OPTIMAL PLACEMENT OF PIEZOELECTRIC ACTUATORS/SENSORS IN SMART STRUCTURES USING IMPULSE RESPONSE

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Abstract. *Optimal placement of actuators and sensors is an important issue for control design and, usually, the positioning of these devices has been governed through distributed parameters, mainly, based on controllability and observability approaches criteria. While, this paper presents the optimal placement of piezoelectric actuator and sensor using a technique based on the impulse response. Numerical and experimental results in a beam-like structure are shown in order to verify the proposed methodology. The* L_∞ *norm of the impulse response is used and the performance indices of actuators/sensors are appraised for each vibration mode.*

Keywords: *optimal placement, smart structures,* L_∞ *Norm, Impulse Response.*

1. INTRODUCTION

Materials science and structural engineering have entered a new age brought about by the development of adaptive materials and their applications in intelligent structures. A smart structure is "a non-biological physical structure having a definite purpose, means and imperative to achieve that purpose and a biological pattern of functioning" (Spillman et. al. 1996). When an engineer develops an active vibration control design using actuators, sensors and a control technique; and connects them on the mechanical structure it is possible to say that is a smart structure.

In particular, the control of structural vibrations, through active techniques, had attracted the attention of great part of studious. Usually, excessive vibrations can compromise the performance of machines and structures. Before the control design, mainly when it has a great number of candidate positions, the problem of placement of sensors and actuators deserves the biggest attention. The placement can define the efficiency of the control; therefore, in function of the position where they are placed, the actuators can compromise the controllability of the system, or demand high levels of energy to get the desired result. On the other hand, if located in optimal locations, the necessary number of actuators and sensors can be reduced, thus diminishing the cost of instrumentation, the processing of signals and the necessary energy for the control of the structure.

An important kind of piezoelectric actuators are named PZT stack (piezoelectric actuator stack). These actuators exhibit an effect whereby they expand or contract in the presence of an applied electric field. Many works present active vibration control design using these actuators (Flint et al., 1995; Wang et al., 1996; Carvalhal, 2005). A piezoelectric stack actuator is supposed to be placed between structures or, also, between a structure and a rigid foundation; so, the relative movement of the structure could be controlled or sensed by the PZT stack. It is possible to control the actuator stroke using an external load applied to the actuator and with this setup a PZT stack can be used in active vibration control design, as shown in Fig. 1. This picture shows a PZT stack connected on bearing supported shaft structure.

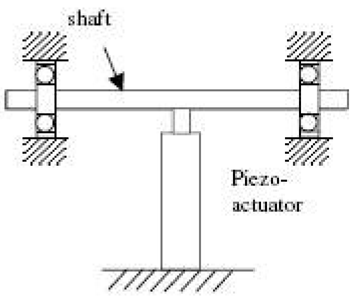


Figure 1. PZT stack acting on bearing supported shaft structure (Li et al., 2006).

In this context, this paper searches the answer for the following question: where is the best location to put a piezoelectric stack actuator for obtaining an optimal vibration control design? The performance indices are based on L_∞ norm of the impulse response. Numerical and experimental results are shown and the proposed methodology is compared with the classical technique that involves controllability concepts.

2. STRUCTURAL MODELING

It is possible to describe the dynamical behaviour of a structure in terms of mass, stiffness and damping matrices, and displacement and velocity vectors as

$$\begin{aligned}&\ddot{\mathbf{q}}(t)+\mathbf{M}^{-1}\mathbf{D}_{\mathbf{a}}\dot{\mathbf{q}}(t)+\mathbf{M}^{-1}\mathbf{K}\mathbf{q}(t)=\mathbf{M}^{-1}\mathbf{B}_0\mathbf{u}(t)\\&\mathbf{y}(t)=\mathbf{C}_{\mathbf{oq}}\mathbf{q}(t)+\mathbf{C}_{\mathbf{ov}}\dot{\mathbf{q}}(t)\end{aligned}\tag{1a,b}$$

