LOW TENSION CABLES: THE ANALYTICAL SOLUTION AND A POWER SERIES APPROACH

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Abstract. *As is known, low tension cables (or chains) adopt a catenary curve when subjected to self weight. The dynamic problem governing equations are derived in which the static problem of the slack cable is included. The cable is fixed at one end (A) while the other end (B), not fixed, is at a different level. The slope at the fixed end is assumed arbitrary. When this type of cable or chain is employed as a mooring device, it is necessary to know the forces for each geometric configuration specially at end B. First, the analytical solution is stated and from it values of the horizontal and vertical forces at end B may be found for different values of static displacements (q, horizontal and h, vertical) at that end. On the other hand, being this the main goal, a power series approach was employed to obtain explicit expansions of the horizontal and vertical forces in terms of q and h. The availability of these expansions allows us to introduce these forces in a quasi-static analysis of the general problem of a floating platform. The series may be introduced in the dynamic differential equations that govern the strongly non-linear motion of the platform.*

1. Introduction

Cables are employed in different type of structures such as roofs, bridges and as mooring devices (see for instance Esmailzadeh and Goodarzi (2001), Irvine (1992), Sannasiraj *et al.* (1998), Smith and MacFarlane (2001) and Tibert (1999)). Since cable or chain structures are in general very flexible, a geometrically nonlinear problem should be solved. Their highly non-linear behavior hardly can be modeled using a standard Galerkin technique. In structural analysis the finite element method is an extended tool. However special elements for cables or chains are often not available in commercial finite element programs. Usually the single cable is modeled by other elements but this approach may lead to numerical instabilities of the algorithm. Alternatively, analytical solutions may provide of more accurate and robust results. The cable non-linearity arises from the very low bending stiffness of the cable deriving in a strongly geometric nonlinearity rather than a material type. This situation results in difficulties when modeling both cable statics and dynamics.

It is thought that Galileo has been the first in address the curve of a suspended cable under its own weight by mid. S.XVII. However the equation of the *catenaria* (the curve assumed by a perfectly flexible cable supported at its ends and subjected to the gravitational forces) was derived by Leibniz, Huygens and John Bernoulli by 1691, responding to a challenge put out by James Bernoulli to find an expression of the chain curve, as Irvine (1992) refers.

The present paper deals with a power series approach to the solution of the cable end forces and the exact curve as an alternative to the catenary expression. The configuration studied is the following. The cable is assumed inextensible and fixed at one end (A) while the other end (B), not fixed, is at a different level. The slope at the fixed end is assumed arbitrary. When this type of cable or chain is employed as a mooring device, it is necessary to know the forces for each geometric configuration specially at end B. The motivation of this solution is to couple it to the study of the dynamics of a floating platform moored with slack cables. The latter was addressed with a cable quasi-static model for instance by Esmailzadeh and Goodarzi (2001) and Rosales and Filipich (2006). In both references the restriction of a null slope at one end was included. This assumption leads to a simplified algebra. This limitation is herein overcome since the tangent at the left end is assumed arbitrary.

First, the dynamic problem of the above-described cable is stated. The static case is obviously included in the governing equations. The well-known analytical solution arise from its solution. The finding of the forces at end B is of interest. On the other hand, being this the main goal, a power series approach was employed to obtain explicit expansions of the horizontal and vertical forces in terms of q and h (horizontal and vertical displacements, respectively). The authors have used this approach systematically in strong non-linear problems (Filipich *et al.* (2004) and Rosales and Filipich (2006)). The availability of these expansions allows us to introduce these forces in the quasi-static analysis of the general problem of a floating platform. The series may be introduced in the dynamic differential equations that govern the strongly non-linear motion of the platform.

2. Dynamics of a slack cable: governing equations

Figure 1 describes the cable under study where (X, Y) are fixed axes and (x, y) are related to the first system by $x = X + u$ and $y = Y + v$. u and v are displacements of a point $P(X, Y)$ ((arbitrarily) located at the reference axis $Y = 0$) at time t_0 to an instant position $P(x, y)$.

