

STATISTICAL DAMAGE DETECTION THROUGH TIME SERIES ANALYSIS ON A SMART STRUCTURE

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***Abstract**. This paper proposes a structural health monitoring procedure using active-sensing of piezoceramics coupled in lightweight structures. The approach is based on time-series analysis from the input-output voltages signals obtained by patches of piezoceramics (PZTs). The auto-regressive moving average with exogenous input (ARMAX) model is used for linear prediction. The damage-sensitive index was defined by the residual ARMAX error. In order to establish a threshold value to recognize the actual integrity state of the structure the one-way analysis of variance (ANOVA) is performed bended with the Tukey's multiple comparison procedure. Tests were made in an aluminum portal frame structure with three bonded PZTs. The structure is assembled with angle brackets and bolts and, the damages were simulated by loosening and tightening different bolts. The diagnoses, which were reached, showed the efficacy of the proposed methodology to detect and locate minor structural changes with statistical confidence and advantages against classical procedures.*

***Keywords**: damage detection, smart structures, time series analysis, ANOVA, multiple comparison procedures.*

1. INTRODUCTION

Due to several reasons, like safe and economical purposes, the structural health monitoring (SHM) reached an enormous importance in modern engineering, as for instance, aircrafts, bridges, oilrigs, etc, see Inman et al. (2005) for details. Additionally, with the high development of the smart material technology, the concepts of smart structures became a reality, mainly with the utilization of piezoceramics actuators (PZTs) and electrical impedance measurements. A recent review about the benefits caused by SHM and the challenges to become it in a "plug-n-play" system in real-world structures can be found in Silva et al. (2007[a]). Among them, Worden and Dulieu-Barton (2004) mention that the detection with statistical confidence of whether damages are presents or not is the most fundamental issue.

In order to implement robust structural health assessment procedures to separate healthy and damaged conditions without deep knowledge about the structural mathematical model, the statistical methodologies have been used, mainly to verify hypotheses if groups of indexes are similar or not. Several structural applications have been employing these hypotheses testing in damage detection, like the one-way analysis of variance (ANOVA) and the Tukey's multiple comparison, (Silva et al., 2007[b]). Silva et al. (2007[b]) applied the ANOVA to detect damage using an index obtained by electric impedance measurements in frequency-domain of patches of PZTs.

However, nowadays there is a trend to investigate indexes more complex, mainly based in time-series input-output from PZTs. An example is the contribution of Lynch (2004) that fitted an auto-regressive model with exogenous input (ARX) from collected data in the time-domain in a beam structure with bounded PZTs. In the present work is proposed a different feature damage-sensitive obtained through ARMAX models. A portal frame structure was utilized as experimental test-bed. The ANOVA and multiple comparison tests were handled to provide a clear diagnostic about the actual state of the structure. Finally, the results obtained are discussed and further directions are pointed out.

2. ARMAX DAMAGE-SENSITIVE INDEX

The first step in this approach is devoted to the construction of an ARMAX model, for each input voltage, u[k], and output voltage, x[k], from healthy states considering the measurements of each PZT. The ARMAX(na,nb,nc) model is written by, (Ljung, 1998):

$$A\left(q^{-1}\right)x[k] = B\left(q^{-1}\right)u[k - n_k] + C\left(q^{-1}\right)e_x[k] \tag{1}$$

where $e_x[k]$ is the error between the measured signal and the output from the prediction model and n_k is the time delay set to unity. $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ are the polynomials in the delay operator q^{-1}. This model is called reference base. The Burg method or prediction error method (PEM) can be used for estimating the coefficients of the polynomials that describe this model: $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$. The orders are considered n_a, n_b and n_c, respectively. These orders, in general, are unknown *a priori*. However, there are several criteria to estimate them. The classical Akaike's information theoretic criteria (AIC) is used in the present paper.

The next step in this approach consists in to using the model associated with eq. (1), in order to investigate the input-output data in unknown condition:

$$A(q^{-1})y[k] = B(q^{-1})u_y[k-n_k] + C(q^{-1})e_y[k] \tag{2}$$

If the ARMAX model, represented by equation (2), is not a good prediction for the input voltage, $u_y[k]$, and output voltage, $y[k]$, that are under continual monitoring, then, the residual error $e_y[k]$ and its probability distribution should be changed statistically comparing with the reference base. It is an indication of a structural variation. The damage-sensitive index proposed in this analysis considers the ratio of Euclidean norm between the unknown residual error $e_y[k]$ and $e_x[k]$ reference error.

