APPROACH TO DETERMINE THE PARAMETRIC ERRORS OF CO-ORDINATE MEASURING MACHINES (CMMs) USING RESPONSE SURFACE METHODOLOGY

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Abstract. In this paper, an approach to establish the volumetric accuracy of CMMs, which permits interpolation between measured points, is presented. Mathematical and statistical techniques, such as Response Surface Methodology, have been used to represent the relationship between each volumetric error component (Ex, Ey and Ez) at a point, within the measuring volume, and the co-ordinates of the point (X_i, Y_i, Z_i). Also, a method to derive the parametric errors of a CMM from the volumetric error data was developed and applied. The measured data obtained by applying a modular space frame on a three-axis Coordinate Measuring Machine (CMM) are used to fit mathematical models that represent each volumetric error component of the machine under test, then, the parametric errors of that machine are derived from the volumetric error data.

Keywords: Coordinate Measuring Machines, Parametric Errors, Response Surface Methodology

1. INTRODUCTION

Coordinate measuring machines (CMMs) are in widespread use as precision measurement tools which are particularly useful when the component to be measured has a complex shape (Peggs et.al. 1999) [1]. To satisfy traceability requirements of most industrial quality system, CMM must be periodically evaluated. That is an essential condition for analysing whether the CMM maintains the manufacturer specifications. Also, evaluation of CMM performance is necessary for obtaining correct measuring results. However, it is important to note that CMM performance evaluation is rather complicated as CMMs are more complex measuring device then most conventional measuring instruments (Silva and Burdekin, 2002).This paper present a methodology to deriving the parametric errors of a co-ordinate measuring machines from the mathematical models fitted to represent the volumetric error components. These error components are obtained by measuring a novel form of space frame, which has been designed and manufactured as part of this research work (Silva and Burdekin, 2002). This space frame has the form of a tetrahedron, which contains a sphere at each apex. The base of the tetrahedron comprises a ball plate that contains three spheres. Each tetrahedron contains three magnetic ball links. A simple magnetic ball link comprises a link, connecting magnetically, to two spheres. One sphere is located on the ball plate and the other at a space point where three links are connected together, figure 1.

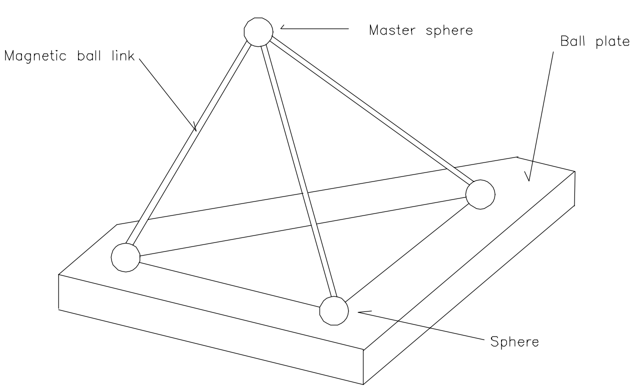

Figure 1. Modular space frame

2. GEOMETRIC ERRORS OF CMMS.

The geometric error of a CMM is the error induced at the probe position caused by dimensional and form errors of the machine components such as a slide. The geometric deflections of movement of a machine component can be defined by 6 parameters. This is similar to a body in a space that has six degrees of freedom, as defined by Weck (1980) . These are: 3 translational and 3 rotational errors as shown in figure 2.

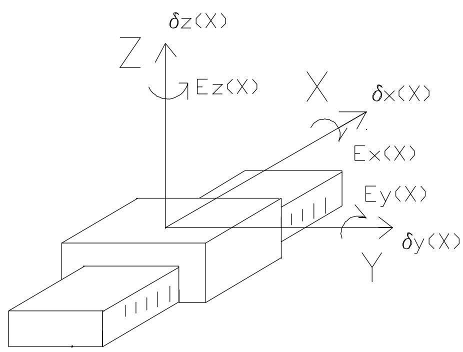

Figure 2. Parametric errors of a guideway moving along X axis.