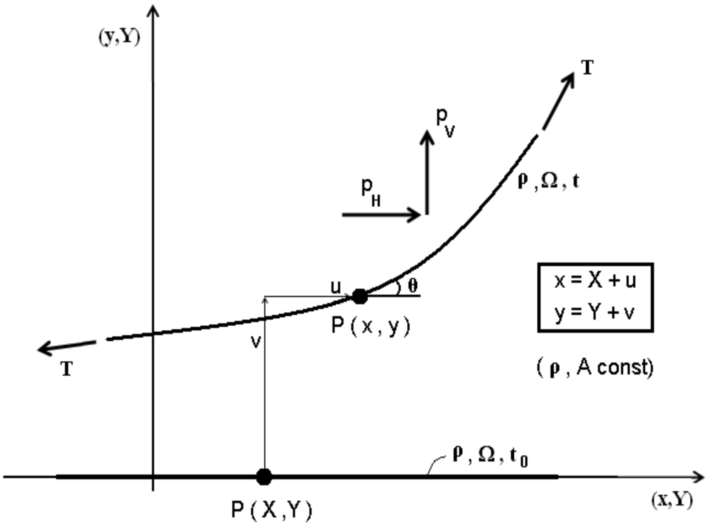

Figure 1. Cable configuration. Mass density per unit length ρ and cross-sectional area Ω are assumed constant.

Let us analyze a cable portion Δs as shown in Figure 2. Since the inextensibility of the cable is assumed, it is true that $\Delta s = \Delta X$. Also it is convenient to define $H = T \cos\theta$ (horizontal component) and $V = T \sin\theta$ (vertical component). The application of Newton's second law and the limit for $\Delta s \to 0$, lead the following equations that govern the dynamic equilibrium of an inextensible cable,

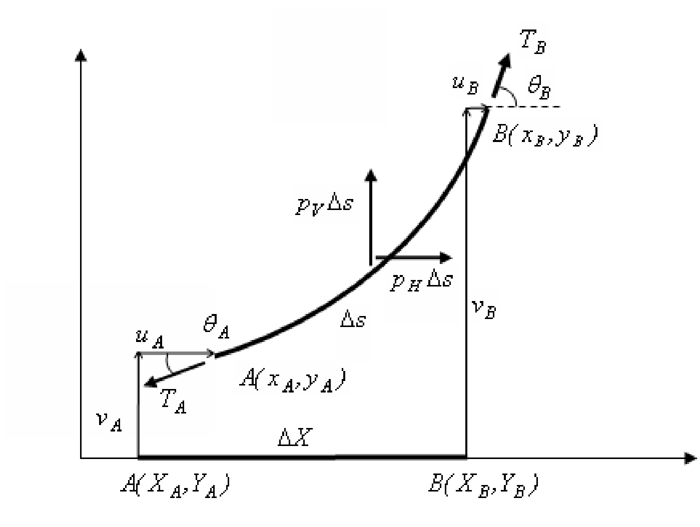

Figure 2. Cable portion Δs. Originally portion ΔX at $Y = 0$.

$$\begin{cases} H_X &= \rho\Omega\ddot{u} - p_H \\ V_X &= \rho\Omega\ddot{v} - p_V \end{cases} \tag{1}$$

where $(\cdot)_X \equiv \partial(\cdot)/\partial X$ and $\dot{(\cdot)} \equiv \partial(\cdot)/\partial t$. The horizontal and vertical components of an eventual (conservative) load are denoted p_H and p_V respectively. The vertical load may be separated into q_V, an eventual vertical external load and self-weight $\gamma\Omega$, i.e. $p_V = q_V - \gamma\Omega$. After geometric considerations, it may be deduced that