In this paper the ARMAX model reference is the most representative of the set of healthy data. It means that the model using this polynomials $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ are able to predict with confidence an enormous set of variations conditions. In order to choose this reference model is utilized in this paper the analysis of variance and comparison procedures.

3. ANALYSIS OF VARIANCE AND MULTIPLE COMPARISON PROCEDURES

The ANOVA procedure evaluates the hypothesis that the all index-features samples have the same mean. The test considers the following assumptions about the data: every sample populations are normally distributed; every sample populations have equal variance and all observation is mutually independent. Lilliefors hypothesis test and F Levene test can be performed to check the initial considerations, (Hogg and Ledolter, 1987). Basically, the ANOVA procedure separates the variability in two types: the within and between group sums of squares. Summarily, the results are used to assembly the ANOVA table, which can be viewed in the table 1. Details about this variables and its means can be found in Hogg and Ledolter (1987) or any basic statistics-engineering book.

Table 1. ANOVA table for completely randomized design.

Source of Variability	Sum of Squares	degrees of freedom	Mean Squares	F statistic
Between groups	$SS_{bg} = \sum_{i=1}^{k} n_i (\overline{Y}_i - \overline{Y})^2$	k-1	$MS_{bg} = \frac{SS_{bg}}{k-1}$	$F = \frac{MS_{bg}}{MS_{wg}}$
Within groups	$SS_{wg} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (Y_{ij} - \overline{Y}_i)^2$	N-k	$MS_{wg} = \frac{SS_{wg}}{N-k}$	
Total	$\sum_{i=1}^{k} \sum_{j=1}^{n_i} (Y_{ij} - \overline{Y})^2$	N-1		

However, it is necessary tests for qualifying which pairs of mean values are significantly different. This information is used to characterize the damage situation with statistical confidence. The t-test or z-test are comparison procedures, but unfortunately, they are used to compare one particular pre-selected set. If there are k groups, one can make many possible paired for comparison; as a matter of fact, there are k(k-1)/2 of them and there are some pitfalls in this procedure. Fortunately, in this case, the Tukey's honestly significant difference criterion can be used to perform a multiple comparison, (Hochberg and Tamhane, 1987).

4. EXPERIMENTAL TEST DESCRIPTION

Figure 1 shows an aluminum portal frame with a rigid base that was used in the experimental tests. The top beam is connected to two vertical beams using aluminum angle brackets and bolts. Three bonded PZTs in the lightweight structure were used as sensors/actuators, called PZT1, PZT2 and PZT3. Table 2 shows the geometric dimensions of the structure and of the PZT patches. Figure 2 shows a schematic diagram of the measurement setup with the positions of PZTs and positions where damages were induced.

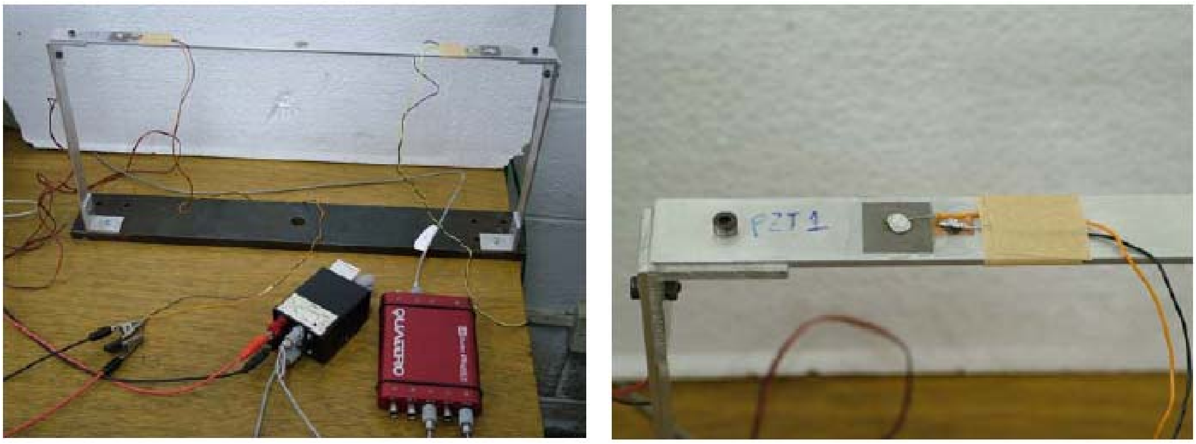

Figure 1. Smart portal frame and experimental setup.