where $\mathbf{q}(t)$ is the n-length displacement vector, $\mathbf{u}(t)$ is the s-length input vector, $\mathbf{y}(t)$ is r-length output vector, $\mathbf{M}$ is the n x n mass matrix, $\mathbf{D}_\mathbf{a}$ is the n x n damping matrix, and $\mathbf{K}$ is the n x n stiffness matrix. $\mathbf{B}_\mathbf{0}$ is the n x s input matrix, $\mathbf{C}_\mathbf{oq}$ and the r x n output displacement matrix, and $\mathbf{C}_\mathbf{ov}$ is the r x n output velocity matrix. The mass matrix is positive definite, and the stiffness and damping matrices are positive semi-definite, n is the number of degrees of freedom of the system (linearly independent coordinates describing the finite-dimensional structure), r is the number of outputs and s is the number of inputs. Using the classic procedure of modal analysis (Maia *et al.*, 1996), it is possible to write the equations of motion in modal coordinates, $\mathbf{q}_\mathbf{m}(t)$. Thus, the modal model of second order is given by

$$\begin{aligned}&\mathbf{q}(t)=\mathbf{\Phi}\mathbf{q}_{\mathbf{m}}(t)\\&\ddot{\mathbf{q}}_{\mathbf{m}}(t)+2\mathbf{Z}\mathbf{\Omega}\dot{\mathbf{q}}_{\mathbf{m}}(t)+\mathbf{\Omega}\mathbf{q}_{\mathbf{m}}(t)=\mathbf{B}_{\mathbf{m}}\mathbf{u}(t)\\&\mathbf{y}(t)=\mathbf{C}_{\mathbf{mq}}\mathbf{q}_{\mathbf{m}}(t)+\mathbf{C}_{\mathbf{mv}}\dot{\mathbf{q}}_{\mathbf{m}}(t)\end{aligned}\tag{2a,b,c}$$

where $\mathbf{\Phi}$ is the modal matrix and $\mathbf{Z}$ is the matrix of damping coefficients (ζ_i), given by

$$\mathbf{Z}=0.5\mathbf{M}_{\mathbf{m}}^{-1}\mathbf{D}_{\mathbf{m}}\mathbf{\Omega}^{-1}=0.5\mathbf{M}_{\mathbf{m}}^{-1/2}\mathbf{K}_{\mathbf{m}}^{-1/2}\mathbf{D}_{\mathbf{m}}\tag{3}$$

where $\mathbf{\Omega}^2=\mathbf{M}_{\mathbf{m}}^{-1}\mathbf{K}_{\mathbf{m}}$ is the matrix of natural frequencies. The matrices $\mathbf{M}_\mathbf{m}$, $\mathbf{K}_\mathbf{m}$ and $\mathbf{D}_\mathbf{m}$ are diagonal matrices of modal mass, stiffness and damping, respectively, which are given by

$$\begin{aligned}&\mathbf{M}_{\mathbf{m}}=\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}\\&\mathbf{K}_{\mathbf{m}}=\mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}\\&\mathbf{D}_{\mathbf{m}}=\mathbf{\Phi}^T\mathbf{D}_{\mathbf{a}}\mathbf{\Phi}\end{aligned}\tag{4a,b,c}$$

The matrix $\mathbf{D}_\mathbf{a}$ is assumed to be proportional to mass and stiffness matrices, with α and β constants, so, that

$$\mathbf{D}_{\mathrm{a}}=\alpha\mathbf{M}+\beta\mathbf{K}\tag{5}$$

Matrix $\mathbf{B}_\mathbf{m}$ in Eq. (2b) is the input modal matrix, or participation modal matrix and is given by

$$\mathbf{B}_{\mathbf{m}}=\mathbf{M}_{\mathbf{m}}^{-1}\mathbf{\Phi}^T\mathbf{B}_{\mathbf{0}}\tag{6}$$

$\mathbf{C}_\mathbf{mq}$ and $\mathbf{C}_\mathbf{mv}$ are the output displacement and velocity modal matrices given by

$$\begin{aligned}&\mathbf{C}_{\mathbf{mq}}=\mathbf{C}_{\mathbf{oq}}\mathbf{\Phi}\\&\mathbf{C}_{\mathbf{mv}}=\mathbf{C}_{\mathbf{ov}}\mathbf{\Phi}\end{aligned}\tag{7a,b}$$