$$\cos\theta = \frac{dx}{ds} = \frac{dx}{dX}; \quad ; \quad \sin\theta = \frac{dy}{ds} = \frac{dy}{dX} \tag{2}$$

but since $x = X + u$ and $y = Y + v$, the following is true

$$\cos\theta = 1 + u_X; \qquad \sin\theta = v_X \tag{3}$$

If we define $(\cdot)' = \frac{d(\cdot)}{dx}$, we know that $(\cdot)_X = (\cdot)' x_X = (\cdot)'(1 + u_X) = (\cdot)'\cos\theta$ and the next statements can be deduced

$$\cos\theta = \frac{1}{1 - u'} \quad (a); \qquad \tan\theta = v' \quad (b) \tag{4}$$

Also, after using Pitagoras, we have

$$\cos^2\theta + \sin^2\theta = 1 \Rightarrow \qquad u_X + \frac{1}{2}(u_X^2 + v_X^2) = 0 \Rightarrow \qquad u' + \frac{1}{2}(v'^2 - u'^2) = 0. \tag{5}$$

The second of Eqs. (5) may be seen from other point of view. In effect this is equivalent to set the inextensibility condition $\epsilon_X = 0$. Recall that, in general, $0 < 1 + \epsilon_X = (1 + 2E_{XX})^{1/2}$, being $E_{XX} = u_X + \frac{1}{2}(u_X^2 + v_X^2)$. Then if $\epsilon_X = 0 \Rightarrow E_{XX} = 0$. The system of equations for the particular case of $p_V = -\gamma\Omega$ and $p_H = 0$ is written as

$$\begin{cases} H_X - \rho\Omega\ddot{u} = 0 \\ V_X - \rho\Omega\ddot{v} = \gamma\Omega \\ 2u_X + u_X^2 + v_X^2 = 0 \\ Hv_X - V(1 + u_X) = 0 \end{cases} \tag{6}$$

The last of Eqs. (6) is derived from the fact that at point P (see Fig. 1) $V/H = \tan\theta$ and making use of Eqs. (3). The unknowns of the problem are $u = u(x,t)$, $v = v(x,t)$, $H = H(x,t)$ and $V = V(x,t)$. The authors are at present working on the solution of this DAE (Differential-Algebraic equations) (Eqs. (6)). Since we are dealing with partial differential equations, a separation of variables is performed with a methodology named WEM (Whole Element Method) (Rosales and Filipich, 2002) which makes use of expanded series of trigonometric functions. A popular approach is the finite element method to solve the dynamic problem. However particular problems should be overcome when dealing with slack cables (see for instance Tibert (1999)).

In the next section, the static problem derived from the above-stated dynamic one, will be presented and solved in both a classical way and using a power series expansion.

3. Statics of a slack cable: derivation of the governing equations and solutions

If we assume $p_H = 0$ and $q_V = 0$ as well as neglect the inertia terms (i.e. $\ddot{u} = 0$ and $\ddot{v} = 0$) in Eq. (1), the static problem of a slack cable subjected to its own weight derives. Consequently

$$H_X = 0 \Longrightarrow H = \text{constant} \qquad (a); \qquad V_X = \gamma\Omega \qquad (b) \tag{7}$$

3.1 Analytical solution

From Eq. (7.b) and since $V = H\tan\theta = Hv'$ we obtain

$$(v')_X = \beta \Longrightarrow v" = \frac{\beta}{\cos\theta} = \beta\sqrt{1 + v'^2} \Longrightarrow v'' - \beta\sqrt{1 + v'^2} = 0 \tag{8}$$

where $\beta = \gamma\Omega/H$. After successive integrations and setting the boundary condition $v(0) = 0$, the following well-known solution yields,

$$v = \frac{1}{\beta}[\cosh(\beta x + C) - \cosh C] \tag{9}$$

From the inextensible condition of the cables, (last of Eqs. (5)) we obtain $u' = 1 \pm \sqrt{1 + v'^2}$. After imposing the boundary condition $u(0) = 0$ the solution for displacement $u(x)$ is obtained,

