INVESTIGATION ON HEAT RANSFER IN LAMINAR LIQUID FILM FLOW

Stasys Sinkunas, stasys.sinkunas@ktu.lt
Jonas Gylys, jonas.gylys@ktu.lt
Kaunas University of Technology, Department of Thermal and Nuclear Energy, Donelaicio 20, LT-44239, Kaunas, Lithuania

Algimantas Kiela, algimantas.kiela@tm.kauko.lt
Kaunas College, Department of Technical Sciences, Pramones 20, LT-50468, Kaunas, Lithuania

Abstract. *The most of the film apparatus for the thermal treatment of liquid products consists of vertical tubes with falling films on their external surfaces. In the case when liquid film flows down a vertical tube the curvature of its surface and the film itself effects heat transfer characteristics on the surface. The objective of present study is to obtain comprehension for heat transfer developments in gravitational laminar liquid film flow. Heat transfer in laminar film flow is influenced by the cross curvature of the wetted surface, heat flux on the film surface and boundary conditions on the wetted wall. The analytical study evaluating the influence of cross curvature on heat transfer in laminar film has been carried out. An equations using correction factors for the calculation of heat transfer to laminar liquid film with respect to the cross curvature for different boundary conditions were established. The considerable relative cross curvature for laminar or wavy-laminar film flow it is possible for high viscosity liquids or for high Prandtl numbers. Heat transfer intensity in thermal entrance region of viscous liquid film flow in presence and in absence of isothermal section was estimated experimentally as well. The experiments have been performed with transformer oil film flowing down a vertical surface of small diameter stainless tube. The results of theoretical calculations were compared to the results of the experimental research.*

Keywords*: laminar film, gravitational flow, heat transfer, analytical study, experiment*

1. INTRODUCTION

Heat transfer has a very wide application, which ranges from our everyday activities of various technological equipments to most advanced nuclear and electronic processing facilities. It also has a wide economical and environmental importance because sources of energy are valuable commodities and therefore various processes involving energy transport are to be optimized and meet future demands.

Many industrial processes in chemical, food and power engineering employ gravitational liquid films flows that often take place with heat transfer phenomena. Laminar liquid film flow is of interest in a wide variety of heating and cooling devices used in aerospace, electronics, nuclear engineering, biomechanics, oil and water transport as well in food, glass, polymer and metal processing.

Laminar film flow characterizes the flow at low fluid velocities where inertial forces are relatively small compared to viscous forces. A deeper understanding of flow regularities as well heat transfer phenomena under laminar flow conditions is required in the design of various industrial equipments.

The steady laminar film flow of a viscous liquid along an inclined wavy wall numerically was studied (Malamataris and Bontozoglou, 1999). The dimensionless Navier-Stokes equations have been solved in the whole range of the laminar flow regime. Numerical predictions were compared with available experimental data for very low Reynolds number. The emphasis in the discussion of results was given in the presentation of free surface profiles, streamlines, velocity and pressure distribution along the free surface and the wall.

Hydrodynamics and heat transfer in laminar two-dimensional wavy film flow have been investigated experimentally (Al-Sibai *et al.*, 2002; Leefken and Renz, 2002). The fluid velocity within the film flow was measured by particle image velocimetry with simultaneous measurement of the film thickness and wave velocity by a fluorescence intensity technique. An infrared thermography technique was applied to measure instantaneous local heat transfer rates through the film. The accuracy of the experimental results has been verified by numerical calculations and Nusselt correlation.

A new method to calculate heat and mass transfer coefficients of falling film absorption process over vertical tube or plate type surface employed in absorption refrigeration system was proposed (Fujita and Hihara, 2005). Authors criticize conventional logarithmic mean temperature difference method for lack of physical rationality and for incorrect results from calculations.

Heat and mass transfer in falling laminar film with a parabolic velocity profile was considered (Lokshin and Zakharov, 2001). Exact analytical solutions have been obtained for temperature profiles under general boundary conditions of the third kind, as well for corresponding Nusselt numbers.

The momentum and heat transfer in a laminar liquid film on a horizontal stretching sheet was analyzed (Anderson *et al.*, 2000). In this research the governing time-depending boundary layer equations were reduced to a set of ordinary differential equations by means of an exact similarity transformation.