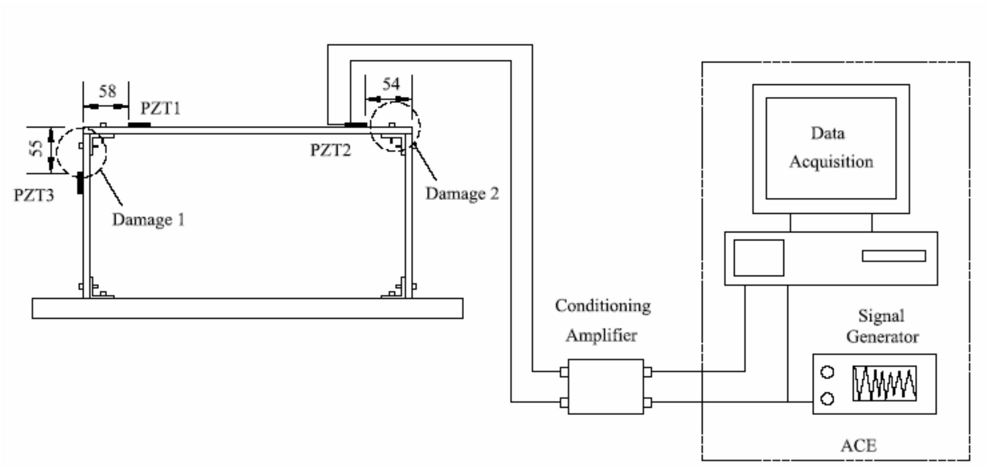

Figure 2. Schematic diagram of the measurement setup, dimensions in mm. It is shown the placement of the PZT patches and the positions of damages.

Table 2. Dimensions of the host structure and the PZT patches, based on PSI-5A-S4 (Piezo Systems®, Inc.).

Property	Top Beam	Vertical Beam	3 PZTs
Length	500 mm	250 mm	20 mm
Width	25 mm	25 mm	20 mm
Thickness	2.5 mm	2.5 mm	0.267 mm

The input generation and data acquisition were led using a commercial system from Data Physics controlled by SignalCalc ACE® software. Two channels were recorded in the time-domain with a sampling rate of 102.4 kHz, producing 8192 time samples each. The white-noise input voltage excitations in the PZT patches and the output voltage from a circuit used to conditional the response were stored as input and output, respectively.

The damage states were considered by uncontrolled loosening of different bolts (damage 1 – close to PZT3 in the vertical beam; and damage 2 – near to PZT 2 in the top beam). After introducing the damage 1, the bolt was handily retightened for the initial condition and in the following the damage 2 was included. Table 3 describes the structural conditions investigated. In each condition were stored eight sets of input-output data from PZT1, PZT2 and PZT3; only in baseline condition was considered twelve samples data. The baseline data means that the bolts joints were all tightened (Healthy condition – test 0). The test 1 was performed to verify false-positive test. These acquisitions were collected in different days in order to include some variability of the environmental into the data. Unfortunately, in this test one does not had a way to exact control the tightening. It was used a mark to restore the system for the initial condition. For further works is been designing a device in order to perform this adjustment.

Table 3. Structural conditions (see fig. 2 to observe the locations).

Tests	Damage Pattern	Location	Number of Samples	Description
0	Undamaged	Undamaged	12	Baseline – Healthy
1	Undamaged	Undamaged	8	False-positive test
2	Damage 1	Near to PZT3	8	Handily tightening and loosening corner bolt
3	Damage 2	Near to PZT2	8	Handily tightening and loosening corner bolt

5. RESULTS

The proposed methodology obtains structural health information based on a non-recursive algorithm. After acquiring a regular number of measurements, corresponding to a defined period, based on knowledge or safety reasons, a set of data is obtained and the SHM process is verified. The samples are, then, classified in groups of undamaged or damaged condition.

For this structure, a previous analysis by AIC led that an ARMAX(6,1,2) model is enough for a suitable prediction, see figure 3a. Figure 3b presents the tests of residuals associated with one case of healthy condition. The residual analysis shows that the correlation between the output and the residual error from model based remains within the confidence interval (99%), except at zero lag. Therefore, the prediction error is close to white noise process. The other undamaged cases were very similar. Hence, this set of model for healthy condition may be considered validated. A representative residual error from PZT1 with damage 1 (reference base 1) is shown in fig. 4a. Figure 4b shows a normal probability plot of ARMAX residual error for this case. Once the data point fall near the line corresponding to the normal probability plot, it was reasonable to assume that the ARMAX residual error is asymptotically normally distributed.