$$u(x) = x - \frac{1}{\beta}[\sinh(\beta x + C) - \sinh C] \tag{10}$$

Let us denote $r \equiv \cosh C$ and $s \equiv \sinh C$, and so $r = \sqrt{1 + s^2}$. Then the two solutions $u(x)$ and $v(x)$ write

$$v(x) = \frac{1}{\beta}[r\cdot(\cosh\beta x - 1) + s\cdot\sinh\beta x] \ ;(a) \qquad u(x) = x - \frac{1}{\beta}[r\cdot\sinh\beta x + s\cdot(\cosh\beta x - 1)] \quad (b) \tag{11}$$

At this point, the values of β and C remain unknown. It is true that at $x = a$, $v(a) = b$ and $u(a) = -(L_c - a)$ (see Figure 3), where $a = A + q$ and $b = B + h$, q and h are arbitrary right end displacements.

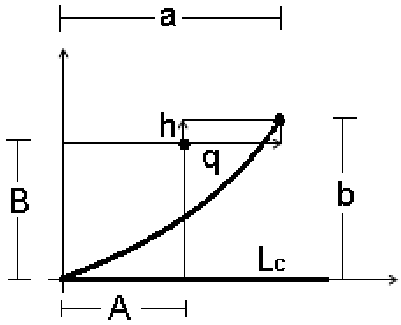

Figure 3. Cable positions. L_c is the cable length. A, B are the coordinates of the right end of the cable once positioned. a, b are the coordinates of the right end for arbitrary displacements q, h at this end.

From the Eqs. (11), we have:

$$r\cosh\beta a + s\sinh\beta a = \beta b + r \qquad (a) \qquad\qquad r\sinh\beta a + s\cosh\beta a = \beta L_c + s \qquad (b) \tag{12}$$

Now, in order to find β and C we will solve the following system in r and s (see Eqs. (12))

$$\begin{pmatrix} \cosh a - 1 & \sinh\beta a \\ \sinh\beta a & \cosh\beta a - 1 \end{pmatrix} \cdot \begin{pmatrix} r \\ s \end{pmatrix} = \beta \cdot \begin{pmatrix} b \\ L_c \end{pmatrix} \tag{13}$$

After some steps we end up with the next relationship that must be satisfied by β

$$\frac{\sinh\varphi}{\varphi} = \frac{\sqrt{L_c^2 - b^2}}{a} \tag{14}$$

in which $\varphi \equiv \beta a/2$ was introduced. Given $a = A + q$ and $b = B + h$ for certain values of q and h (see Fig. 3), the above transcendental equation is solved for φ and then from φ and β definitions, one is able to obtain the value of the horizontal component H of the tension at the right end of the cable. From the solution of system (13) it is possible to find C with the next expression

$$C = \text{arcsinh}\left(-\frac{\beta}{2}(L_c + pb)\right) \quad ; \qquad \text{with } \; p = -\coth\frac{\beta a}{2} \tag{15}$$

Once C is known the value of the vertical component V of the tension at the right end of the cable is obtained by calculating the following

$$V = Hv' \qquad\qquad \text{where} \qquad\qquad v' = \sinh(2\varphi + C) \tag{16}$$

With the steps above described, the problem of finding the horizontal and vertical H and V components of the tension T at the right boundary is solved. Obviously, the tension may be found at any point of the cable as well as the position of the cable for a given configuration (for any q and h) (Eqs. (9) and (10)).

