NUMERICAL SIMULATIONS OF FLUID VIBRATIONS EXCITED BY A NON-IDEAL POWER SOURCE

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Abstract. *In this research we analyze free surface oscillations of a fluid in a cylinder tank excited by a non-ideal power source, an electric motor with limited power supply. We investigate the possibility of parametric resonance in this system, showing that the excitation mechanism can generate chaotic response. Additionally, the dynamics of parametrically excited surface waves in the tank can reveal new characteristics of the system. The fluid-dynamic system is modeled in such way as to obtain a nonlinear differential equation system. Numerical experiments are carried out to find the regions of chaotic solutions. Simulation results are presented as phase-portrait diagrams, Poincaré Maps, bifurcation diagrams and basins of attraction graphs to characterize the resonant vibrations of free fluid surface and the existence of several types of chaotic attractors during transition from regular to chaotic motion.*
Keywords*: parametric resonance, non-ideal power sources, nonlinear dynamics, free-surface*

1. INTRODUCTION

The study of resonance oscillations in a partially filled tank with liquid is important because there is several problems in industry associate with elevated water and liquefied natural gas tanks. In such cases, the motion of a liquid surface (sloshing) in the containers appears due to nonlinearity of the liquid inertia force. The characteristic of sloshing of liquid has been a concern in a number of engineering fields. Many papers examined this kind of nonlinear behavior of liquid sloshing in tanks that are excited horizontally and vertically (Abramson, 1966; Abramson, et al., 1966; Dodge, et al. 1965; Hutton, 1963; Ibrahim, et al. 2001; Ikeda and Murakami, 2005; Krasnopolskaya and Shvets, 1993 and 1994; Miles, 1976, 1984a, 1984b and 1984c, Miles and Henderson, 1990).

In this paper we examine the free surface oscillations of liquid sloshing in a tank that is vertical excited by a non-ideal power source, i.e, an electric motor with limited power supply. This excitation mechanism can generate chaotic response. We investigate that the dynamics of parametrically excited surface waves in the tank can reveal resonance in the system. The fluid-dynamic system is modeled in such way as to obtain a nonlinear differential equation system. Numerical experiments are carried out to find the regions of chaotic solutions. Simulation results are presented to characterize the resonant vibrations of free fluid surface and the existence of several types of chaotic attractors during transition from regular to chaotic motion.

2. MATHEMATICAL MODEL

We investigate wave dynamics on the surface of the fluid in the tank, vibrated by an electric motor with a limited power-supply. The system is modeled by a cylindrical tank of radius *R* partially filled with a liquid considered inviscid and incompressible (Fig. 1). A shaft of an electric motor and a crank mechanism connect with the platform of the tank. The crank turns by the angle σ and the base of the tank moves vertically with velocity $v(t) = \dot{x}(t)$, where $x(t) = x_0 \cos\sigma(t)$ and x_0 is the crank arm. The free surface of the liquid is described by $z = \eta(r,\theta,t)$. The fluid has a density ρ is assumed inviscid and incompressible. A detailed Lagrangian description of the fluid surface is performed by (Krasnopolskaya and Shvets, 1993, 1994), Miles (1976, 1984b).

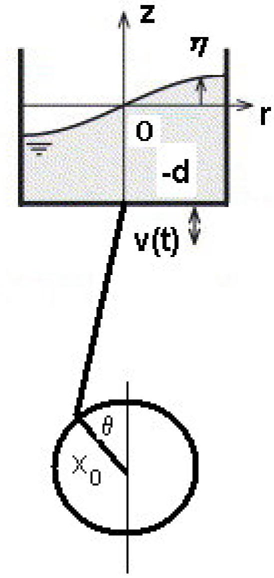

Figure 1.System composed by liquid in tank and electric motor.