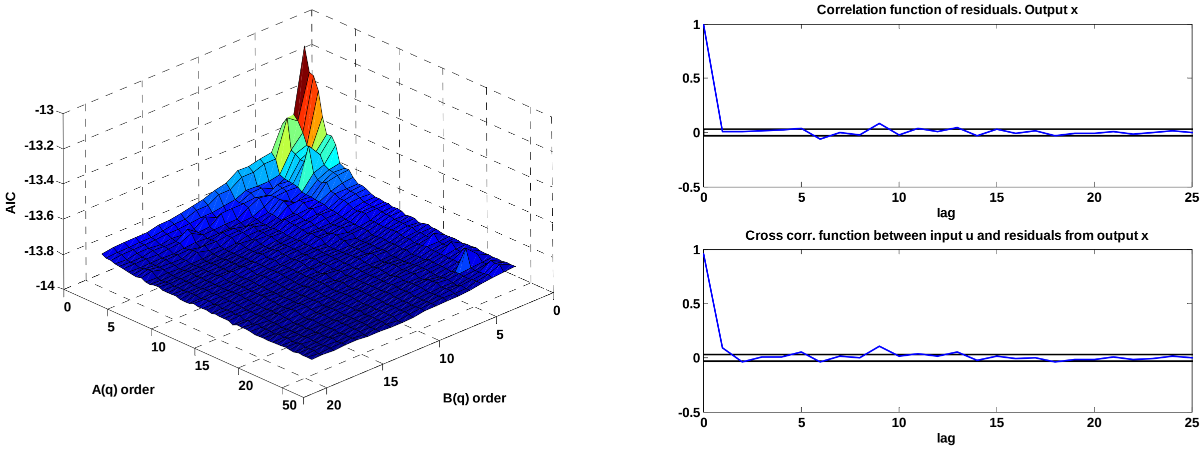

Figure 3. (a) AIC. (b) Residual analysis from model with healthy measurements from PZT1.

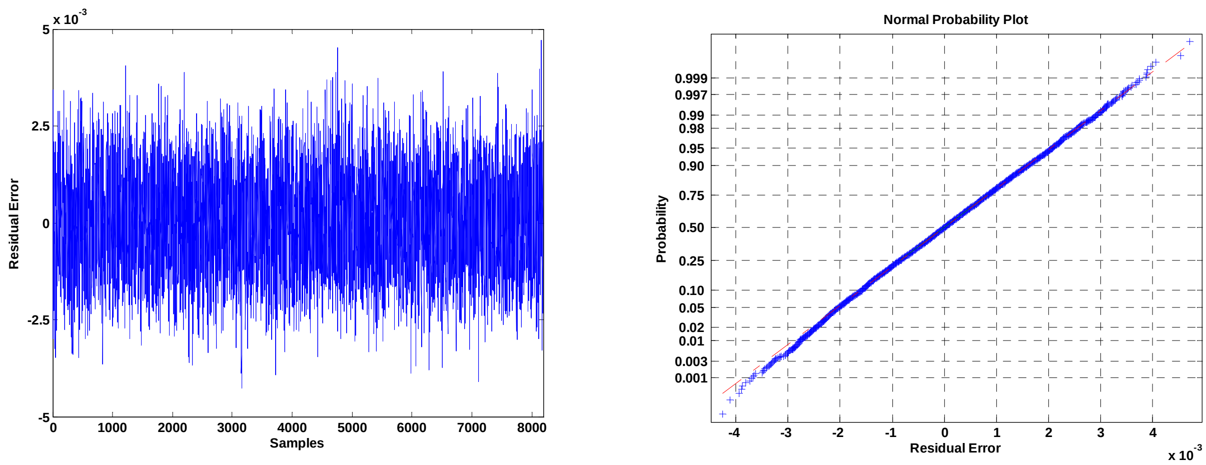

Figure 4. (a) Residual error from PZT1 with damage 1 (reference base 1). (b) Normal probability plot of ARMAX residual error with damage 1 for PZT1.

One had twelve reference bases (signals from test 0), so it was obtained twelve indexes for each structural condition analyzed. An ANOVA procedure was driven for choosing the better reference base for each PZT, in order to detect structural variations. Figure 5a shows the results of this computation from PZT1 with all possible values of the index in the reference base. Figure 5b presents the multiple comparisons using the ANOVA results for this set.