When a machine component moves on the machine guideway it will experience geometric errors as result of existing geometric inaccuracy between the machine component and the guideway. In this research, for convenience of notation, the translational errors is represented by δ and the rotational errors by E. In addition, two characters are used. The first one indicates the direction of the error and the second one indicates the direction of movement. For instance, $\delta_x(Y)$ means the straightness error component in the X direction when moving along the Y axis.

a) Positional errors

Generally, the positional error is the scale error plus the Abbe error (Bryan, 1979) which arise from the Abbe offset and associated angular errors. Hence, the magnitude and direction of the positional error depend upon the location of measurement within the measuring volume.

b) Straightness errors

The out straightness of the guideway gives straightness error in the movement of the machine element that is moving on the guideway. The straightness errors are influenced by the associated rotational errors on the machine. There are two cases: the horizontal straightness error, $\delta_y(X)$, and the vertical straightness error $\delta_z(X)$, respectively along the X axis. Similarly, $\delta_x(Y)$, $\delta_z(Y)$ and $\delta_x(Z)$, $\delta_y(Z)$ are defined along the Y and the Z axis, respectively. The 6 straightness errors are thus considered in a 3 axis machine.

c) Rotational (or angular) errors.

There are 3 rotational errors along the guideway (or the X axis). If the rule of the right hand screw is adopted to describe the rotational movements, the direction of feed determines the rotation axis. The Roll error, $E_x(X)$, is associated with the rotation about the guideway. The pitch error, $E_y(X)$, is associated with the rotation about the horizontal transverse direction (direction of the Y axis). The Yaw error, $E_z(X)$, is associated with the rotation about the vertical axis (the Z axis). It should be that these rotational errors contribute to the total volumetric error with the Abbe offset. Similarly, the rotational errors can be introduced along the Y and Z axes. They are Roll errors ($E_y(Y)$, $E_z(Z)$), Pitch errors ($E_x(Y)$, $E_y(Z)$) and Yaw errors ($E_z(Y)$, $E_x(Z)$), along the Y and Z axis, respectively. Thus, 9 rotational errors are considered in a 3 axis machine.

d) Squareness errors

When the multi axis movement is introduced, the misalignment of each axis gives the squareness error (or orthogonality error). In the three axis machine, three squareness (or orthogonality) errors are defined in the XY, YZ and XZ plane. The planar squareness error, α, between X and Y axis, is defined as the out of squareness between axes X and Y. The vertical squareness error, β_1, is defined as the out of squareness between the axes X and Z. The vertical squareness error, β_2, is defined as the out of squareness between the axes Y and Z. All the squareness errors are considered as positive (+) when they are outward from the right angle (90 degree).

Therefore, twenty-one error components have to be considered in a three axis machine. They are: three positional errors, six straightness errors, nine rotational errors and three squareness errors. Each error component, such as yaw, roll, pitch, straightness, squareness and positioning error, is measured by conventional measuring equipment, for example, laser interferometer, electronic level, straight edge and square. It is important to note that the conventional techniques used to measured the parametric error components of a machine is time consuming, requires expensive equipment and special skill to operate that equipment, for instance, laser interferometer system and not included the errors of software and probe (Lee and Ferreira, 2002); (Umetsu et al., 2005) ; (Schwenke et al., 2005).

Therefore, a critical need exists in order to overcome disadvantages that existing techniques, to measured the parametric errors of CMMs, present. In this regard, it is necessary to developed a new technique that is capable of determining the parametric errors from the volumetric error components E(x), E(y) and E(z). Also, the new technique should require a minimum number of mechanical transfer standard and should be simple to use and measure. This paper presents a new approach to determine the parametric errors of CMMs by using mathematical models fitted to represent the volumetric error components of the machine under test.

3. FITTING MATHEMATICAL MODELS TO REPRESENT THE VOLUMETRIC ACCURACY

Response Surface Methodology (RSM) has been applied to fit a mathematical model to represent the volumetric errors of CMMs. RSM, is a collection of mathematical and statistical techniques that are useful for the modelling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimise this response (Montgomery, 1991),(Box et. al. 1978). By comparing the calibrated and measured tetrahedron configurations of the modular space frame it is possible to determine the volumetric error components (Ex_i,Ey_i,Ez_i) of the machine under test. The volumetric error at each point defined by the modular space frame is given as follows:

$$Ex_i=X_{mi} - X_i \tag{1}$$
$$Ey_i=Y_{mi} - Y_i \tag{2}$$
$$Ez_i=Z_{mi} - Z_i \tag{3}$$

where,

X_i,Y_i,Z_i, are the calibrated co-ordinates of the points generated by the calibrated modular space frame.