3.2 Power series solution

Since our interest is to find H and V in terms of q and h, a power series approach will be proposed. Let us denote the right hand member of Eq. (14.a) as $f = f(q, h)$

$$f = f(q,h) = \frac{\sqrt{L_c^2 - b^2}}{a} = \frac{\sqrt{L_c^2 - (B+h)^2}}{A+q} = \frac{\sqrt{1 - \left(\frac{B+h}{L_c}\right)^2}}{\frac{A+q}{L_c}} \tag{17}$$

where the values of L_c, A, B (see figure 3) are data. Two variables w_n and w_d are introduced

$$w_n = \sqrt{1 - \left(\frac{B+h}{L_c}\right)^2} = \sum_{k=0}^{M} N_k h^k \qquad (a) \qquad ; w_d = \frac{1}{\frac{A+q}{L_c}} = \sum_{k=0}^{M} D_k q^k \qquad (b)$$

With this notation it is possible to find the coefficients of a series in terms of q and h

$$f(q,h) = w_n w_d = \sum_{i=0}^{M}\sum_{j=0}^{M} K_{ij} q^i h^j \tag{18}$$

where the K_{ij} may be found by a Taylor expansion. On the other hand, with $\varphi = \frac{\beta a}{2}$,

$$\frac{\sinh\varphi}{\varphi} = f(q,h) = \sum_{i=0}^{M}\sum_{j=0}^{M} K_{ij} q^i h^j. \tag{19}$$

A Taylor's expansion of the left-handmost member yields

$$F(\varphi) \equiv \frac{\sinh\varphi}{\varphi} = 1 + \frac{\varphi^2}{3!} + \frac{\varphi^4}{5!} + \frac{\varphi^6}{7!} + \dots \tag{20}$$

In turn, as is observed, an expansion of powers of φ is needed in terms of q and h. This is written as follows

$$\varphi^k = \sum_{i=0}^{M}\sum_{j=0}^{M} R_{kij} q^i h^j. \tag{21}$$

In particular $\varphi = \sum_{i=0}^{M}\sum_{j=0}^{M} R_{1ij} q^i h^j$. It is possible to find

$$R_{2ij} = \frac{A_{ij} - \sum_{k=4,6,\dots} \gamma_k R_{kij}}{\gamma_2} \tag{22}$$
$$\text{where} \quad \gamma_k = \frac{1}{(k+1)!}, \quad k = 2,4,6,\dots \quad \text{and} \quad A_{ij} \equiv (K_{ij} - \delta_{i0}\delta_{j0})$$

The coefficients R_{2ij}'s may be found by means of an iteration procedure and afterwards the R_{1ij}'s are found from the following expressions

$$R_{(n+2)ij} = \sum_{r=0}^{i}\sum_{s=0}^{j} R_{nrs} R_{2(i-r)(j-s)}, n = 2,4,6\dots \qquad R_{2ij} = \sum_{r=0}^{i}\sum_{s=0}^{j} R_{1rs} R_{1(i-r)(j-s)} \tag{23}$$

After working out this expression, the following expressions are found

$$\begin{cases} i = j = 0: \\ R_{100} = \sqrt{R_{200}} \\ \\ i = 0: \\ R_{10j} = \left[R_{20j} - \sum_{s_1}^{j-1} R_{10s} R_{10(j-s)}\right]/2R_{100} \\ \\ j = 0: \\ R_{1i0} = \left[R_{2i0} - \sum_{r_1}^{i-1} R_{1r0} R_{1(i-r)0}\right]/2R_{100} \\ \\ i \neq 0, j \neq 0: \\ R_{1ij} = \{R_{2ij} - [2R_{10j}R_{1i0} + S_{0s} + S_{r0} + S_{rs}]\}/2R_{100} \end{cases} \tag{24}$$

where the summations S_{lm} are

$$S_{0s} \equiv \sum_{s1}^{j-1}(R_{10s}R_{1i(j-s)} + R_{1is}R_{10(j-s)}) \ ; \qquad S_{r0} \equiv \sum_{r1}^{i-1}(R_{1r0}R_{1(i-r)j} + R_{1rj}R_{1(i-r)0})$$
$$S_{rs} \equiv \sum_{r1}^{i-1}\sum_{s1}^{j-1}(R_{1rs}R_{1(i-r)(j-s)})$$

This iterative procedure allows finding φ and its powers.

