CHAOS CONTROL OF A NONLINEAR PENDULUM USING A MULTI-PARAMETER METHOD

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Abstract. *Chaos control is based on the richness of responses of chaotic behavior. A chaotic attractor has a dense set of unstable periodic orbits (UPOs) and the system often visits the neighborhood of each one of them. Moreover, chaotic response has sensitive dependence to initial condition, which implies that the system's evolution may be altered by small perturbations. Therefore, chaos control may be understood as the use of tiny perturbations for the stabilization of an UPO embedded in a chaotic attractor, which makes this kind of behavior to be desirable in a variety of applications, since one of these UPO can provide better performance than others in a particular situation. The OGY method is a discrete technique that considers small perturbations promoted in the neighborhood of the desired orbit when the trajectory crosses a specific surface, such as some Poincaré section. This contribution proposes a multi-parameter semi-continuous method in order to control chaotic behavior. As an application of the general formulation, it is investigated a two-parameter atuation of a nonlinear pendulum control employing uncoupled actuations. Analyses are carried out considering signals that are generated by numerical integration of the mathematical model.Results show that the procedure can be an efective good alternative for chaos control since it provides a more effective UPO stabilization.*

Keywords*: Chaos, control, nonlinear dynamics, nonlinear pendulum.*

1. INTRODUCTION

Chaos control may be understood as the use of tiny perturbations for the stabilization of unstable periodic orbits (UPOs) embedded in a chaotic attractor. Chaos control methods may be classified as discrete or continuous techniques. The first chaos control method was proposed by Ott *et al.* (1990), nowadays known as the OGY method as a tribute of their authors (Ott-Grebogi-Yorke). This is a discrete technique that considers small perturbations promoted in the neighborhood of the desired orbit when the trajectory crosses a specific surface, such as some Poincaré section (Grebogi & Lai, 1997; Shinbrot *et al.*, 1993). On the other hand, continuous methods are exemplified by the so called delayed feedback control, proposed by Pyragas (1992), which states that chaotic systems can be stabilized by a feedback perturbation proportional to the difference between the present and a delayed state of the system. There are many improvements of the OGY method that aim to overcome some of its original limitations, as for example: control of high periodic and high unstable UPO (Otani & Jones, 1997, Ritz *et al.*, 1997 and Hübinger *et al.*, 1994) and control using time delay coordinates (Dressler & Nitsche, 1992; So & Ott, 1995 and Korte *et al.*, 1995).

This contribution considers a multi-parameter chaos control method based on the semi-continuous method built upon the OGY method, named SCC-OGY (Pereira-Pinto *et al.*, 2004, 2005; Savi *et al.*, 2006). The idea is to define different control parameters in order to perform the UPO stabilization. A general formulation is presented and after that, an uncoupled two-parameter control of a nonlinear pendulum is carried out. Results show that the procedure can be a good alternative for chaos control since it provides a more effective UPO stabilization.

2. MULTI-PARAMETER CHAOS CONTROL METHOD

A chaos control method may be understood as a two stage technique. The first step is known as learning stage where the unstable periodic orbits are identified and some system characteristics are evaluated. After that, there is the control stage where the desirable UPOs are stabilized.

The multi-parameter chaos control (MPCC) method considers different control parameters, however, in a specific section only one of those actuates. Under this assumption, the map $F^{(n,n+1)}$ that establishes the relation between the system in control section n and n+1, depends on all control parameters, p_i. Although each parameter actuates in different sections, it is assumed its influence based on their positions in section Σ_n .

$$\xi^{n+1} = F^{(n,n+1)}(\xi^n, P^n) \tag{1}$$

where P^n is a vector with all control parameters. Using a first order Taylor expansion, one obtains the linear behavior of the map $F^{(n,n+1)}$ in the neighborhood of the control point ξ_C^n and around the control parameter reference.

$$\delta\xi^{n+1} = D_{\xi^n} F^{(n,n+1)}(\xi^n, P^n)\Big|_{\xi^n=\xi_C^n, P^n=P_0} \delta\xi^n + D_{P^n} F^{(n,n+1)}(\xi^n, P^n)\Big|_{\xi^n=\xi_C^n, P^n=P_0} \delta P^n \tag{2}$$