X_{mi},Y_{mi},Z_{mi}, are the measured co-ordinates of the points generated by the measured modular space frame.

The general equation that represents each volumetric error component (Ex, Ey, Ez) can be written either by using a first-order mathematical model, that is,

$$E_k(X_1, X_2, X_3) = \beta_{ko} + \beta_{k1}X_1 + \beta_{k2}X_2 + \beta_{k3}X_3 \tag{4}$$

or by using a second-order mathematical model,

that is,

$$\begin{aligned} E_k(X_1, X_2, X_3) = \beta_{ko} + \beta_{k1}X_1 + \beta_{k2}X_2 + \beta_{k3}X_3 + \beta_{k4}X_1^2 + \beta_{k5}X_2^2 + \beta_{k6}X_3^2 \\ + \beta_{k7}X_1X_2 + \beta_{k8}X_1X_3 + \beta_{k9}X_2X_3 \end{aligned} \tag{5}$$

where, k=x, y or z

In both equation (1) and (2) the coefficients β's are to be estimated by the method of least squares where the basic formula is given by the following equation:

$$\beta_k = (XX^T)^{-1} X^T Y_k \quad (6)$$

where,
k=x, y or z and it is related to X, Y and Z direction , respectively.
Y$_k$= the vector of error component Ex, Ey or Ez in X, Y or Z direction, respectively.
X^T= Transposed of matrix **X**
X= the matrix of independent or predictor variables X_1, X_2 and X_3.
X_1, X_2, X_3 = coded co-ordinates of the ith experimental point in the X, Y and Z direction, respectively.

3.1 Deriving the parametric errors from the mathematical models fitted to represent the volumetric error components.

Two cases have been considered to represent the volumetric error component in the X, Y and Z direction. Initially a first-order mathematical model was fitted. Next, a second-order mathematical model was developed. In both cases the residuals (Y_i - Y_{if}) were plotted against the fitted value, Y_{if}, obtained from the fitted mathematical model. By analysing the residual plots as shown in Silva and Burdekin (2002) it was found that the second-order model more adequately represents the measured data. The analysis of variance concerning the second-order mathematical model was established and it was observed that the overall regression is statistically significant. Therefore, the second-order mathematical model has been selected to represent the volumetric error component in the X, Y and Z direction (Silva and Burdekin , 2002).

Once the mathematical models that represent the volumetric error components (Ex, Ey, Ez) of a CMM has been fitted, it is possible to derived the parametric errors of the machine under test. To achieve this objective a particular method has been developed and applied as part of this research. This method is founded on the physical meaning of each parametric error component. Basically, the following steps must be performed when applying the proposed method. First, a reference plane within the measuring volume of the machine is selected. Second, measurement lines on the reference plane are defined. These measuring lines are defined by taking into account the physical meaning of each parametric error to be determined. Third, the boundary conditions are applied on the fitted equations that represent the volumetric error components(Ex, Ey, Ez). The parametric errors can be derived as following:

- Positioning error in X direction, $\delta_x(X)$, at plane Z=0.
  Boundary conditions: Z=0, Y=0, 0<=X<=X_{max}, Then,

$$\delta_X(X)\big|_{Z=0;Y=0} = Ex(X, Y=0, Z=0) \quad (7)$$

- Positioning error in Y direction, $\delta_y(Y)$, at plane Z=0.
  Boundary conditions: Z=0, X=0, 0<=Y<=Y_{max} , Then,

$$\delta_Y(Y)\big|_{Z=0;X=0} = Ey(X=0, Y, Z=0) \quad (8)$$

- Positioning error in Z direction, $\delta_z(Z)$, at plane X=0.
  Boundary conditions: X=0, Y=0, 0<=Z<=Z_{max} , Then,