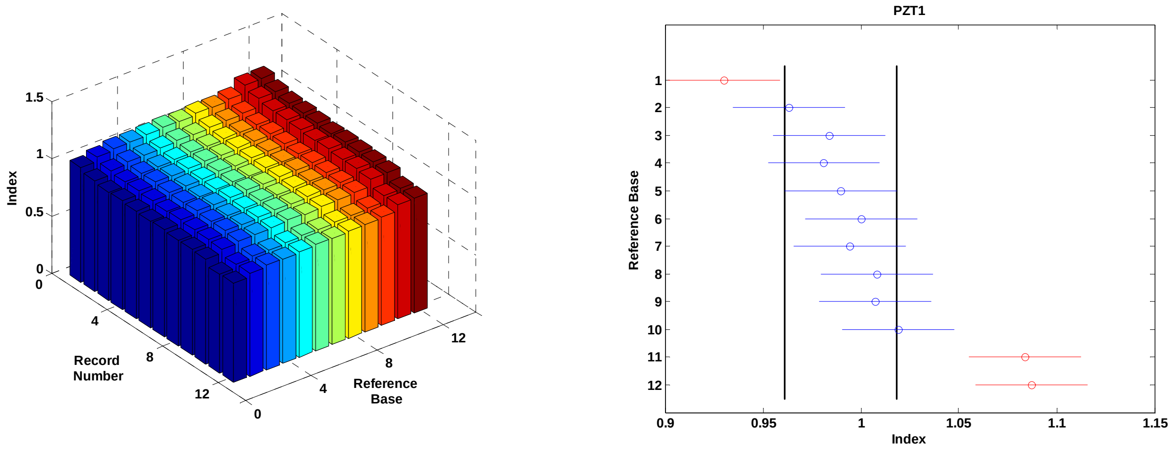

Figure 5. (a) Set of indexes from PZT1 assuming 12 different reference bases for each healthy measurement. (b) Tukey's multiple comparison procedure for choosing which base will be used to predict the series through PZT1 data.

Analyzing figure 5b, one can observe that the signals in case 1,11 and 12 are the most representative to be used as reference for measurements from PZT1, because its variability within groups were bigger face the other cases. It was chosen the case 12 as reference for PZT1. A similar procedure was performed for the others PZTs and, it was obtained the cases 9 and 4 for PZT2 and PZT3, respectively. The plots for these cases were omitted in this text for clearness. The normalized damage metric charts to all PZTs considering all damages cases are shown in fig. 6. In this plot was considered the previous analysis to choose the most representative references. It was assumed that in each structural condition group (four tests showed in tab. 3) there was not variation and, the signals were acquired uniformly. These cases correspond the baseline (12 first samples), the undamaged (false-positive test – 13 to 20 record data), damage 1 (21 to 28 samples) and damage 2 (29 to 36).

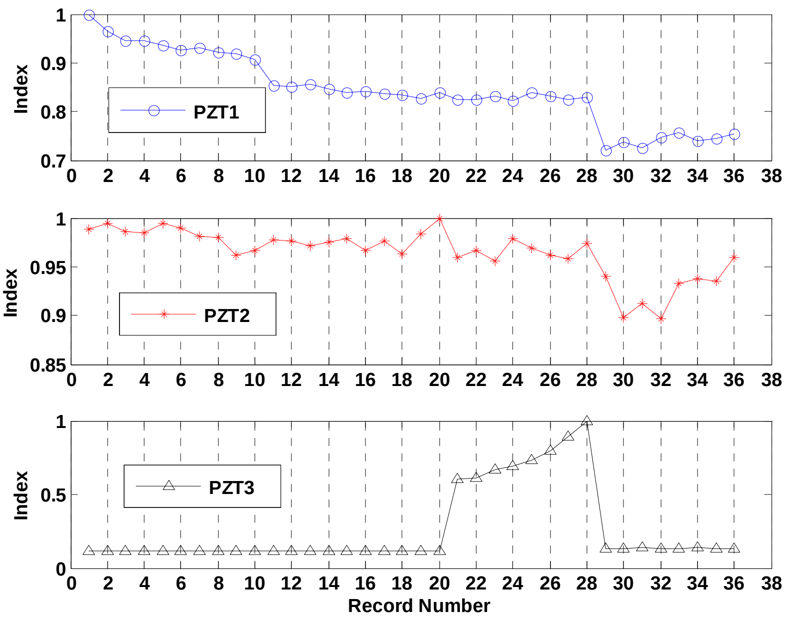

Figure 6. Normalized damage metric chart evolution for each PZT - Ratio between the Euclidean norm of the residual error for healthy (base reference) and for unknown condition. The twelve first values are in the undamaged set (database).