2. HEAT TRANSFER IN LAMINAR FILM FLOW ON A VERTICAL SURFACE

In the case of laminar film flow the conduction may be viewed as a mode of heat transfer only. Thus, the equation known as Fourier's law can be used to compute the amount of energy being transferred per unit time

$$q = -\lambda gradT \tag{1}$$

By substituting the expression of temperature gradient for Eq. (1) and integrating, we obtain

$$\int_{T_w}^{T} dT = -\frac{q_w}{\lambda}\int_0^y \frac{q}{q_w}dy \tag{2}$$

or

$$T_w - T = \frac{q_w}{\lambda}\int_0^y \frac{q}{q_w}dy \tag{3}$$

where q_w and q - heat flux density on the wetted surface and in the film respectively, W/m^2; T_w - temperature of the wetted surface, K; λ - thermal conductivity, W/(m·K); y - distance from the wetted surface, m.

The heat flux from a liquid film can be determined as follows

$$Q = cT_f\rho\overline{w}b\delta = \int_0^{\delta} cT\rho wbdy \tag{4}$$

where c - specific heat, J/(kg·K); T_f - temperature of the film, K; ρ - liquid density, kg/m^3; $\overline{w}$ - average film velocity, m/s; b - elementary width of the film, m; δ - liquid film thickness, m; w - film velocity, m/s.

The mean temperature is defined

$$T_f = \frac{\int_0^{\delta} wTdy}{\overline{w}\delta} \tag{5}$$

By substituting Eq. (3) for Eq. (5), we obtain

$$T_f = T_w - \frac{\frac{q_w}{\lambda}\int_0^{\delta} w\left(\int_0^y \frac{q}{q_w}dy\right)dy}{\overline{w}\delta} \tag{6}$$

By defining the temperature difference between a wetted surface and the mean temperature of the film as

$$T_w - T_f = \frac{\frac{q_w}{\lambda}\int_0^{\delta} w\left(\int_0^y \frac{q}{q_w}dy\right)dy}{\overline{w}\delta} \tag{7}$$

we obtain the following expression for the calculation of local heat transfer coefficient

$$\alpha = \frac{\overline{w}\,\delta\lambda}{\int\limits_0^{\delta} w\left(\int\limits_0^{y} \frac{q}{q_w}\,dy\right)dy} \tag{8}$$

In order to define the regularity $q/q_w = f(y)$, we employ the energy equation

$$c\rho w\frac{\partial T}{\partial x} + \frac{dq}{dy} = 0 \tag{9}$$

By integrating the Eq. (9) within the limits from 0 to *y*, we obtain the ratio of heat flux densities in the film

$$\frac{q}{q_w} = 1 - \frac{c\rho}{q_w}\int\limits_0^{y} w\frac{\partial T}{\partial x}\,dy \tag{10}$$

Assume that $\partial T/\partial x = dT_f/dx$ (Kays, 1972). The ratio dT_f/dx can be determined by equation of heat balance in the film

$$\frac{dT_f}{dx} = \frac{q_w}{c\rho\overline{w}\,\delta} \tag{11}$$

Substitution of Eq. (11) by Eq. (10) leads to the following relationship

$$\frac{q}{q_w} = 1 - \frac{\int\limits_0^{y} w\,dy}{\overline{w}\,\delta} \tag{12}$$

By substituting Eq. (12) for Eq. (8) and by employing the Nusselt number Nu_d, we obtain

$$Nu = \frac{\overline{w}\,\delta d}{\int\limits_0^{\delta} w\left(\int\limits_0^{y}\left(1 - \frac{\int\limits_0^{y} w\,dy}{\overline{w}\,\delta}\right)dy\right)dy} \tag{13}$$

Velocity in the laminar film flow one can calculate by the following equation

$$w = \frac{g\delta y}{\nu}\left(1 - 0.5\frac{y}{\delta}\right) \tag{14}$$

and the mean velocity respectively

$$\overline{w} = \frac{g\delta^2}{3\nu} \tag{15}$$

By taking Eqs. (14) and (15) into account, we from Eq. (13) obtain the following expression

$$Nu_d = \frac{8w\delta^2}{3\int_0^{\delta} w\left(\int_0^{y}\left(1-\frac{3\int_0^{y} wdy}{2w}\right)dy\right)dy} \tag{16}$$

The above-mentioned equation could be solved numerically and the following result is obtained

$$Nu_d = 8.2353 \tag{17}$$

In case of heat transfer calculations, it is more reasonable to use the modified Nusselt number Nu_M. Therefore (Gimbutis *et al.*, 1993), the following equation is used

$$Nu_M = 2.27Re^{-1/3} \tag{18}$$

where $Re = 4\Gamma(\rho\nu)$ - Reynolds number; Γ - wetting density, kg/(m·s); ν - kinematic viscosity, m^2/s.