The state-space equations can be written in a vector-matrix format through the triple ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$); it allows the equations to be manipulated more easily. The related matrices are given by

$$\mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{\Omega}^2 & -2\mathbf{Z}\mathbf{\Omega}\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}\mathbf{0}\\ \mathbf{B}_{\mathrm{m}}\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}\mathbf{C}_{\mathrm{mq}} & \mathbf{C}_{\mathrm{mv}}\end{bmatrix}\tag{8a,b,c}$$

Equations (8) are not a modal state representation (although it was obtained using modal displacements, $\mathbf{q}_\mathrm{m}$). The modal state-space representation has a triple ($\mathbf{A}_\mathrm{m}$, $\mathbf{B}_\mathrm{m}$, $\mathbf{C}_\mathrm{m}$) characterized by the block-diagonal dynamic matrix, $\mathbf{A}_\mathrm{m}$, and the related input and output matrices (Gawronski, 1998)

$$\mathbf{A}_{m} = \mathrm{diag}\left(\mathbf{A}_{mi}\right), \qquad \mathbf{B}_{m} = \begin{bmatrix} \mathbf{B}_{m1} \\ \mathbf{B}_{m2} \\ \vdots \\ \mathbf{B}_{mn} \end{bmatrix}, \qquad \mathbf{C}_{m} = \left[\mathbf{C}_{m1} \quad \mathbf{C}_{m2} \quad \cdots \quad \mathbf{C}_{mn}\right] \tag{9a,b,c}$$

where i=1,2,…,n; $\mathbf{A}_{mi}$, $\mathbf{B}_{mi}$ and $\mathbf{C}_{mi}$ are 2 x 2, 2 x *s* and *r* x 2 blocks, respectively. These blocks can take several different forms and also it is possible to convert from one form to another by a linear transformation. One possible form to block $\mathbf{A}_{mi}$ is:

$$\mathbf{A}_{mi} = \begin{bmatrix} -\zeta_i \omega_i & \omega_i \\ -\omega_i(\zeta_i^2 - 1) & -\zeta_i \omega_i \end{bmatrix} \tag{10}$$

The state vector $\mathbf{x}(t)$ in modal coordinates consists of *n* independent components, $\mathbf{x_i}(t)$, that represent a state of each mode. The $\mathbf{x}_i(t)$ (*i*th state component), related to Eq. (10), is given by (Kailath, 1980).

$$\mathbf{x}_i(t) = \begin{Bmatrix} \mathbf{q}_{mi}(t) \\ \mathbf{q}_{moi}(t) \end{Bmatrix}, \quad \mathbf{q}_{moi}(t) = \zeta_i \mathbf{q}_{mi}(t) + \dot{\mathbf{q}}_{mi}(t) \big/ \omega_i \tag{11}$$

3. OPTIMAL PLACEMENT OF ACTUATORS AND SENSORS

Many studies have been done on the optimal placement of actuators and sensors. An usual approach is maximizing controllability and observability properties using the Grammian matrices (Bruant and Proslier, 2005; and Jha and Inman, 2003). In this paper is proposed to use the L_∞ norm of the impulse response to obtain the best positions for actuators and sensors on a structure. Actuator and sensor placement are solved independently and the indices are obtained for each vibration mode. The L_∞ norm is compared with the traditional methodology to verify the results.

3.1. L_∞ Actuator and Sensor Indices

The L_∞ actuator and sensor indices are based on L_∞ norm of a signal. The L_∞ norm of a signal is defined as follows:

$$\|y\|_{L\infty} = \sup_{t \ge 0} |y(t)| \tag{12}$$

where sup denotes the supremum (Jeffreys and Jeffreys 1988). The supremum $\sup_{x \in S} x$ for S a subset of the extended real number $\overline{R} = R \cup \{\pm\infty\}$ is the smallest value $y \in \overline{R}$ such that for all $x \in S$, $x \le y$. The L_∞ norm of a signal provides an alternative method of characterizing whether a signal is small of large (Clark et al., 1997).