Summary, the description of the fluid surface is in the form of the sum of eigenmodes $\eta(r,\theta,t) = \eta_n(t)\psi_n(r,\theta)$ where the summation is carries out for identical indexes *i* and *j*,

$$\eta(r,\theta,t) = \sum_{i,j}\left[q_{ij}^{c}(t)k_{ij(r)}\cos(i\theta) + q_{ij}^{s}(t)k_{ij(r)}\sin(i\theta)\right]. \tag{1}$$

In this equation we can characterize the amplitudes of the fundamental and secondary modes that represent an approximation of the oscillations of the free fluid surface. A detailed study connecting Eq. (1) with (2, below) was performed by Krasnopolskaya and Shvets (1993 and 1994) invoking the Lagrangian averaging procedure over time. These authors assume $\eta_n \propto [p_n(\tau)\cos(\sigma(t) + q_n(\tau)sin(\sigma(t)], n = 1,2$ for dominant modes, where p_n, q_n, are amplitudes. Following these authors, we can write the following system of evolution equations for these amplitudes of the dominant modes:

$$\begin{cases} \dfrac{dp_1}{d\tau} = -\alpha p_1 - (\beta + AE - 2)q_1 + BMp_2 \\ \dfrac{dq_1}{d\tau} = -\alpha q_1 + (\beta + AE + 2)p_1 + BMq_2 \\ \dfrac{dp_2}{d\tau} = -\alpha p_2 - (\beta + AE - 2)q_2 - BMp_1 \\ \dfrac{dq_2}{d\tau} = -\alpha q_2 + (\beta + AE + 2)p_2 - BMq_1 \\ \dfrac{d\beta}{d\tau} = N_2 - N_1\beta - \mu(p_1q_1 + p_2q_2) \end{cases} \tag{2}$$

where τ is slow time (see Miles 1984a; Krasnopolskaya and Shvets, 1993, 1994), p_1, q_1, p_2, q_2 are amplitudes of the dominant modes, α is the coefficient of additional viscous damping forces acting on the liquid oscillations, and β is a tuning parameter, which measures the offset of frequencies. *A* and *B* are constant coefficients (Miles, 1984a) characterized by physical geometry, whose values depend on the diameter of the tank and the depth *d* (Fig.1) of the filled liquid in the tank. For example, if we assume that the tank is filled by fluid to the depth $d > 3a$, so, as shown by

Miles (1984b), A= 1.112 and B = -1.531. E and M are the energy and the angular momentum respectively of the vibrations of the fluid in the fundamental models:

$$E = E_1 + E_2$$
$$M = p_1 q_2 - p_2 q_1 \tag{3}$$

with

$$E_n = \frac{1}{2}\left(p_n^2 + q_n^2\right). \tag{4}$$

The last equation in the system (2) is obtained from the equation for the rotation of the shaft of the electric motor. We investigated the steady-state response and according Krononenko (1969), Krasnopolskaya and Shvets (1993 and 1994) we can write an approximation of the static characteristics of the engine. In this last equation N_1 is a constant of the linear static performance curve of the motor, N_2 is a function of the natural frequency of the fundamental of the free surface oscillations, and μ is a parameter in function of the natural frequency and physical characteristics of the motor. As we are interested in the steady-state response, the parameters (N_1, N_2, μ) are obtained of the static characteristic of the electromotor (Krononenko, 1969).

3. NUMERICAL ANALISYS

In this section we analyze the steady solutions of the equation system (2), which may represent equilibrium states, periodic, almost-periodic and chaotic solutions corresponding, respectively, in the five-dimensional phase-space (p_1, q_1, p_2, q_2, β) asymptotically to a point, a limit cycle, a limit torus and a chaotic attractor.