As it is shown (Gimbutis *et al.*, 1993), the application of Eq. (18) can be expanded using the corresponding multipliers. The curvature of the film flowing down the outside surface of vertical tube and heat transfer between the film surface and surrounding medium of gas or vapour, can be evaluated by correction factor C_{Rq}. Multiplier ε_{Pr} may estimate the variability of liquid physical properties. Then, the modified Nusselt number can be calculated by the following equation

$$Nu_M = 2.27Re^{-1/3}C_{Rq}\varepsilon_{Pr} \tag{19}$$

The correction factor C_{Rq} can be determined by equations: when $q_w = const$

$$C_{Rq} = \frac{136}{136+39\varepsilon_R} + \left(0.52-0.03\varepsilon_q\right)\varepsilon_R \tag{20}$$

and when $T_w = const$

$$C_{Rq} = \frac{136}{136+39\varepsilon_R} + \left(0.52-0.03\varepsilon_q\right)\varepsilon_R \tag{21}$$

where $\varepsilon_R = \delta/R$ - relative cross curvature of the film; R - cross curvature of wetted surface (tube external radius), m; $\varepsilon_q = q_s/q_w$ - ratio of heat flux densities; q_s - heat flux density on the film surface.

The variations of correction factor C_{Rq} upon the range of relative cross curvature and external heat exchange between the film surface and surroundings medium according to Eqs. (20) and (21) shown in Fig. 1.

The variation of physical properties of the liquid across the film must be taken into account in a case of high value of temperature difference. The multiplier ε_{Pr} in Eq. (19) can be determined as an exponential function of the following ratios Pr_f/Pr_w and μ_f/μ_w. The results of numerical calculations showed that ratio Pr_f/Pr_w unambiguously does not evaluate the influence of physical properties variation for different liquids and the character of this influence practically is not dependent upon the values of ε_q and ε_R. This research showed that for $-0.5 \le \varepsilon_R \le -1$ and $0.4 \le Pr_f/Pr_w \le 2$ the multiplier $\left(Pr_f/Pr_w\right)^{0.25}$ can be used. The multiplier ε_{Pr} must be determined as a function of parameters μ_f/μ_w for more accurate evaluation of liquid physical properties variation. This parameter evaluates practically the influence of physical properties variation for water, transformer oil and compressor oil. The multiplier ε_{Pr} in such case can be calculated by the following equation

$$\varepsilon_{Pr} = \left(\mu_f/\mu_w\right)^n \tag{22}$$

where $n = 0.315(2+\varepsilon_R)^{-0.49}$, for $0.1 \le \mu_f/\mu_w \le 1$ and $n = 0.325(2+\varepsilon_R)^{-0.24}$, for $1 \le \mu_f/\mu_w \le 10$; Pr_f and Pr_w - Prandtl number referred to the film and wetted surface respectively; μ_f and μ_w - dynamic viscosity referred to the film and wetted surface respectively, (Pa·s) . The similar data (Gimbutis *et al.*, 1993) for $\varepsilon_R = 0$ have showed that boundary conditions on the wetted surface have no influence to the character of heat transfer dependence on variability of the liquid physical properties.