The four first samples in the database were not considered to produce the matrix of data. This procedure is made because the ANOVA used in this work is valid only for equal samples size. Tables 4 to 6 show the one-way analysis of variance results. Analyzing the tables, the p-value is zero in all cases, so it is no doubt that one or more of the tests have different mean values.

Table 4. ANOVA results for PZT1.

Source of Variability	Sum of Squares	Degrees of freedom	Mean Squares	F statistic	p-value
Between groups	0.1259	3	0.0420	103.2032	0
Within groups	0.0114	28	4.0661e-004		
Total	0.1373	31			

Table 5. ANOVA results for PZT2.

Source of Variability	Sum of Squares	Degrees of Freedom	Mean Squares	F statistic	p-value
Between groups	0.0141	3	0.0047	23.7057	0
Within groups	0.0056	28	1.9875e-004		
Total	0.0197	31			

Table 6. ANOVA results for PZT3.

Source of Variability	Sum of Squares	Degrees of Freedom	Mean Squares	F statistic	p-value
Between groups	2.3550	3	0.7850	163.4773	0
Within groups	0.1345	28	0.0048		
Total	2.4895	31			

It is worth to point out that the variability within groups is smaller than the variability between groups in all PZTs. Here, each group represents one structural condition. The biggest variability between groups was reached for PZT3 and, it was due to the application of the induced damage by loosening and tightening handily the bolt near to this field. One can also verify that the variability between groups in PZT1 is bigger than PZT2. It is credited by the fact that PZT1 is closer to the damaged region in the damage 1 (see fig. 2) than PZT2. Another reason is because all bolts were retightened after each measurement test and, it could be difficult to assembly in the same situation, or else, a residual damage in these bolts could have remained. Figure 7 shows the box-whisker plot for PZT1, PZT2 and PZT3. In figure 8 are plotted the multiple comparisons using one-way ANOVA results to determine which estimates are significantly different using the Tukey's criterion.

So important as to detect the damage, is to avoid the false-positive (false alarming of fault). The measurements of test 1 were obtained after a time period without any structural modification induced. The analysis of figures 7 and 8 permits to recognize these healthy conditions. However, the test 1 using the PZT1 was distant from the baseline (test 0). It could represent an alarm of damage, but if the damage had been included, it would be hoped that the index value in the PZT3 changed its value in the test 1 due to PZT3 to be close the field of PZT1 (see figure 2). The PZT2 also presented a little difference, but the index remained within of the control limits. The results permit conclude that this approach seems to be robust to false alarming in the PZT2 and PZT3 and in the PZT1 with more careful in the analysis of diagnosis in this sensor.

The damage 1 (test 2) was induced near to PZT3 in the vertical beam. It is clear that the most variation occurred in this PZT index. It is also worth to observe that the indexes from PZT1 and PZT2 in this test overlap the limit of control. This result is difficult to observe by simple visual analysis of fig. 6, proving that ANOVA approach help us to obtain a clearer decision. Hence, this analysis procedure was able to detect and locate the damage. PZT3 gave a clear indication of the damage and PZT1 can also be useful to confirm the damaged region once that the variation in the PZT2 was smaller than PZT1. It was occurred because the PZT2 is more distant from damaged field (see fig. 2).

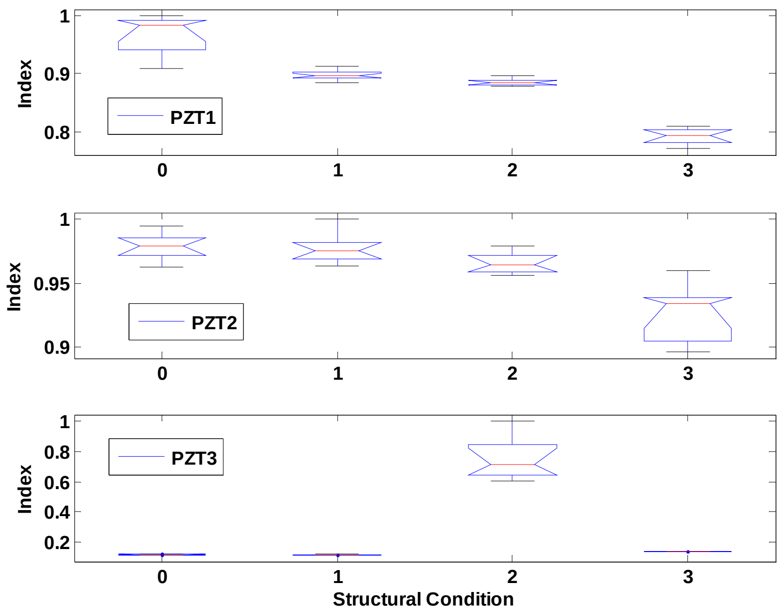

Figure 7. Box-and-whisker display of data set for PZT1, PZT2 and PZT3.