$$\delta_Z(Z)\big|_{X=0;Y=0} = Ez(X=0, Y=0, Z) \quad (9)$$

b) Straightness errors

- Straightness error in the X direction when moving along the Y axis, $\delta_x(Y)$, at plane Z=0.
  Boundary conditions: Z=0, X=0, 0<=Y<=Y_{max} Then,

$$\delta_X(Y)\big|_{Z=0;X=0} = Ex(X=0, Y, Z=0) \quad (10)$$

- Straightness error in the direction Y when moving along the X axis, $\delta_Y(X)$, at plane Z=0.
  Boundary conditions: Z=0, Y=0, 0<=X<=X_{max} , Then,

$$\delta_Y(X)\big|_{Z=0;Y=0} = Ey(X, Y=0, Z=0) \quad (11)$$

- Straightness error in the direction X when moving along the Z axis, $\delta_X(Z)$, at plane Y=0.
Boundary conditions: X=0, Y=0, 0<=Z<=Z_{max} ,Then,

$$\delta_X(Z)\Big|_{Y=0;X=0} = Ex(X=0,Y=0,Z) \tag{12}$$

- Straightness error in the Y direction when moving along the Z axis, $\delta_Y(Z)$, at plane X=0.
Boundary conditions: X=0, Z=0, 0<=Y<=Y_{max} , then,

$$\delta_Y(Z)\Big|_{X0;Y=0} = Ey(X=0,Y=0,Z) \tag{13}$$

- Straightness error in the direction Z when moving along the X axis, $\delta_Z(X)$, at plane Y=0.
Boundary conditions: Y=0, Z=0, 0<=X<=X_{max} , then,

$$\delta_Z(X)\Big|_{Y0;Z=0} = Ez(X,Y=0,Z=0) \tag{14}$$

- Straightness error in the direction Z when moving along the Y axis, $\delta_Z(Y)$, at plane X=0.
Boundary conditions: X=0, Z=0, 0<=Y<=Y_{max} , then,

$$\delta_Z(Y)\Big|_{X=0;Z=0} = E_Z(X=0,Y,Z=0) \tag{15}$$

c) Angular errors

- Pitch and Yaw errors

In this research, a method to determine pitch and yaw errors has been applied. This method consists in deriving the positioning error along two measuring lines that are parallel to the axis of motion. A distance, h, in the appropriated orthogonal distance, separates these measuring lines. Pitch and yaw errors can be calculated by using the following equations:

- Pitch error in X axis

$$\begin{aligned} E_Y(X) &= \frac{\delta_X(X)\big|_{Z=h,Y=0} - \delta_X(X)\big|_{Z=0,Y=0}}{h} \\ &= \frac{Ex(X,Y=0,Z=h) - Ex(X,Y=0,Z=0)}{h} \end{aligned} \tag{16}$$

Pitch errors in Y and Z axes can be determined following similar procedure.

- Yaw error in X axis

$$\begin{aligned} E_Z(X) &= \frac{\delta_X(X)\big|_{Y=h,Z=0} - \delta_X(X)\big|_{Y=0,Z=0}}{h} \\ &= \frac{Ex(X,Y=h,Z=0) - Ex(X,Y=0,Z=0)}{h} \end{aligned} \tag{17}$$

Yaw error in Y and Z-axes can be determined following similar procedure.

- Roll error in X axis

The roll error of the X axis can be derived by considering the straightness error, $\delta_Y(X)$, on two parallel planes, which are separated by an orthogonal distance, h. That error can be calculated by the following equation:

$$E_X(X) = \frac{\delta_Y(X)\big|_{Z=h,Y=0} - \delta_y(X)\big|_{Z=0,Y=0}}{h}$$
$$= \frac{Ey(X,Y=0,Z=h) - Ey(X,Y=0,Z=0)}{h} \qquad (18)$$

Roll errors in Y and Z axes can be determined following similar procedure.