This equation may be rewritten as follows

$$\delta\xi^{n+1} = J^n \delta\xi^n + W^n \delta P^n \tag{3}$$

Where $\delta\xi^{n+1} = \xi^{n+1} - \xi_C^{n+1}$, $J^n = D_{\xi^n} F^{(n,n+1)}(\xi^n, P^n)\Big|_{\xi^n=\xi_C^n, P^n=P_0}$, $W^n = D_{P^n} F^{(n,n+1)}(\xi^n, P^n)\Big|_{\xi^n=\xi_C^n, P^n=P_0}$ and $\delta\xi^n = \xi^n - \xi_C^n$. W^n is the sensitivity matrix and each column is related to a control parameter. In order to evaluate the influence of all parameters actuation in the system response it is built a basis where each term is formed by the influence of a single isolated parameter when the others are fixed at a reference value. Under this assumption, it is assumed that the resultant actuation is represented by a linear combination of these basis vectors. Therefore, $\delta P^n = \beta^n \delta p^n = \beta^n (p^n - p_0)$, where β^n weights each parameter influence in the system response, p^n is a vector with all parameter positions, p_0 is a vector with all the reference parameter positions and δp^n is a vector that contains the real parameter actuations.

By assuming that each parameter actuates in only one control section it is possible to define active parameter, represented by subscript a, $\delta P_a^n = \beta_a^n \delta p_a^n$ - actuates in section section Σ_n, and passive parameters, represented by subscript p, $\delta P_p^n = \beta_p^n \delta p_p^n$ - that does not actuate in section Σ_n. Here, it is assumed β_a^n and β_p^n as scalars, meaning that there is a contribution to all passive parameters and a different one to the active parameters. Therefore,

$$\delta\xi^{n+1} = J^n \delta\xi^n + W^n \delta P_a^n + W^n \delta P_p^n \tag{4}$$

At this point, it is necessary to align the vector $\delta\xi^{n+1}$ with the stable direction v_s^{n+1}:

$$\delta\xi^{n+1} = \alpha v_s^{n+1} \tag{5}$$

where $\alpha \in \Re$, needs to be satisfied as follows:

$$J^n \delta\xi^n + W^n \delta P_a^n + W^n \delta P_p^n = \alpha v_s^{n+1} \tag{6}$$

Therefore, the unknown variables are α and the non-vanishing term of the vector δP_a^n, resulting in the following system:

$$\begin{bmatrix} \delta P_a^n \\ \alpha \end{bmatrix} = -[W^n \quad -v_s^{n+1}]^{-1} [J^n \quad W^n] \begin{bmatrix} \delta\xi^n \\ \delta P_p^n \end{bmatrix} \tag{7}$$

The solution of this system furnishes the necessary values for the system stabilization: α and δp_i^n, where δp_i^n is related to the non-vanishing element of the vector δP_a^n.

A particular case of this control procedure has uncoupled control parameters meaning that each parameter returns to the reference value when it becomes passive. Moreover, since there is only one active parameter in each control section, the system response to parameter actuation is the same as when it actuates alone. Under this assumption:

$$\beta_p^n = 0 \text{ and } \beta_a^n = 1 \tag{8}$$

Therefore, the map $F^{(n,n+1)}$ that establishes the relation between the system in control section n and n+1 is just a function of the active parameter $\xi^{n+1} = F^{(n,n+1)}(\xi^n, P_a^n)$:

$$\delta\xi^{n+1} = J^n \delta\xi^n + W^n \delta P_a^n \tag{9}$$

where the sensitivity matrix W^n is the same of the previous case. Moreover, since $\beta_a^n = 1$, it follows that $\delta P_a^n = \delta p_a^n$, thus the value of δP_a^n obtained from (10) correspond to the real perturbation necessary to stabilize the system. In order to align the vector $\delta\xi^{n+1}$ with the stable direction, the following system is obtained:

$$\begin{bmatrix} \delta P_a^n \\ \alpha \end{bmatrix} = -[W^n \quad -\nu_s^{n+1}]^{-1} J^n \delta\xi^n \tag{10}$$

3. NONLINEAR PENDULUM

As a mechanical application of the general chaos control procedure here presented, a nonlinear pendulum is considered. The motivation of the proposed pendulum is an experimental set up discussed in De Paula *et al.* (2006). Here, a mathematical model is developed to describe the dynamical behavior of the pendulum while the corresponding parameters are obtained from the experimental apparatus. Numerical simulations of such model are employed in order to obtain time series related to the pendulum response. Finally, some unstable periodic orbits are identified with the close return method and their control simulated employing the MPCC method.