In this paper is proposed to compute the L_∞ norm of the impulse response of the system to obtain the optimal positions of actuators and sensors. The optimal location of actuators can be obtained considering sensors in arbitrary locations and computing the L_∞ norm for each actuator position. In the following, with the actuators in optimal location, compute the L_∞ norm for each sensor position. The actuator (sensor) indices for each vibration mode can be computed using equation (1) or in form of modal state-space representation. The L_∞ placement index $\sigma_{L\infty\text{-}ij}$ evaluates the *j*th actuator (sensor) at the *i*th mode. It is defined for every mode and every admissible actuators or sensors, as:

$$\sigma_{L\infty-ij} = \|y_{ij}\|_{L\infty}; \quad j = 1, \cdots, S \text{ and } i = 1, \cdots, n \tag{13}$$

S is the number of actuator positions (or j =1,…, R and R is the number of sensors). The L_∞ norm was computed using the software Matlab® with the command "linfnorm".

Computing $\sigma_{L\infty}$ to all actuators positions and vibration modes is possible to define the L_∞ matrix for placement of actuators (the similar matrix can be defined for the sensors)

$$T_A = \begin{array}{cl} \begin{bmatrix} \sigma_{L\infty-11} & \cdots & \sigma_{L\infty-1j} & \cdots & \sigma_{L\infty-1S} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sigma_{L\infty-i1} & \ddots & \sigma_{L\infty-ij} & \ddots & \sigma_{L\infty-iS} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sigma_{L\infty-n1} & \cdots & \sigma_{L\infty-nj} & \cdots & \sigma_{L\infty-nS} \end{bmatrix} & \begin{array}{l} \\ \\ \Leftarrow \textit{ith} \quad \textit{mode} \\ \\ \\ \end{array} \\ \begin{array}{c} \Uparrow \\ \textit{jth} \quad \textit{actuator} \end{array} & \end{array} \tag{14}$$

3.2. Grammian Actuator Indices

It is possible to define a performance index for optimal placement of actuators using controllability concepts. Controllability and observability are structural properties that carry useful information for structural testing and control, yet are often overlooked by structural engineers (Gawronski, 1998). A structure is controllable if the installed actuators excite all the structural modes of interest. It is observable if the installed sensors detect the motion of all the modes of interest. This information, although essential in many applications, is too limited; it answers the question of excitation or detection in terms of yes or no. The quantitative answer is supplied by the controllability and observability of each mode.

Controllability, as a coupling between the input and the states, involves the system matrix **A** and the input matrix **B**. A linear system, or the pair (**A**, **B**), is controllable at t_0 if it is possible to find a piecewise continuous input $\mathbf{u}(t)$, t Є $[t_0, t_1]$, that will transfer the system from initial state, $\mathbf{x}(t_0)$, to the origin $\mathbf{x}(t_1) = 0$, at finite $t_1 > t_0$. If this is true for all initial $\mathbf{x}(t_0)$ the system is completely controllable. Otherwise, the system, or the pair (**A**, **B**) is uncontrollable.

There are many criteria to determine system controllability and observability (Kailath, 1980; Zhou, 1995). A linear time-invariant system (**A**, **B**, **C**), with s inputs is completely controllable if and only if the N x sN matrix of

$$\mathbf{C_o} = \left[\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B} \quad \ldots \quad \mathbf{A}^{N-1}\mathbf{B}\right] \tag{15}$$

has rank N = size(**A**).

An alternative approach uses Grammians to determine the system properties. Grammians express the controllability properties qualitatively, and avoid numerical difficulties. The controllability Grammian is defined as (Kailath, 1980)

$$\mathbf{W}_c(t) = \int_0^t \exp(\mathbf{A}t)\mathbf{B}\mathbf{B}^{\mathbf{T}} \exp(\mathbf{A}^{\mathbf{T}}t)dt \tag{16}$$

Alternatively, it can be determined from the following system of differential equations

$$\dot{\mathbf{W}}_c(t) = \mathbf{A}\mathbf{W}_c + \mathbf{W}_c\mathbf{A}^{\mathbf{T}} + \mathbf{B}\mathbf{B}^{\mathbf{T}} \tag{17}$$