In the parameter space (α, A, B, N_2, N_1, μ) of the equation system (2), numerical experiments were carried out to find the regions of existence of chaotic solutions, and to investigate the transition from regular to chaotic regimes. The computational numerical method of solution used was the fourth-order Runge-Kutta. The system of equations (2) has six parameters, which together with the initial conditions determine its behavior in the steady regimes. We our simulations we assume these parameters and initial conditions equal to α = 0.8, A = 1.112, B = -1.531, N_2, = -0.25, μ = 4.5 and $p_1(0)=q_1(0)=0.1$, $p_2(0)=q_2(0)=1.0$, $\beta(0)=0$. The parameter N_1 was varied to determine all possible classes of asymptotic trajectories (point, curve, torus, attractor). The magnitude this parameter determines the energy losses in the electromotor. Varying the values of the parameter N_1, as for Krasnopolskaya and Shvets (1993 and 1994) the four main classes of steady-state regimes have been obtained. For example at N_1 = 1.95 and integration time τ =1,000, the phase-plane (p_2,q_2) projections of asymptotic trajectories are presented in Fig. 2 with the chaotic attractor has a two-cycle arrangement.

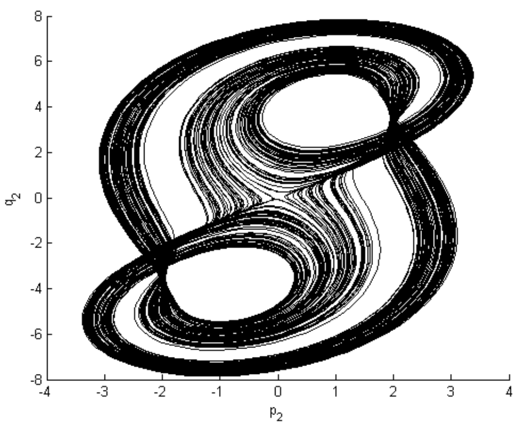

Figure 2.Phase-plane (p_2,q_2) projections of the trajectories at N_1 = 1.95.

Figure 3 shows the several classes of asymptotic trajectories when we vary the parameter N_1 for some values in the range from 0.1 to 5.0 with integration time of τ =500. We can see structures with several types of chaotic attractors during transition from regular to chaotic motion.

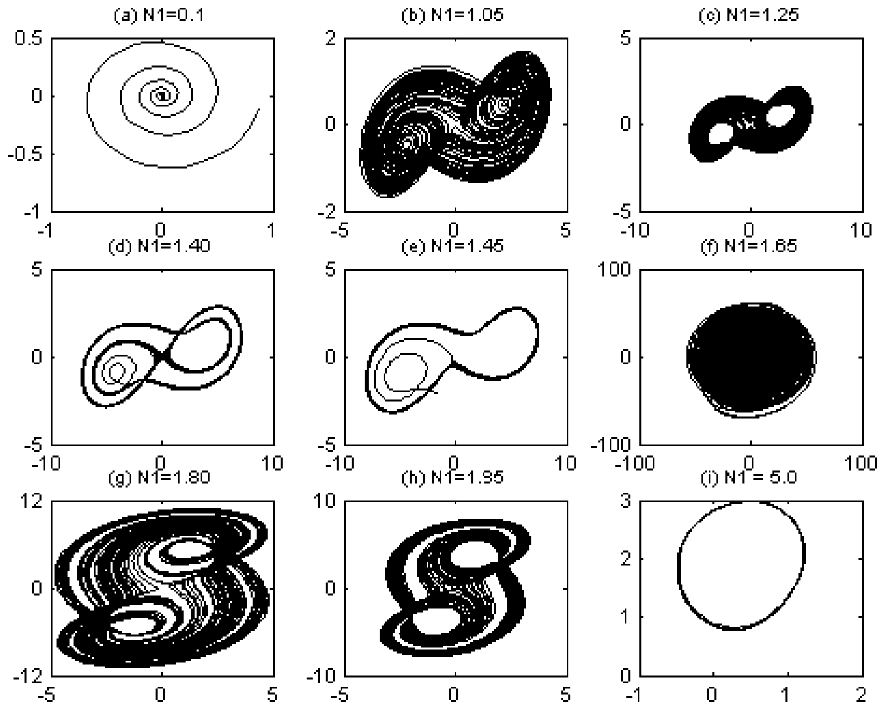

Figure 3.Phase-plane (p_2,q_2) projections of the trajectories at N_1: (a) 0.1; (b) 1.05; (c) 1.25; (d) 1,4; (e) 1.45; (f) 1.65; (g) 1.80; (h) 1.95; (i) 5.0.