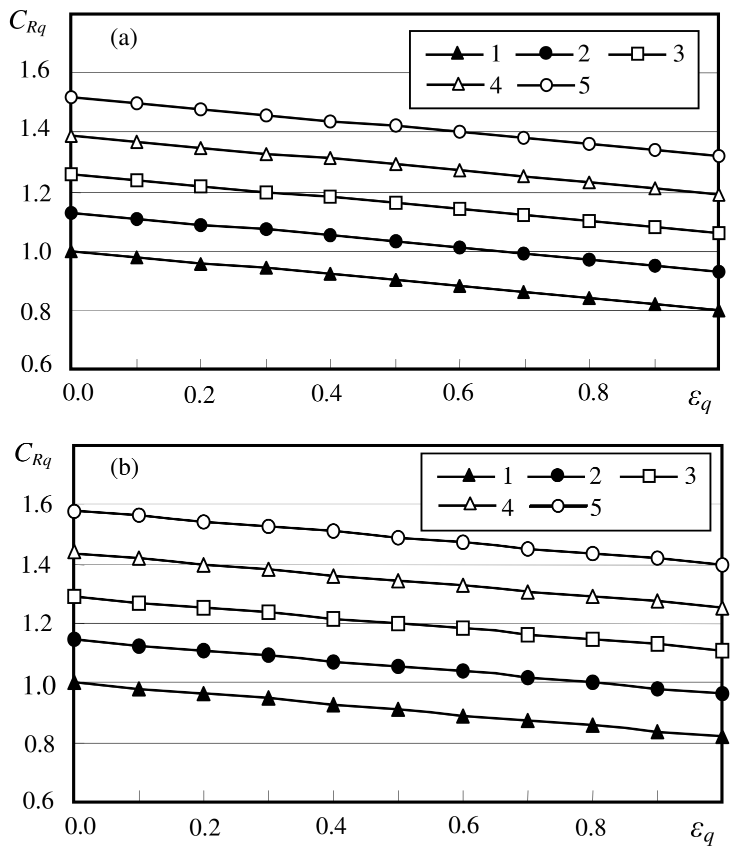

Figure 1. Variations of correction factor on film cross curvature and external heat transfer: (a) boundary condition $q_w = const$ and (b) boundary condition $T_w = const$ *1 – 5* - $\varepsilon_R = 0$; 0.25; 0.50; 0.75 and 1.0 correspondingly

3. EXPERIMENTAL SET-UP AND PROCEDURE

The scheme of experimental set-up is presented in Fig. 2. Transformer oil was used as a working liquid and with the aid of feed pump it has been supplied to the liquid tank, which includes a slot distributive mechanism. The oil film from the distributor fell down a vertical tube (calorimeter). The stainless steel tube 3.8 mm in outside diameter with the length of 546 mm was used in the experiment as a calorimeter. The fixing bolts at the end of tested tube allowed the possibility to regulate and to guarantee verticality of the tube. After flowing down the calorimeter, transformer oil was gathered back to the reservoir. The gutter at calorimeter end ensured a smooth falling of the oil film into the reservoir. The temperature of falling down film was measured by two calibrated thermocouples. The location of thermocouple in the liquid distributor ensured the measurement of oil film temperature in the inlet. The thermocouple installed at the end of calorimeter determined oil film temperature at the exit correspondingly. In order to keep the necessary temperature of transformer oil the cooler was installed in the reservoir.

The electric current supplied for the calorimeter provided a steady heat flux on the experimental section. The current could have been ranged from 300 to 500 amperes and voltage from 5 to 10 volts respectively. In order to convert an alternating current into a direct current, the rectifier was used. Voltages drop on the known value resistor fitted in as a shunt determined the electric strength in the circuit. Its readings were taken from the millivoltmeter. The resistance of the test section has come to 0.012 ohm/m, while the resistance of transformer oil film did not exceed $36 \cdot 10^6$ ohm/m respectively. So, the heat dissipation within the oil film was ignored on the assumption that the whole heat flux determined by voltage drop and current has been carried through the surface of tested section exclusively.

As heat flux along the tested section did not change, so to the demanded accuracy of the experiment it was assumed that the bulk mean temperature of the liquid film conform linear regularity. That circumstance allowed determining the oil film temperature at any cross-section of the tested section accurately. Measurement accuracy for the determination of film flow parameters was of the range 2.5 – 7.1%.