For a stable system, the stationary solutions of the above equations are obtained by assuming $\dot{\mathbf{W}}_c = 0$. In this case, the Grammian matrix is determined from the following Lyapunov Equation

$$\mathbf{A}\mathbf{W}_c + \mathbf{W}_c\mathbf{A}^{\mathbf{T}} + \mathbf{B}\mathbf{B}^{\mathbf{T}} = \mathbf{0} \tag{18}$$

For a stable **A**, the Grammian $\mathbf{W_c}$ is positive definite. Considering $\mathbf{B}_{ij}$ the input matrix of *i*th mode and *j*th actuator position and $\mathbf{A}_i$ the dynamic matrix of *i*th mode, is possible to solve the correspondent Eq. (18) and to obtain the *Actuator Grammian Index* ($\sigma_{AG\text{-}i,j}$)

$$\sigma_{AG-i,j} = \mathrm{tr}\left(\mathbf{W}_{c-i,j}\right) \quad i = 1,\ldots,n \quad j = 1,\ldots,S \tag{19}$$

where $\mathbf{W}_{c\text{-}i,j}$ is the controllability Grammian matrix considering the *i*th mode and *j*th actuator, tr is the trace of the matrix, n is the number of modes and S is the number of actuator positions. This index characterizes the performance of the actuator in the *j*th position on *i*th structural mode.

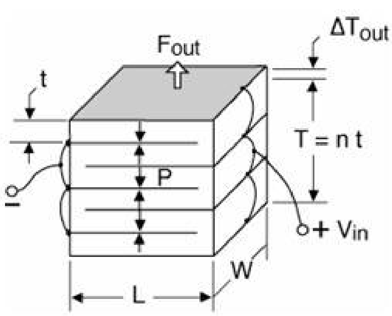
Computing σ_{AG} to all actuators positions and vibration modes, it is possible to define the Grammian Matrix for Placement of Actuators

$$T_A = \begin{array}{cl} \begin{bmatrix} \sigma_{AG-11} & \cdots & \sigma_{AG-1j} & \cdots & \sigma_{AG-1S} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sigma_{AG-i1} & \ddots & \sigma_{AG-ij} & \ddots & \sigma_{AG-iS} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sigma_{AG-n1} & \cdots & \sigma_{AG-nj} & \cdots & \sigma_{AG-nS} \end{bmatrix} & \Leftarrow ith \quad mode \\ \Uparrow & \\ jth \quad actuator & \end{array} \qquad (20)$$

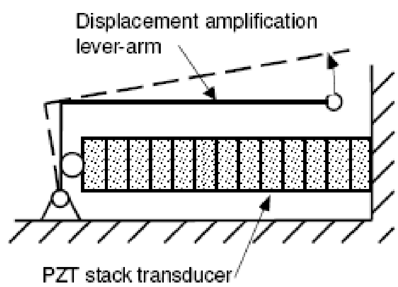
where the kth column consists of indices of the kth actuator for each mode, and the ith row is a set of the indices of the ith mode for each actuator. In a similar way, it is possible to define the sensor indices and Grammian matrix for placement of sensors using the observability Grammian matrix.

4. NUMERICAL AND EXPERIMENTAL TESTS

In numerical application the piezoelectric stack actuator (PZT) was appraised in three different points of the structure (nodes 2, 11 and 18); Fig. 3a. PZT stack actuator was analyzed to obtain the best position for vibration attenuation. These actuators exhibit an effect whereby they expand or contract in the presence of an applied electric field. This "induced strain", or change in length, occurs as electrical dipoles in the piezoelectric material rotate to align with an orientation that more closely aligns with the direction of the applied electric field. The change in the length is generally proportional to the field strength as applied via the device actuator voltage. A stack consists of "n" thin layers of PZT ceramic laminated together and electrically connected in parallel (Fig. 2a). In this way, it is possible to attenuate vibrations using these transducers as displacement amplification lever-arm (Fig. 2b) when they act in opposite direction of the vibration. Figure 2a shows the configuration of a stack PZT where V_{in} is the input voltage, F_{out} is the output force, L is the width, W is the thickness, and T is the total length (t is the length of each ceramic).