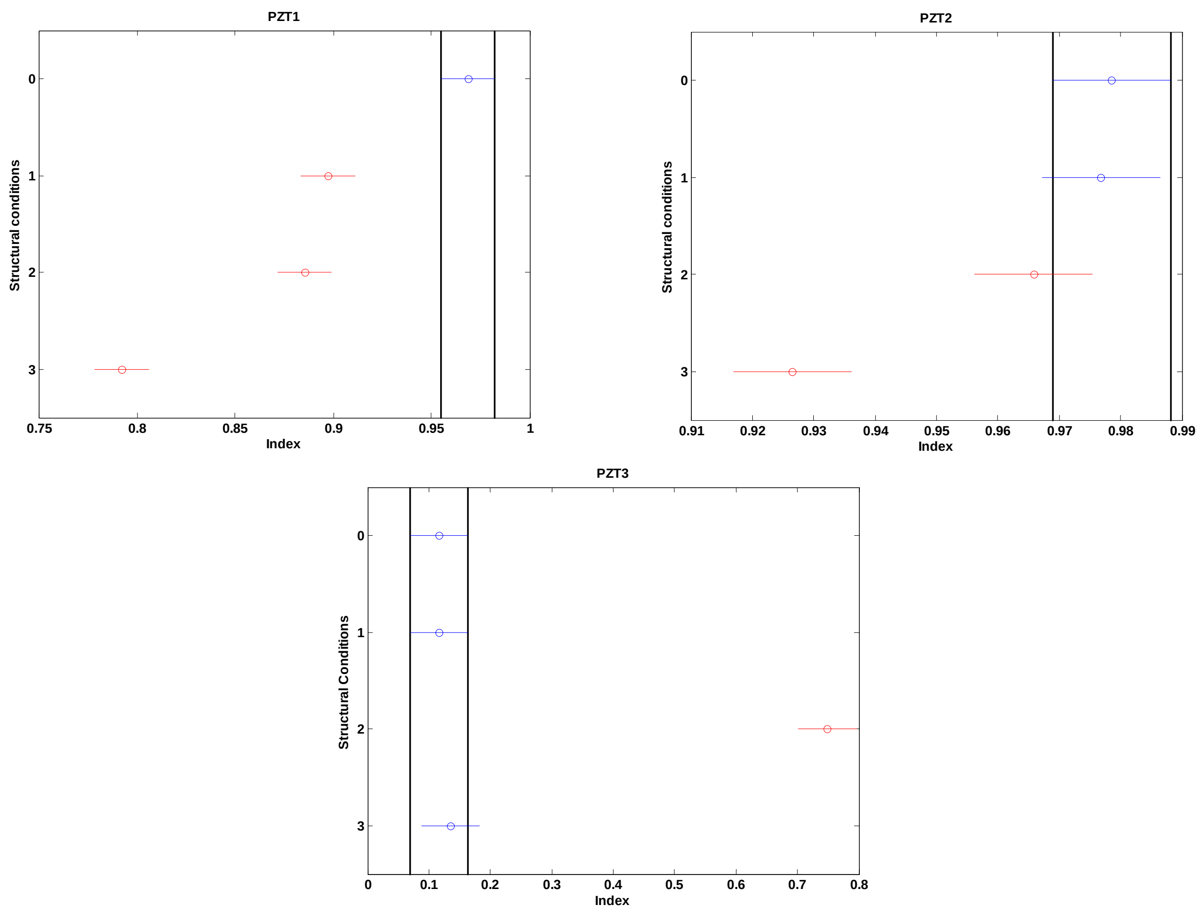

Figure 8. Multiple comparisons for each PZT using one-way ANOVA results and the Tukey's honestly significant difference criterion.

On the other hand, when the damage 2 was induced (test 3) the bigger variations were observed in PZT 1 and 2. However, the value of the index in PZT1 is smaller than PZT2. For clearness, fig. 9 shows the box-and-whisker plot for test 3 (damage 2) considering the index from PZTs 1 and 2. The median value is bigger in the PZT2 than PZT1. Moreover, the F- statistic is bigger in PZT1 than PZT2 (see ANOVA tables) and the distribution of values in PZT2 is asymmetric with more values down to median, while in the PZT1 is symmetric. These evidences are an indicative of damage in the corner bolt close to PZT2.