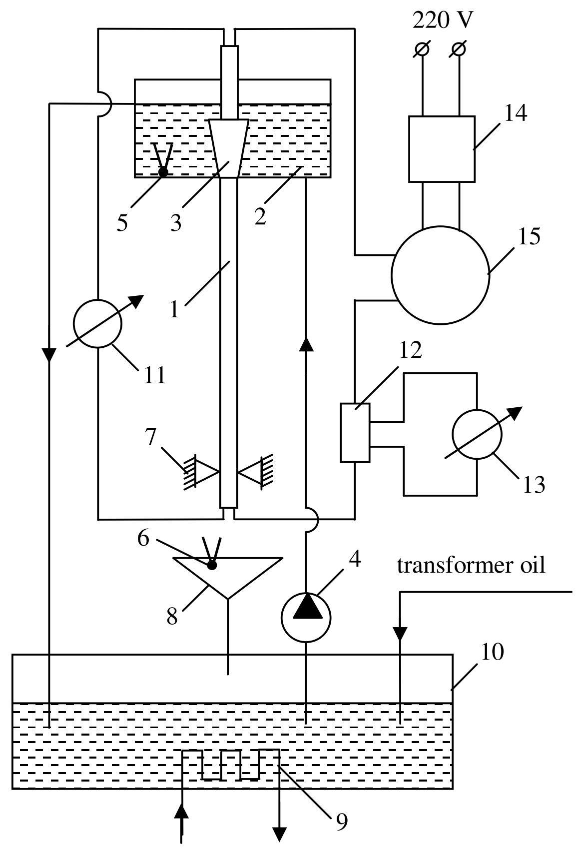

Figure 2. Schematic diagram of experimental set-up: *1* - calorimeter; 2 - liquid tank; *3* - slot distributive mechanism; *4* - feed-pump; *5* - inlet thermocouple; *6* - outlet thermocouple; *7* - centering bolts; *8* - gutter; *9* - cooler; *10* - liquid reservoir; *11* - voltmeter; *12* - shunt; *13* - millivoltmeter; *14* - voltage regulator; *15* - rectifier

4. HEAT TRANSFER IN THE THERMAL ENTRANCE REGION OF LIQUID FILM FLOW

Heat transfer in the entrance region was studied experimentally for boundary condition $q_w = const$. The results were obtained for the transformer oil film flow on vertical stainless steel tube with outside diameter 3.8 mm. The length of the tested tube was equal 546 mm. The experiments were provided in presence and in absence of isothermal section with 105 mm length.

The considerable relative curvature ε_R for laminar or wavy-laminar film flow is possible with high viscosity liquid or with high Prandtl number. In such case the thermal entrance region is much longer than hydrodynamic one.

The results of local heat transfer coefficient in the entrance region for laminar film flow on vertical plane surface can be determined by the expression

$$Nu_{df} = 8.24\left[1 + 0.0011\left(Pe_f\, d/x\right)^{4/3}\right]^{1/4}\left(Pr_f / Pr_w\right)^{1/4} \tag{23}$$

where $Pe_f = \left(Re_f Pr_f\right)$ - Peclet number referred to the film; d - hydraulic diameter of the film, m; x - longitudinal coordinate, m.

For stabilized heat transfer, evaluating the cross curvature of wetted surface, Eq. (23) can be written as follows

$$Nu_{df} = 8.24 C_{Rq}\left(Pr_f / Pr_w\right)^{1/4} \tag{24}$$

In this case the thickness of thermal boundary layer is equal to the film thickness and can be determined by the following equation

$$\delta = 1.67R\left(\sqrt{1 + 1.09\left(Re/Ga_R\right)^{1/3}} - 1\right) \tag{25}$$

where $Ga_R = gR^3/\nu^2$ - Galileo number; g - acceleration of gravity, m²/s.

For laminar film flow we can assume that local heat transfer coefficient is inversely proportional to the thickness of thermal boundary layer $\alpha/\alpha_{stab} = \delta/\delta_t$. By taking into account this precondition, from Eqs. (23) and (24) it follows that

$$\frac{\delta_t}{\delta} = \frac{\alpha_{stab}}{\alpha} = \left[1 + 0.0011\left(Pe_f d/x\right)^{4/3}\right]^{-1/4} \tag{26}$$

On this correlation basis, the relative cross curvature of the laminar film one can express as follows

$$\varepsilon_R = \frac{\delta}{R}\left[1 + 0.0011\left(Pe_f d/x\right)^{4/3}\right]^{-1/4} \tag{27}$$

In absence of external heat transfer $(q_s = 0)$, the correction factor evaluates the film curvature

$$C_{Rq} = C_R = 1 + 0.52\varepsilon_R \tag{28}$$

External heat transfer does not influence the heat transfer intensity in the thermal entrance region and Eq. (27) is valid. The local heat transfer coefficient in thermal entrance region for laminar film falling down outside surface of vertical tube can be determined by the following expression

$$Nu_{df} = 8.24\left[1 + 0.0011\left(Pe_f d/x\right)^{4/3}\right]^{1/4} C_R\left(Pr_f/Pr_w\right)^{1/4} \tag{29}$$

Variation of local heat transfer coefficient in the thermal entrance region of transformer oil film flow is presented in Fig. 3.