Now the value of H at the right end of the cable will be expressed in power series of q and h

$$H(q,h) = \sum_{i=0}^{M}\sum_{j=0}^{M} H_{ij} q^i h^j. \tag{25}$$

The H_{ij}'s may be found taking into account that $H = (\gamma\Omega)(A+q)/(2\varphi)$ and recalling φ and β definitions. The steps are analogous to the ones described above to find the R_{1ij}.

The vertical component V at the right end B of the cable may be obtained from expression $V_B = \gamma\Omega L + H\sinh C$. In order to arrive to an expression like $V = \sum_{i=0}^{M}\sum_{j=0}^{M} V_{ij} q^i h^j$ and find the V_{ij}'s one has to first expand the function $\sinh C$ with a power series in terms of q and h. For the sake of brevity the detailed algebra is not presented herein though the methodology is completely similar to the one employed up to this point. On the other hand the numerical results are presented for both component of the tension.

4. Numerical Results

The static problem of a cable of length $L = 47$ m, $A = 40$ m, $B = 20$ m, $\gamma\Omega = 50$ N/m is analyzed with both the analytical and the power series solution. Figure 4 depicts the variation of the components H and V and its resultant T found with the analytical solution.

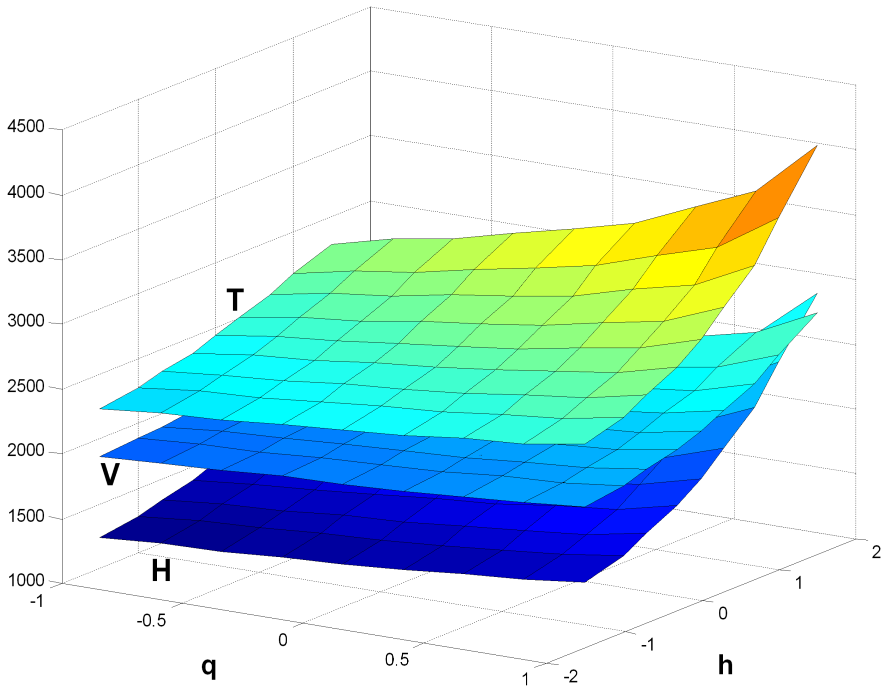

Figure 4. Values of horizontal (H) and vertical (V) components of the tension (T) at right end of the cable.

Now, and in order to validate the power series solution, a comparison of values of the variable H for different right-end displacements q, h, with results from the analytical solution is made. The results found with 6 terms ($M = 6$) are shown in Table 1. Next, and to show the strong rate of convergence of the power series, the values of H are found with the series taking 20 terms. The results are tabulated in Table 2. As may be observed, the error clearly decreases.