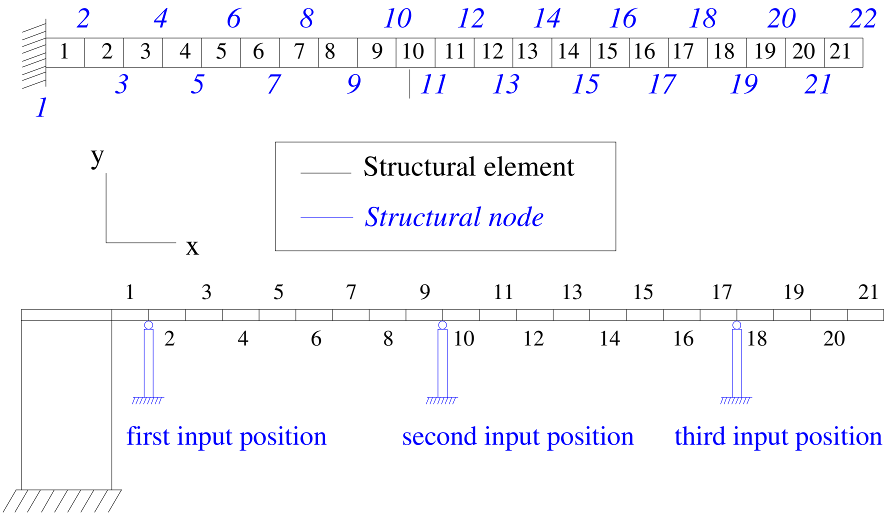
(a) PZT stack configuration



(b) displacement lever arm*

Figure 2. Illustration of a piezoelectric actuator stack.
*http://www.dynamic-structures.com/pdf/intro_piezo_actuation.pdf, access in march/04/07

The proposed methodology of optimal placement was applied numerically in a beam-like structure, as shown in Fig. 3. The beam was modeled through finite element method (FEM) with 21 elements (using Euller-Bernoulli element) and 22 structural nodes. A cantilever beam was considered with 44 degree of freedom (2 dofs per node). The physical and geometric properties of the beam are given in Table 1.



(a)

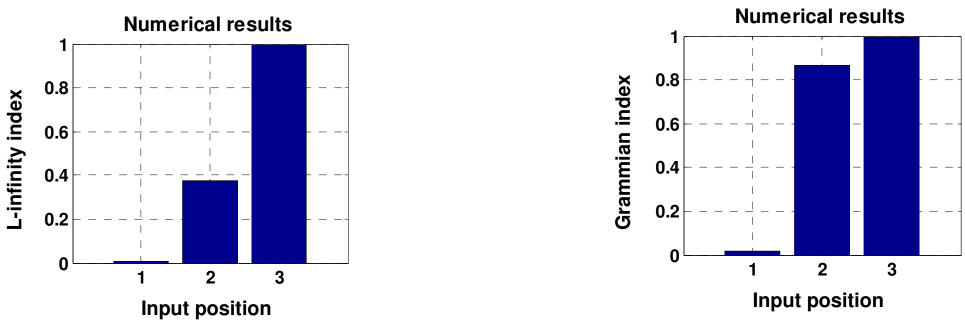
(b)

Figure 3. (a) structural nodes and elements; (b) input positions.

Table 1 – Physical and geometric properties of the cantilever beam.

Young's Modulus (GPa)	210
Width (mm)	37
Thickness (mm)	5
Length L (mm)	420
Density (Kg/m^3)	7800

In practical situations, the number of actuators and sensors is limited and there is also restriction in the placement. So, it was considered a sensor (accelerometer) on node 22 (Fig. 3a) and that the actuator can be positioned in any three input positions as shown in Fig.3b. Figure 4 shows the L_∞ indices for each input excitation position. It is possible to observe that the third input position is more efficient for the PZT stack placement. Similar results were obtained using the Grammian methodology.

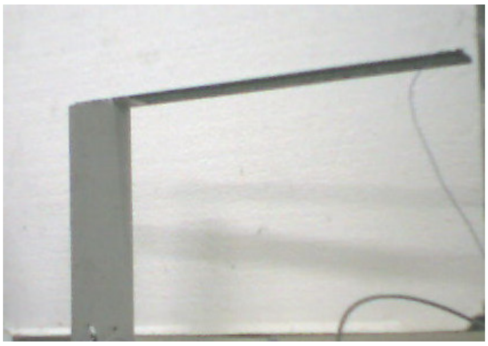


(a) L-infinity index

(b) Grammian Index

Figure 4. Actuator performance indices – numerical results.