d) Squareness errors between axes of motion

In this research, a method to determine the squareness errors α, β_1 and β_2 in the planes XY, XZ and YZ, respectively, has been proposed. Basically, this method uses the straightness errors, which are obtained from the mathematical models fitted to represent the volumetric error components. To describe this method, let us consider the straightness errors $\delta_x(Y)$ and $\delta_y(X)$ at plane XY. The slope of the best fit least square line to the curves defined by $\delta_y(X)$ and $\delta_x(Y)$ are θ_{yx} and θ_{xy} , respectively. The squareness error α is given by the following equation:

$$\alpha=\theta_{yx}-\theta_{xy} \qquad (19)$$

Similarly, by applying the same method the squareness errors β_1 and β_2 can be determined. These errors are defined as follows:

$$\beta_1=\theta_{xz}-\theta_{zx} \qquad (20)$$

$$\beta_2=\theta_{yz}-\theta_{zy} \qquad (21)$$

where,

θ_{xz},θ_{zx},θ_{yz} and θ_{zy} are the slopes of the best fit least squares line to the curves defined by $\delta_x(Z)$, $\delta_z(X)$, $\delta_y(Z)$ and $\delta_z(Y)$, respectively.

3.2 Practical application of the proposed method for determining the parametric errors of a tree-axis CMM

The approach developed in this research was applied on a numerically controlled co-ordinate measuring machine of the moving bridge type. The machine has XYZ travels of 600x500x400 mm, respectively. Basically, the machine construction comprises: a granite surface table which has a matrix of threaded holes (M10) which is used to locate and clamp components to be measured; X axis guideway that consists of a granite straight edge bonded on the CMM table; Y axis guideway and a Z axis spindle which are both made from ceramic material. The volumetric error data obtained from that practical application are used to establish mathematical models to represent the volumetric error components of the CMM. Also, based on the background described by Box (1978) the adequacy of the fitted mathematical models is performed as shown in (Silva and Burdekin , 2002). The mathematical models that have been obtained by applying the proposed approach on a three axis CMM are:

$$\begin{aligned} Ex(X_1,X_2,X_3) = &-4.7342 - 2.9952X_1 - 2.87391X_2 - 3.4347X_3 \\ &+1.4115X_1^2 - 2.5520X_2^2 - 1.4867X_3^2 \\ &+9.3441X_1X_2 - 1.9188X_1X_3 - 1.6712X_2X_3 \end{aligned} \qquad (22)$$

$$\begin{aligned} Ey(X_1,X_2,X_3) = &\ 4.3684 - 4.3144X_1 + 2.7010X_2 + 2.7762X_3 \\ &+1.0181X_1^2 + 1.2166X_2^2 - 1.3125X_3^2 \\ &-2.37081X_1X_2 + 1.8673X_1X_3 + 8.2472X_2X_3 \end{aligned} \qquad \mathbf{(23)}$$

$$Ez(X_1, X_2, X_3) = -5.8200 - 1.3586X_1 - 2.4692X_2 + 2.5301X_3 \\ +5.7018X_1^2 + 0.0191X_2^2 + 0.2181X_3^2 \\ +2.7005X_1X_2 - 4.7821X_1X_3 + 0.1136X_2X_3 \tag{24}$$

The parametric errors are obtained by using the methodology presented in the section 2 along with the mathematical models represented by the equations 22, 23 and 24. Figures 3, 4 and 5 show the positioning errors,$\delta_X(X)$, $\delta_Y(Y)$, $\delta_Z(Z)$ in X, Y and Z direction, respectively. The straightness errors $\delta_Y(X)$, and $\delta_Z(X)$ when moving along the X axis are shown in figures 6 and 7, respectively. The straightness errors, $\delta_X(Y)$ and $\delta_Z(Y)$ when moving along the Y axis are shown in figures 8 and 9. Also, Figures 10 and 11 show, respectively, the straightness error, $\delta_X(Z)$ and $\delta_Y(Z)$ when moving along the Z axis.

In figures 12 and 13 are shown the pitch errors $E_y(X)$ and $E_x(Y)$ when moving along the X and Y axes, respectively. These errors were calculated based on the method described in section 4 which utilizing the positoning errors $\delta_X(X)$ and $\delta_y(Y)$, respectively, calculated at Z=0 and Z=100.

