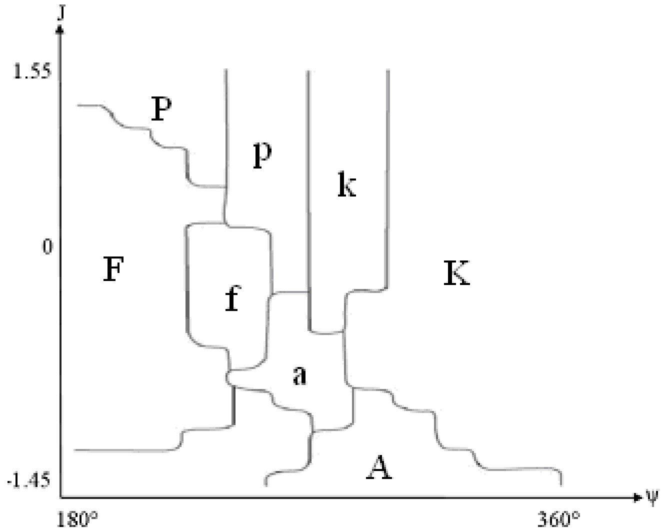

Figure 8. R_p=5R_u

4.5. Results for R_p=10R_u

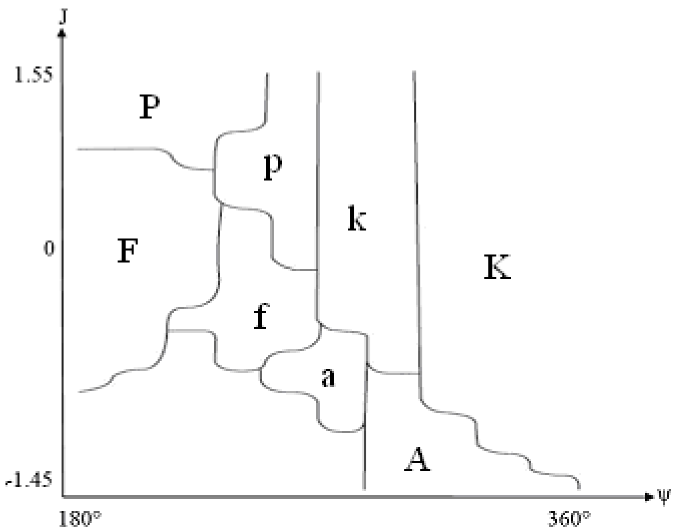

Figure 9. R_p=10R_u

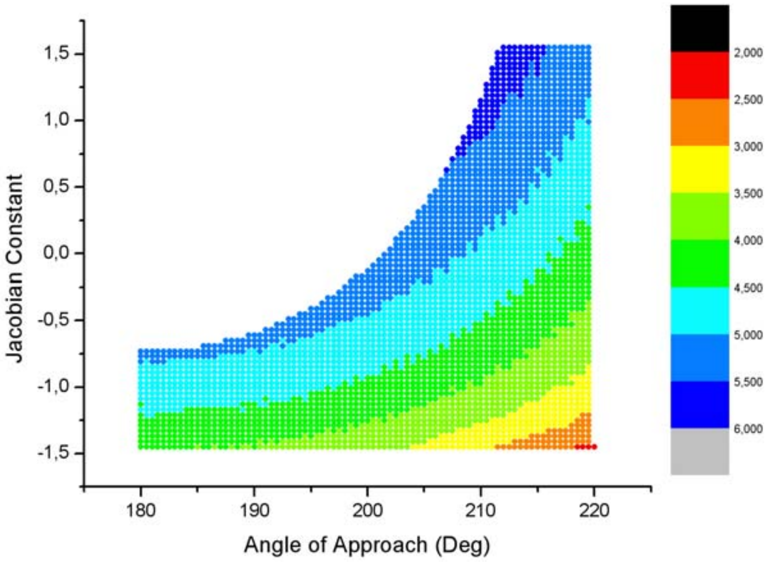

Figure 12. Variation of the Initial Velocity

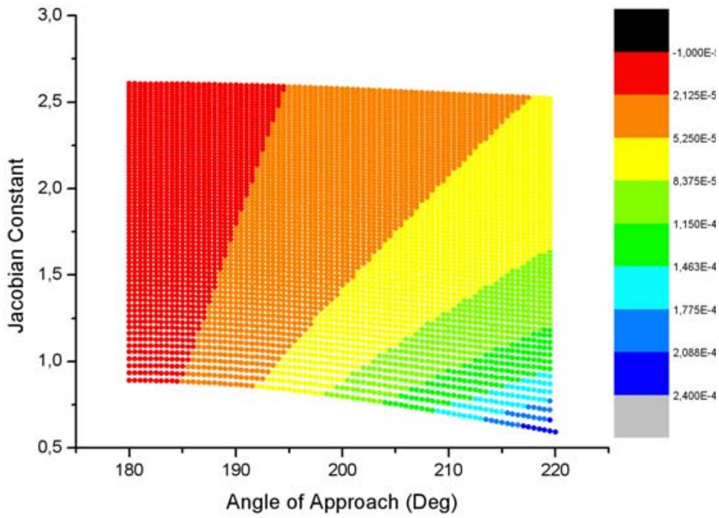

Figure 13. Variation of Energy Using the Two-Body Problem

4.6. Examples of Trajectories in the Rotating and Fixed Reference Frame

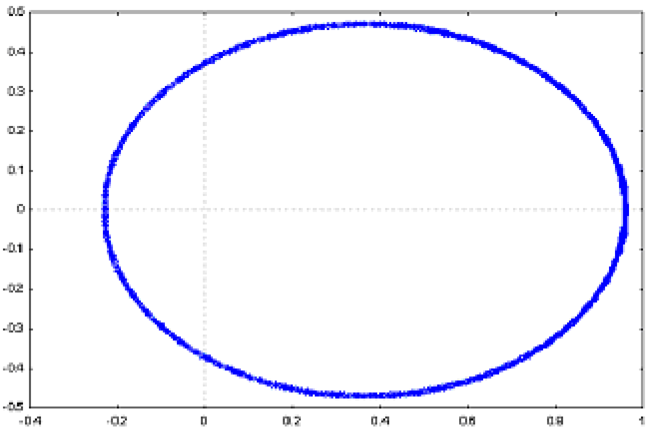

Figure 18. Orbit "A" – Fixed System

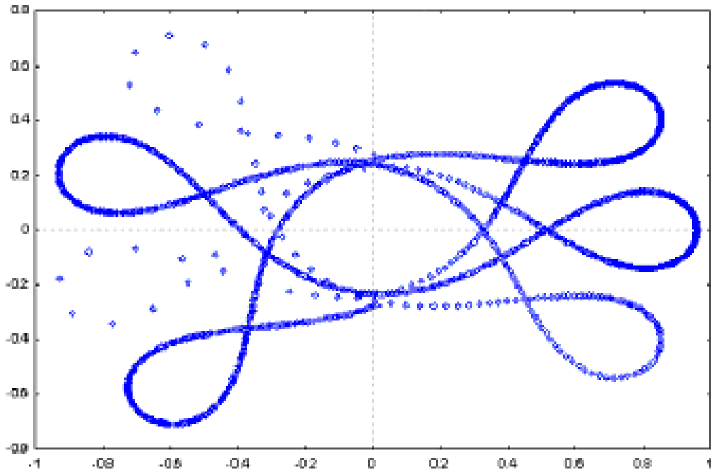

Figure 19. Orbit "A" – Rotating System

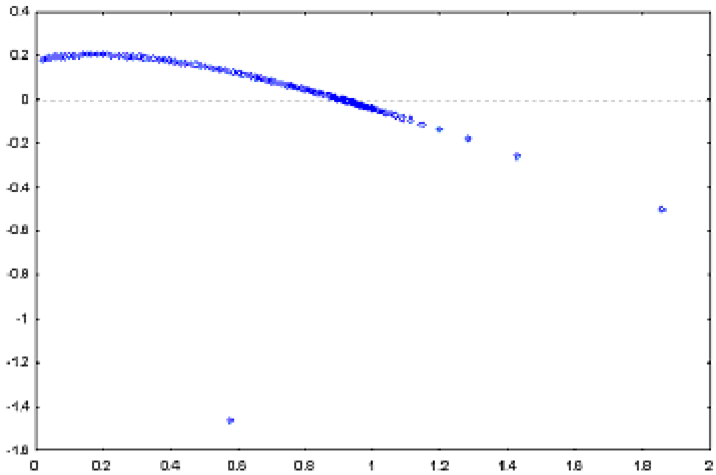

Figure 20. Orbit "p" – Fixed System

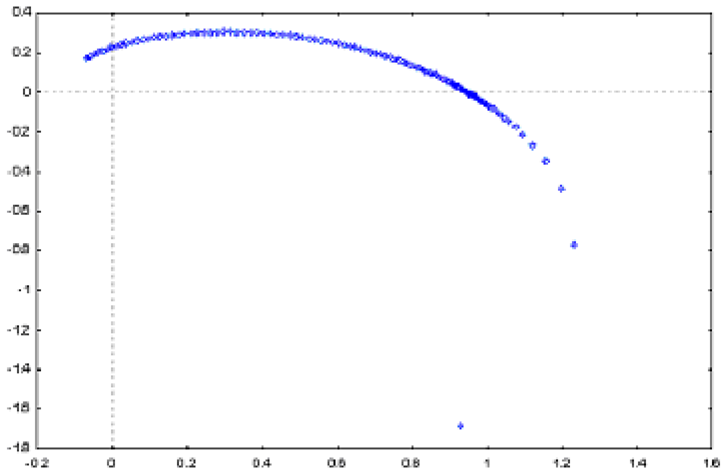

Figure 21. Orbit "p" – Rotating System

5. CONCLUSIONS

A numerical algorithm to calculate the effects of a close approach with Uranus in the trajectory of a spacecraft is developed. Many trajectories are classified and some of them are shown in detail. It is also shown which ones of those trajectories have a potential use for missions involving departures from the Earth or returns to the Earth. The theoretical prediction that for 0°≤ ψ ≤180°, the spacecraft losses energy and for 180°≤ ψ ≤360° the spacecraft gains energy is confirmed.

The outgoing and the incoming excess velocities of the spacecraft with respect to the Earth involved in those transfers are calculated, as well as the flight path angles at the point where the orbits of the spacecraft and the Earth intersect. Those results are sufficient to identify regions of minimum excess velocities for practical maneuvers.

After that, a procedure was developed to study the differences in the effects of the close approach predicted by the two models studied for the dynamics. The results showed that there is a good agreement of the results in the majority of the situations, but there are large discrepancies when the energy before or after the passage is small. In about 5% of the trajectories simulated the differences in semi-major axis predicted by the two models were larger than 10% of the Sun-Uranus distance. Trajectories with differences in the order of several hundreds of Sun-Uranus distances were also encountered. Those trajectories are the main reason to use more complex models to study this problem. In terms of energy change, the largest discrepancies obtained were about 5%.

6. ACKNOWLEDGEMENTS

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