In Fig. 3 we can see solid regions, for example, at N_1 = 1.05 the structure of the attractor becomes solid, without windows. Fig.4 illustrates in detailed the phase-plane (p_2,q_2) of this type of chaotic attractor.

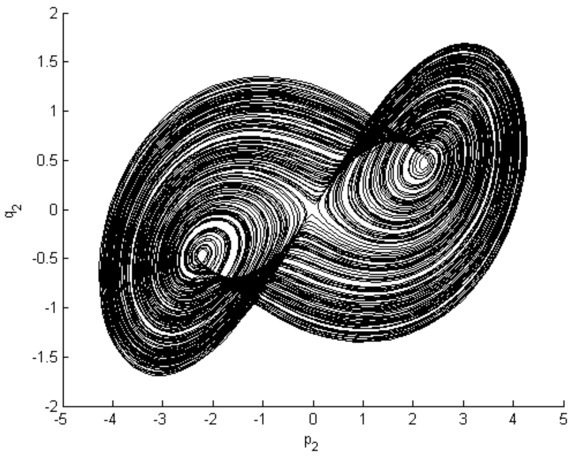

Figure 4.Phase-plane (p_2,q_2) projections of the trajectories at N_1 = 1.05.

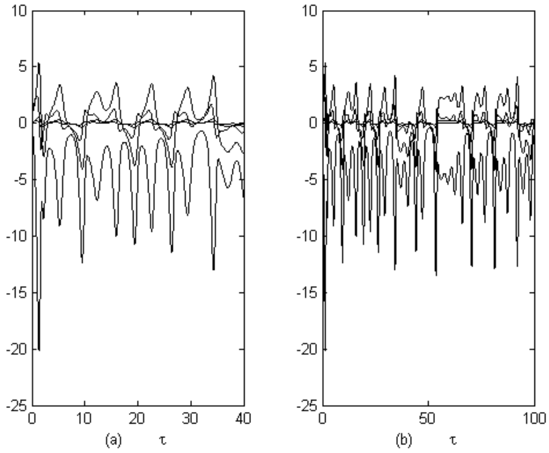

Figure 5. Steady solutions in five-dimensional phase-space (p_1, q_1, p_2, q_2, β) in function of the time (N_1 = 1.05).

Figure 5 illustrates the steady solutions in five-dimensional phase-space (p_1, q_1, p_2, q_2, β) in function of the time. The variables p_1 and p_2, and q_1 and q_2 are duals varying approximately from –0.4 to 0.6, from –4 to 6, from –-0.2 to 0.2, and from –2 to 2, respectively. The amplitudes of variables p_2, q_2 are ten times the values of amplitudes of variables p_1, q_1. The variable β varying from –15.0 to 0, approximately. This similarity between the projections of the phase portraits both for regular and for chaotic attractors is associated with the symmetry of the equation system (2) with respect to the variables p_2, q_2 and p_1, q_1.

The power spectrum log S versus the spectral frequency associated with the variable p_2 of the chaotic attractor at N_1 = 1.05 and an integration time of τ =500 is displayed in Fig. 6. We can see some peaks in this power spectrum that has a broadband character. This is a distinguishing characteristic of a chaotic solution.