The squareness errors of the CMM, on which the modular space frame has been applied, were calculated following the method described in section . The squareness errors of the CMM were found to be:

squareness error in plane XY α= -0.05 arc sec
squareness error in plane XZ β_1= -5.54 arc sec
squareness error in plane YZ β_2= -9.80 arc sec

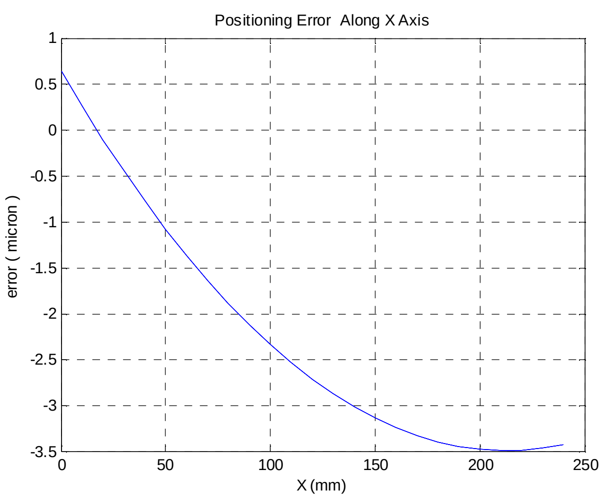

Figure 3 Positioning error, $\delta_x(X)$.

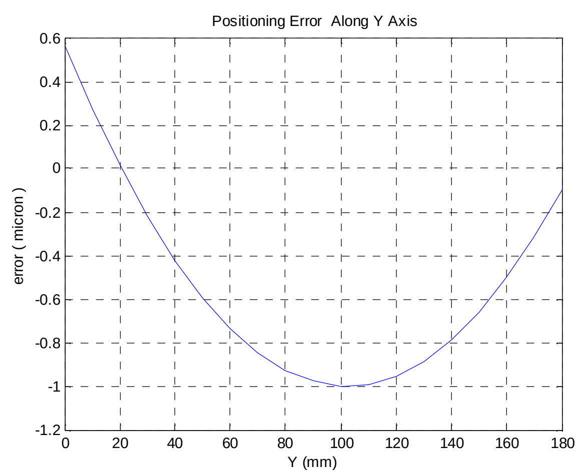

Figure 4 Positioning error, $\delta_y(Y)$.

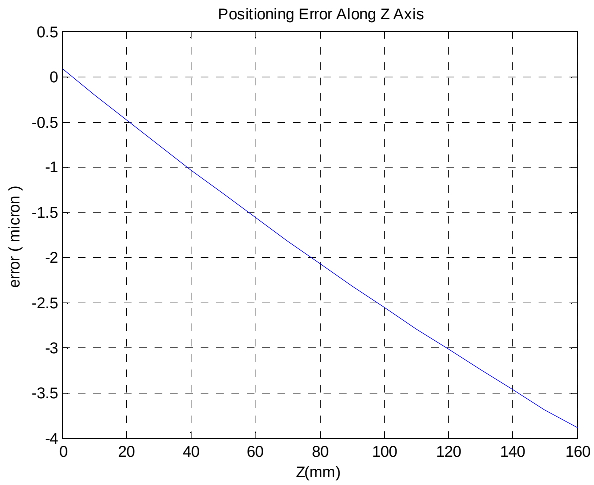

Figure 5 Positioning error, $\delta_z(Z)$.

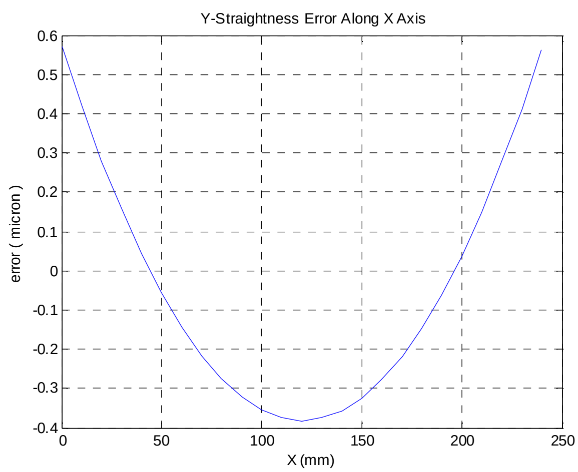

Figure 6 Straightness error, $\delta_y(X)$.