Many works about optimal placement present only numerical results. However, experimental tests are indicated for validating any formulation. In this way, the proposed methodology was applied experimentally in a beam-like structure, as shown in Fig. 4a. The properties of the beam are given in Table 1. Tests were performed by exciting the structure with an impact hammer, Fig. 5b. The output signals were measured with an accelerometer, model 352C22 PCB Piezotronics®. The measurements were obtained five times for each input position to verify the repeatability of the results. In these experiments the software SignalCalc ACE® II was used to realize the data acquisition. The parameters of the system were identified by using the Eigensystem Realization Algorithm (ERA) for all three different input excitation positions. It is not presented information about ERA algorithm; further details can be found in Juang and Minh, 2001 and Bueno et al., 2006. The disposition of the experimental setup is shown in Fig. 6.



(a) beam used in experimental tests

(b) impact hammer

Figure 5. Cantilever beam and equipments used in experimental tests.

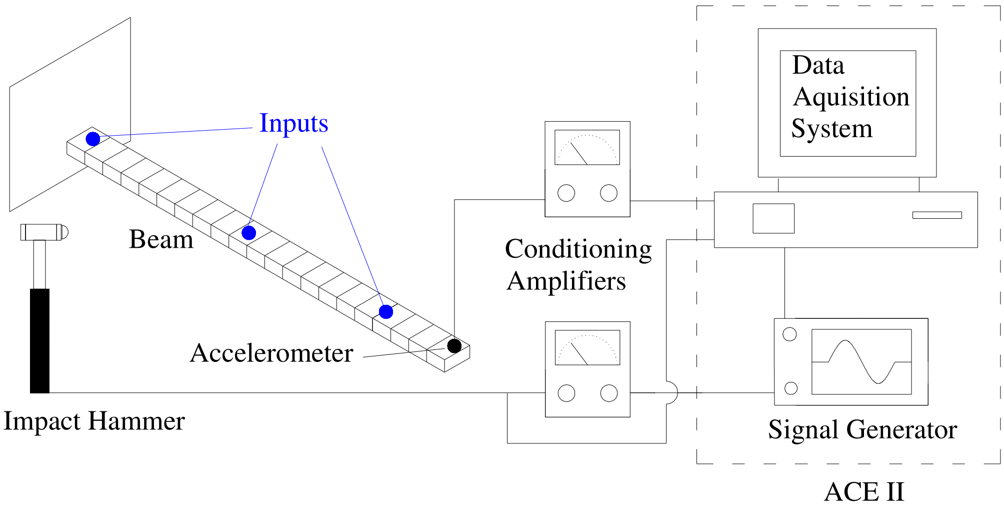


Figure 6. Disposition of experimental setup.

Figures 7 and 8 show the signals in time domain for the input excitation and the output signal, respectively. These signals were obtained with excitation in the first input position; the other signals were similar and they are omitted . Using these signals the system matrices were identified by ERA for each input position. Figure 9 shows the frequency response functions for each input position.

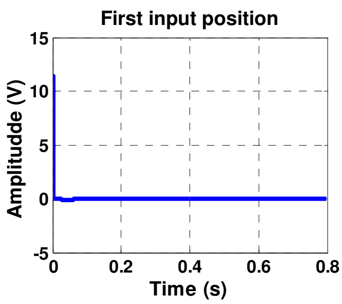


Figure 7. Input signal obtained from first position.

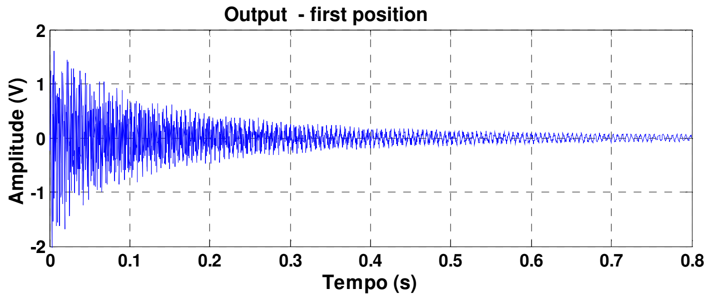


Figure 8. Output signal – input excitation in the first position.