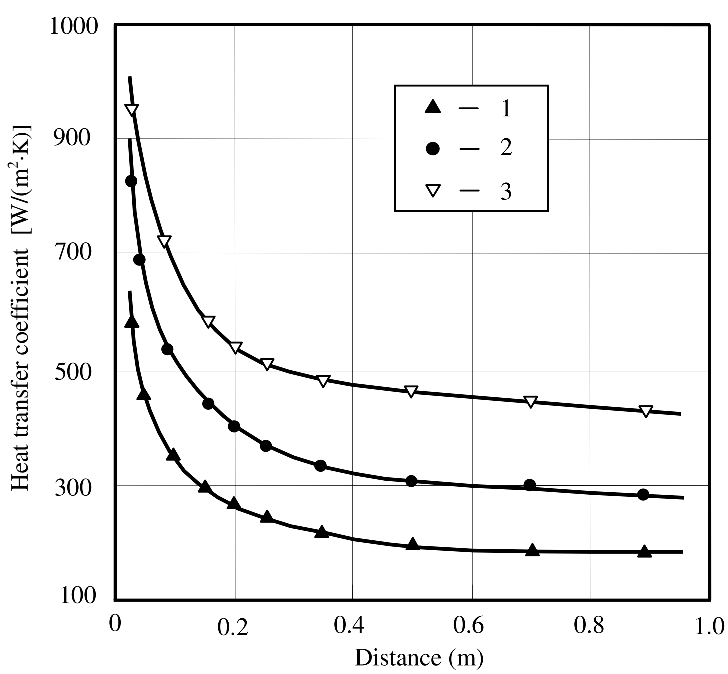

Figure 3. Variation of local heat transfer coefficient in the thermal entrance region of transformer film flows when $q_w = const$: *1* - *Re* = 40 – 44; *2* - *Re* = 300 – 330; *3* - *Re* = 530 – 950

The comparison of theoretical results obtained by Eq. (29) with experimental data is presented in Fig. 4. As we can see, the experimental results are in good agreement with Eq. (29).

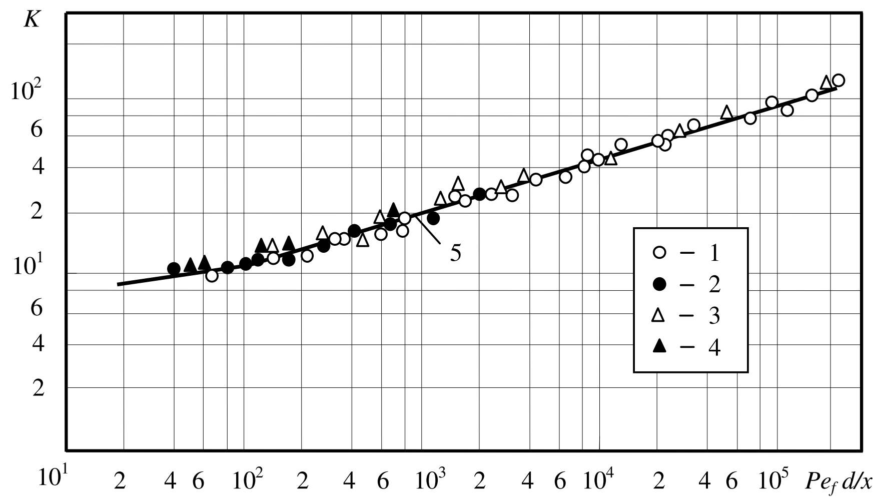

Figure 4. Experimental results of local heat transfer: *1*,*2* - laminar and wavy-laminar flow in absence of the isothermal section correspondingly; *3*,*4* - laminar and wavy-laminar flow in presence of the isothermal section; *5* - Eq. (29); $K = Nu_{df} C_R^{-1} \left(Pr_w / Pr_f \right)^{1/4}$

5. CONCLUSIONS

The cross curvature and external heat exchange of the film has a significant influence on heat transfer rate from the wetted surface in laminar gravitational liquid film flowing down the outside surface of vertical tube. In this case equation for heat transfer intensity calculation in laminar liquid film falling down a vertical plane surface must be supplemented with multiplier C_{Rq} evaluating curvature of the film and heat transfer between the film surface and surrounding medium of gas or vapour and with multiplier ε_{Pr} evaluating the variability of liquid physical properties respectively.

In a case of film flow with high Prandtl number the thermal entrance region is much longer than hydrodynamic one. Therefore, we can assume that heat transfer from beginning takes place at stabilized film flow.

Heat transfer intensity in the thermal entrance region of transformer oil film flow in presence and in absence of isothermal section was estimated experimentally. The experimental results showed that heat transfer stabilization in the thermal entrance region takes place very fast.

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