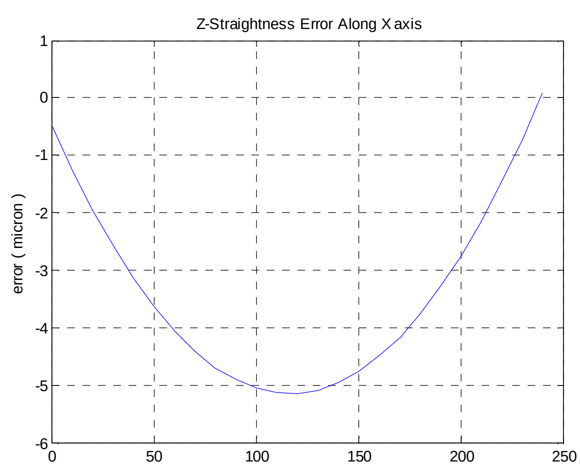

Figure 7 Straightness error, $\delta_z(X)$.

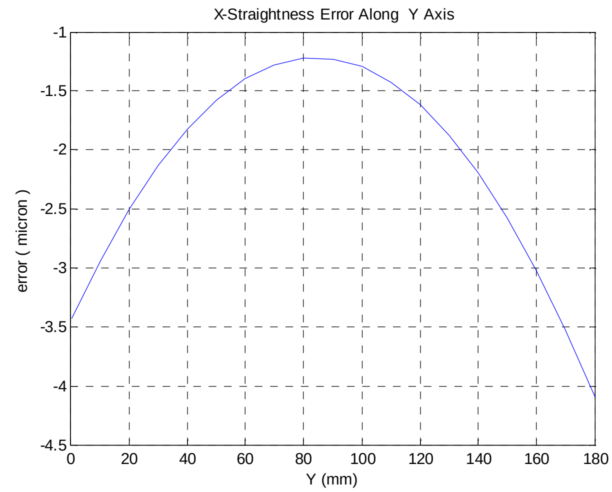

Figure 8 Straightness error, $\delta_x(Y)$.

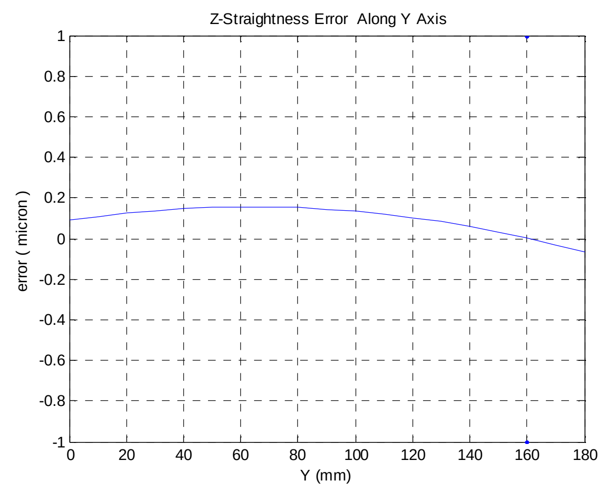

Figure 9 Straightness error, $\delta_z(Y)$.

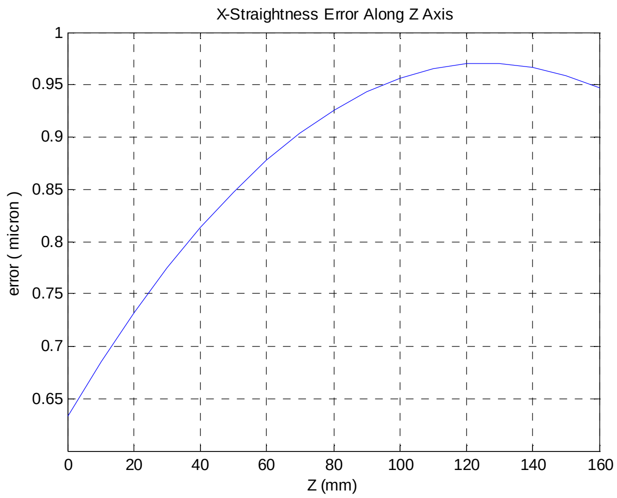

Figure 10 Straightness error, $\delta_x(Z)$.