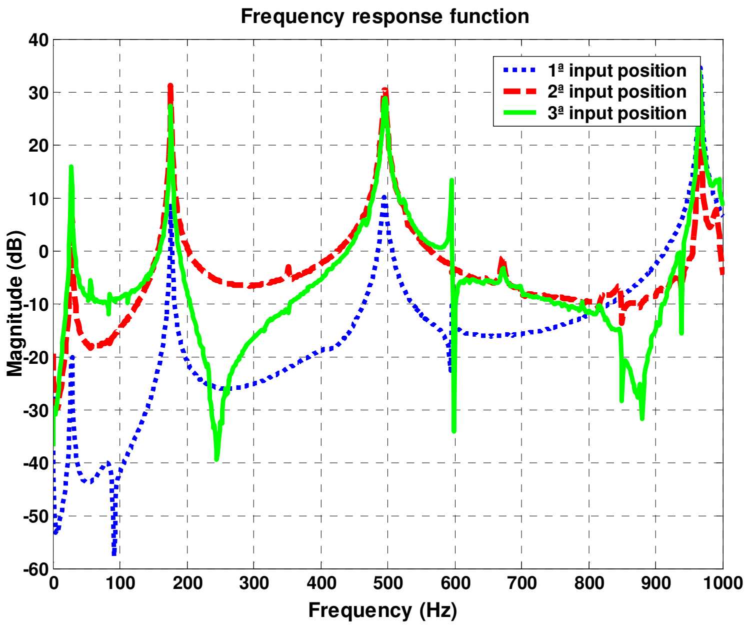
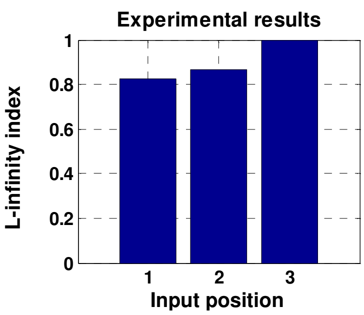
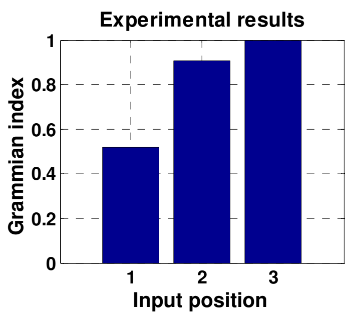


Figure 9. Frequency response functions for all three excitation positions.

Using L_∞ norm of the impulse response, the optimal placement of the actuator was computed. Similar results were obtained through Grammian methodology. In these experiments were used impulses as input excitations, but to compute the L_∞ norm was necessary to identify a state space model, because there was not control in the input force, which cause different amplitudes. Using L_∞ technique as performance index is very important to assure the same input. Figure 10 shows that the third input position is considered the best place for the PZT stack actuator.



(a) L-infinity index



(b) Grammian Index

Figure 10. L_∞ and Grammian performance indices for the experimental tests.

5. FINAL REMARKS

Advanced structures with integrated sensors and actuators are currently being investigated due the vast vantages that they offer in adaptive control. These structures are denominated smart structures and an important stage of the design is the optimal placement of actuators and sensors. In this paper the problem of sensors and actuators locations was solved using a new approach that involves the L_∞ norm of the impulse response. The traditional methodology to obtain the optimal placement of actuators and sensors are based on controllability and observability concepts by Grammian matrices, and it was used for comparison proposal. Techniques based on frequency response function, as H_∞ norm and H_2 norms should also be used. In literature, few papers propose a solution for the optimal placement problem using techniques based on time domain. This paper shows the optimal placement of a PZT stack actuator in a beam-like structure. The methodology is easily implemented and can be used in complex structures with many degrees of freedom.

6. ACKNOWLEDGEMENTS

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