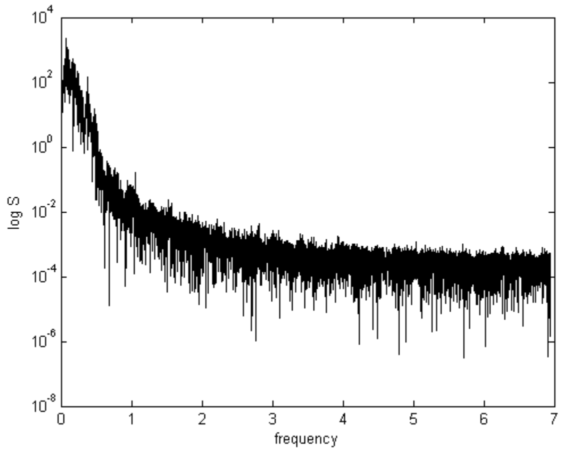

Figure 6. Power spectrum of p_2 at N_1 = 1.05

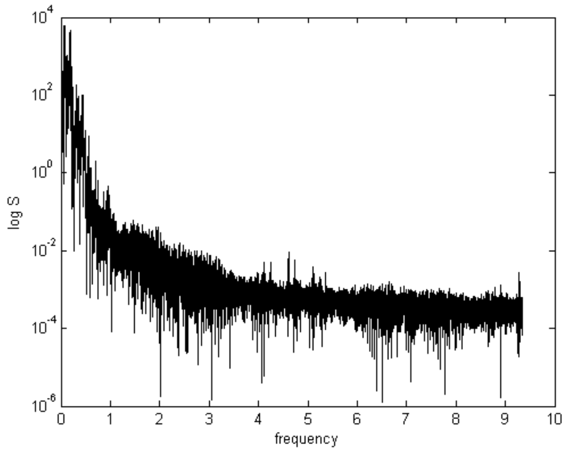

Figure 7. Power spectrum of p_2 at N_1 = 1.40

Figure 7 shows the power spectrum of variable p_2 of the chaotic attractor at N_1 = 1.40 and an integration time of τ =500. We note that the structural differences of the attractors are captured by spectral characteristics. In this case, the spectrum has not a continuous broadband character in all range of frequencies. It has distinct troughs at middle frequencies.

Now the Lyapunov exponents for this chaotic structure (N_1 = 1.80) are determined. Fig. 8 shows the dynamics of Lyapunov exponents. The algorithm employed for this computation was proposed by Wolf et al. (1985). Figure 8 shows the dynamics of Lyapunov exponents until integration time of τ =200. The Lyapunov exponents for the five variables (p_1, q_1, p_2, q_2, β) converge approximately to $\lambda_1 = 0.63$, $\lambda_2 = 0.07$, $\lambda_3 = -0.42$, $\lambda_4 = -2.0$, and $\lambda_5 = -2.54$, respectively.

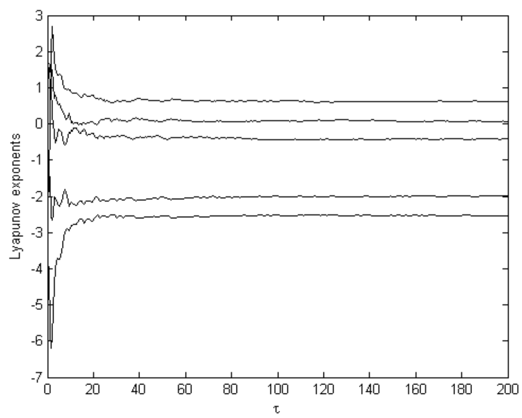

Figure 8. Dynamics of Lyapunov exponents at N_1 = 1.05.

The variation of the principal Lyapunov exponent for the various types of chaotic attractor is relevant because indicates an increase in the rate of divergence of nearby phase trajectories for more randomized chaotic attractors. For example, for some structures in Fig. 3, the principal Lyapunov exponent $\lambda_1 = 5.2$ at N_1 = 1.95. The value of $\lambda_1 = 9.1$ at N_1 = 1.80 and subsequently decreases to $\lambda_1 = 5.4$ at N_1 = 1.45.

4. FINAL CONSIDERATIONS

This paper presents an investigation the free surface oscillations of a fluid in a cylinder tank excited by a non-ideal power source. There is parametric resonance in this system modeled with five equations. As results we show the existence of several types of chaotic attractors during transitions from regular to chaotic motion. Some quantitative and qualitative characteristics of these various types of structures are described in detailed.

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