In order to show the convergence behavior of the series, Figure 5 graphics the rate of convergence found by increasing the number of terms of the power series. Fig. 5 shows the behavior of the vertical component V of the tension at the right end of the cable for end displacements $q = -1m$ and $h = -1.5$. Additionally the relative error (RE) was found for the tension T and its horizontal and vertical components H and V resp., for end displacements $q = 0.5$ m and $h = 0.7$ m ($RE = (F_{powerseries} - F_{analytical})/F_{analytical}$, in which F stands for T, H or V). The plots are shown in Fig. 6

5. Final comments

The governing equations governing the dynamics of a planar cable have been derived. The particular case of the static problems was studied in detail. The algebra to get the well-known analytical solution was included. Since the values of the tension T are of particular interest and specially at the right end of the table, a power series approach was employed to find the horizontal and vertical components H and V respectively, in terms of the horizontal and vertical displacements q and h. These expansions allow the coupling of the cable and other structures such as a floating platform with the advantage of fully consideration of the cable non-linearity without truncations during the derivation of the expressions. The numerical results and comparisons show the excellent behavior and strong convergence rate. The authors are at present dealing with the complete dynamic equations in which the separation of variables will be done by extended trigonometric series.

6. ACKNOWLEDGMENTS

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Table 1. Values of H (horizontal component of the tension at end B). Analytical (first line) and power series (second line) solutions and absolute error (third line). $M = 6$ for power series.

q	**h**				
	-1.5	-0.7	0	0.3	1.0
-1.0	1231.21877600921 1231.16274547663 0.05603053258	1284.48033656466 1283.94306606489 0.53727049977	1340.20934623680 1339.08492477732 1.12442145948	1367.35123947239 1365.74862500379 1.60261446860	1440.30832950405 1436.22914063641 4.07918886764
-0.5	1334.30760171697 1334.20396604485 0.10363567212	1400.67747201123 1400.66541925759 0.01205275364	1471.49261879978 1471.47130514346 0.02131365632	1506.52212261029 1506.49209767896 0.03002493133	1602.56540278897 1602.47586586772 0.08953692125
0	1459.07077679911 1458.78431394112 0.28646285799	1544.32219807111 1544.31793781021 0.00426026090	1637.71957882454 1637.71890155521 0.00067726933	1684.92453865689 1684.92392464405 0.00061401284	1818.09315231654 1818.05217538067 0.04097693587
0.50	1614.94082521587 1613.93424570183 1.00657951404	1729.08746747598 1729.06159743322 0.02587004276	1858.95306615241 1858.92333905975 0.02972709266	1926.70940686528 1926.66590645836 0.04350040692	2126.56788962459 2126.28961594003 0.27827368456
1.00	1818.42506778035 1813.90753613644 4.5175316439099	1980.75497850702 1979.64759339656 1.1073851104600	2176.55546015110 2174.12905389491 2.4264062561897	2284.13883981463 2280.49583546429 3.6430043503401	2627.82023813940 2616.10864292883 11.7115952105701

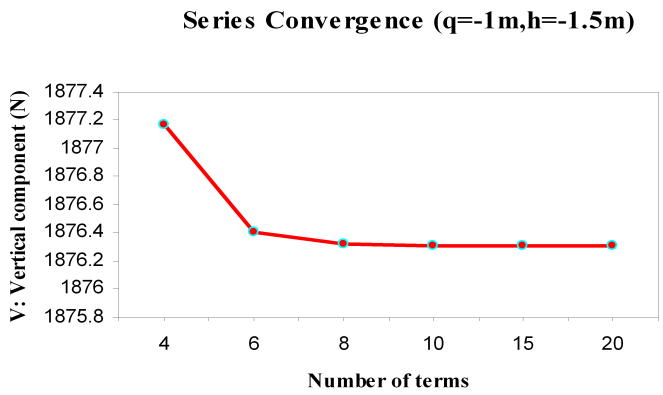

Figure 5. Convergence rate of power series. Vertical component V.