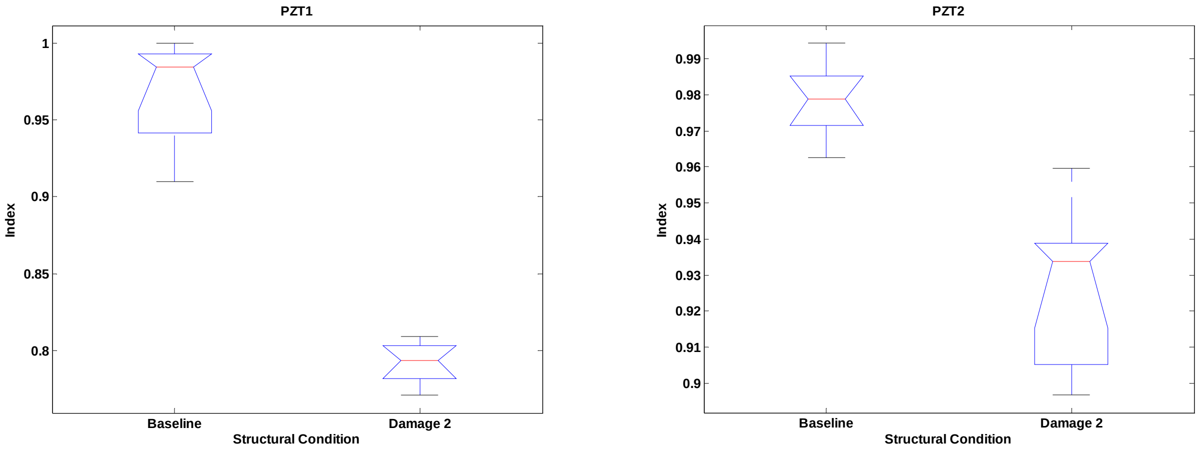

Figure 9. Box-and-whisker display of data set from test 3 (Damage 2) for PZT1 and PZT2.

6. FINAL REMARKS

The approach used in this paper was able to detect and locate structural damages in non-supervised learning mode without mathematical model. The proposed methodology also takes advantage of high frequency structural excitations and smart materials frameworks. Similar results were found in the paper of Silva et al. (2007[b]) by using a root-mean-square deviation (RMSD) based on frequency-domain index. However, the results of this paper are considered more representatives, because the use of ARMAX time-series model fitted using a set of healthy data can provide parametric information useful for a further prognostic step. Moreover, the ARMAX models are basically a set of discrete-time filters easily programmed in a digital signal processor. Hence, this formulation is attractive for implementation in a real monitoring system without human supervisor. Additionally, the threshold value determined by ANOVA and multiple comparison procedures leads a more rigorous thresholds statistical criterion than the simple RMSD used in the classical damage metric chart obtained by electric impedance techniques. The next phase of this research is the investigation of non-linear damage-feature by using NARMAX models.

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8. REFERENCES

Hochberg, Y., and Tamhane, A. C. *Multiple Comparison Procedures*, Willey, 1987.

Hogg, R. V. and Ledolter, J. *Engineering Statistics.* MacMillan Publishing Company, 1987.

Inman, D. J.; Farrar, C. R.; Lopes Jr., V. and Steffen Jr., V. *Damage Prognosis for Aerospace, Civil and Mechanical Systems*, 1st ed. West Sussex, John Wiley & Sons Ltda, 2005, v. 01, 449p.

Ljung, L. *System Identification: Theory for the User*. Prentice Hall PTR, 2nd edition, 1998.

Lynch, J. P. Detection of structural cracks using piezoelectric active sensors. *ASCE Engineering Mechanics Conference, EM2004*, University of Delaware Newark, DE, 2004.

Silva, S.; Dias Jr., M. and Lopes Jr., V. Damage detection in a benchmark structure using AR-ARX models and statistical pattern recognition. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, v. 29, p. 174-184, 2007[a].

Silva, S.; Dias Jr., M. and Lopes Jr., V. Structural damage detection using smart material and analysis of variance. *Submitted to Journal of Sound and Vibrations*, 2007[b].

Worden, K. and Dulieu-Barton, J. M. An overview of intelligent fault detection in system and structures. *Structural Health Monitoring – SHM.* v. 3, n. 1, p. 85-98, 2004.

9. RESPONSIBILITY NOTICE

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