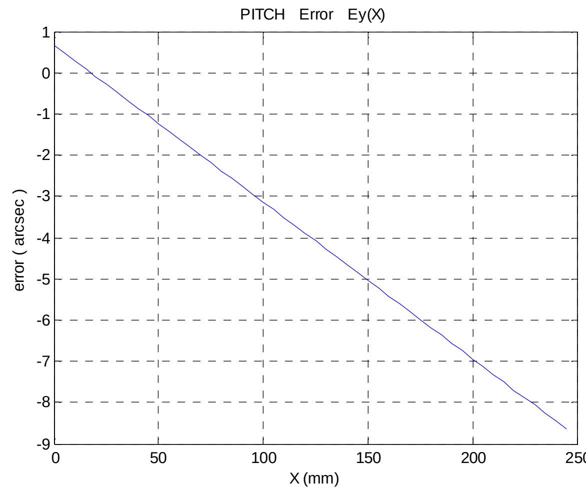

Figure 12 Pitch error, $E_y(X)$.

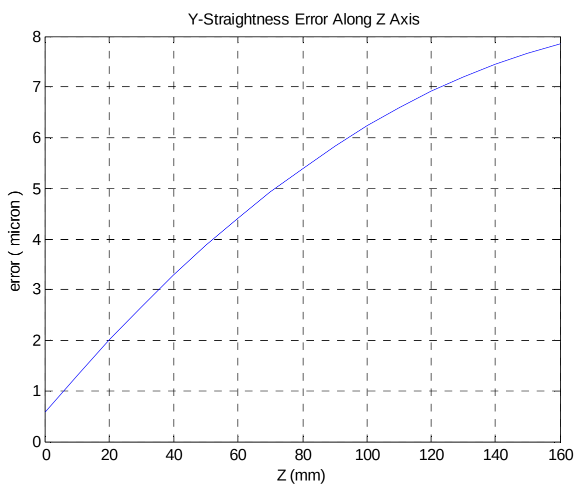

Figure 11 Straightness error, $\delta_y(Z)$.

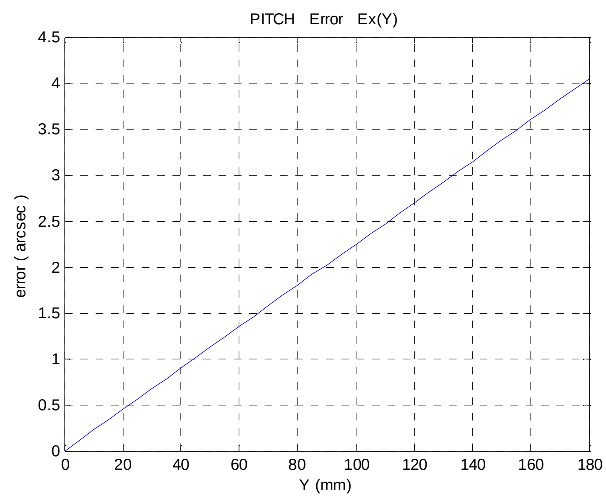

Figure 13 Pitch error, $E_x(Y)$

4. CONCLUSIONS

Response surface methodology (RSM) used in this research to fit mathematical models, which represent the volumetric error components, has proved to be an important and effective technique and that a second-order mathematical model more adequately represents the volumetric error data obtained by measuring the modular space frame. The developed method to derive the parametric errors from the volumetric error data constitutes an efficient and rapid tool in diagnosing the sources of error of a CMM. Additionally, the technique developed in this research does not have to assume that the CMM under investigation has to behave as a rigid body kinematic system and it can be applied to any type of three axis coordinate measuring machines (CMMs). Further research work will be carried out in order to applied the present technique for five-axis CMMs. Therefore, the research will focus on the design and implementation of a new 3D space frame which should be capable to determine the measurement uncertainty of five-axis CMMs..

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