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Table 2. Values of H (horizontal component of the tension at end B). Analytical (first line) and power series (second line) solutions and absolute error (third line). $M = 20$ for power series.

q	**h**				
	-1.5	-0.7	0	0.3	1.0
-1.0	1231.21877600921 1231.21879969065 0.00002368144	1284.48033656466 1284.48034982982 0.00001326516	1340.20934623680 1340.20935316770 0.00000693090	1367.35123947239 1367.35124170462 0.00000223223	1440.30832950405 1440.30823465208 0.00009485197
-0.5	1334.30760171697 1334.30761191396 0.00001019699	1400.67747201123 1400.67747792172 0.00000591049	1471.49261879978 1471.49262219929 0.00000339951	1506.52212261029 1506.52212522238 0.00000261209	1602.56540278897 1602.56540409754 0.00000130857
0	1459.07077679911 1459.07078113885 0.00000433974	1544.32219807111 1544.32220037860 0.00000230749	1637.71957882454 1637.71958002143 0.00000119689	1684.92453865689 1684.92453952841 0.00000087152	1818.09315231654 1818.09315268978 0.00000037324
0.50	1614.94082521587 1614.94082580665 0.00000059078	1729.08746747598 1729.08746823543 0.00000075945	1858.95306615241 1858.95306649124 0.00000033883	1926.70940686528 1926.70940709275 0.00000022747	2126.56788962459 2126.56788969978 0.00000007519
1.00	1818.42506778035 1.818.42488860364 0.00017917671	1980.75497850702 1980.75497845012 0.00000005690	2176.55546015110 2176.55545712564 0.00000302546	2284.13883981463 2284.13882938468 0.00001042995	2627.82023813940 2627.81995970836 0.00027843104

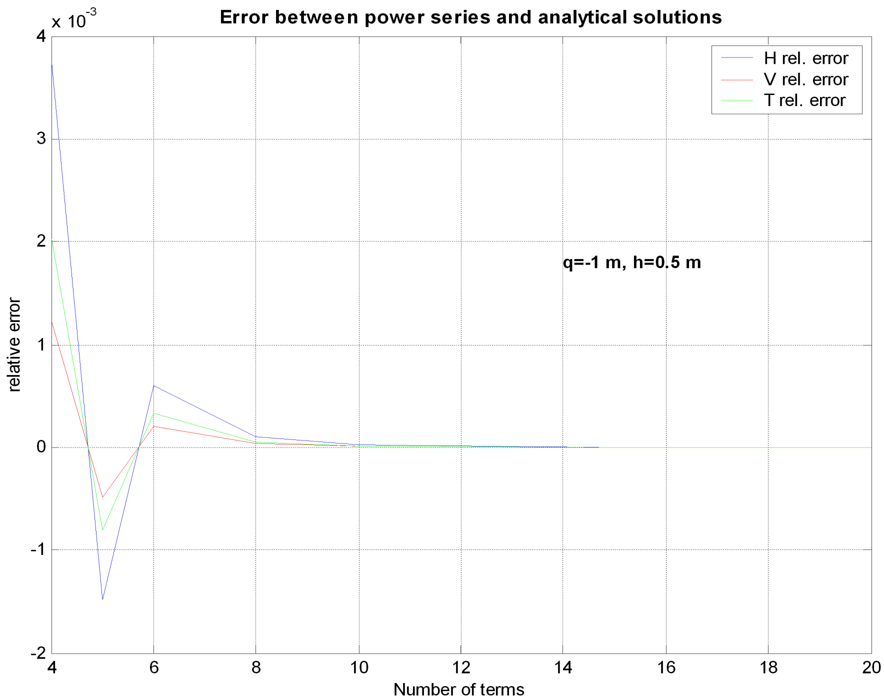

Figure 6. Relative error in tension T, horizontal (H) and vertical (V) components.