The considered nonlinear pendulum is shown in Figure 1. The right side presents the experimental apparatus while the left side shows a schematic picture. Basically, the pendulum consists of an aluminum disc (1) with a lumped mass (2) that is connected to a rotary motion sensor (4). A magnetic device (3) provides an adjustable dissipation of energy. A string-spring device (6) provides torsional stiffness to the pendulum and an electric motor (7) excites the pendulum via the string-spring device. Two actuators are considered in order to provide the necessary perturbations to stabilize this system. Actuator (5) which properly changes the end string length and actuator (8) which changes the string position near the motor. Both actuators can be experimentally implemented by, for example, step motors.

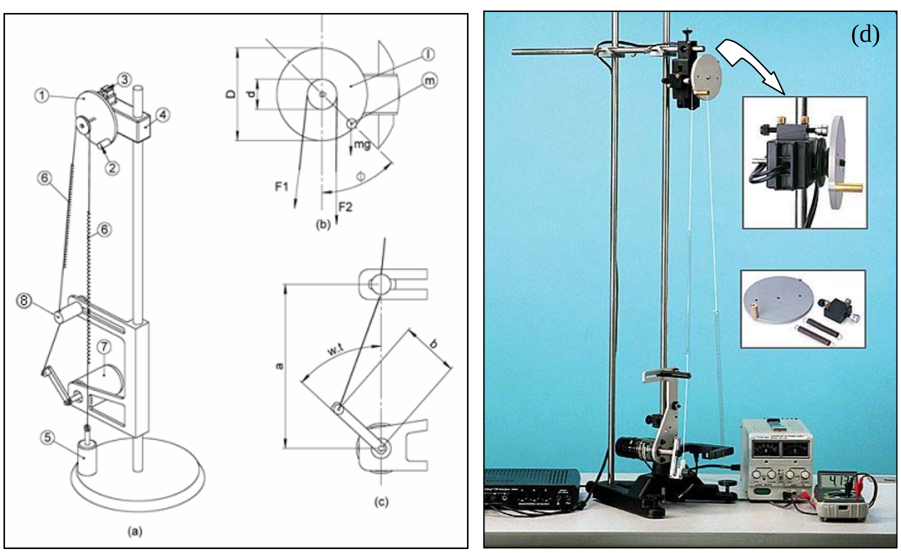


Figure 1. Nonlinear pendulum. (a) Physical Model: the pendulum consists of a metallic disc (1) with a lumped mass (2) that is connected to a rotary motion sensor (4). A magnetic device (3) provides an adjustable dissipation of energy. A string-spring device (6) provides torsional stiffness to the pendulum which is excited by an electric motor (7). Two actuators (5) and (8) are considered.(b) Parameters and forces on the metallic disc. (c) Parameters from driving device. (d) Experimental apparatus.

In order to describe the pendulum dynamics, a mathematical model is proposed. Assuming that ϖ is the forcing frequency, a defines the position of the guide of the string with respect to the motor, b is the length of the excitation arm of the motor, D is the diameter of the metallic disc and d is the diameter of the driving pulley, the equation of motion is given by (De Paula *et al.*, 2006):

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{kd^2}{2I} & -\dfrac{\zeta}{I} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ \dfrac{kd}{2I}(\Delta f(t) - \Delta l_1) - \dfrac{mgD\,\mathrm{sen}(x_1)}{2I} - \dfrac{2\mu}{\pi I}\arctan(q x_2) \end{bmatrix}$$

where $\Delta f(t) = \sqrt{a^2 + b^2 + \Delta l_2{}^2 - 2ab\cos(\varpi t) - 2b\Delta l_2 \sin(\varpi t)} - (a-b)$.

The Δl_1 parameter is the length variation in the string provided by the linear actuator (5), while Δl_2 is the variation of the string position provided by actuator (8). This model represents the pendulum dynamics and its numerical simulations are in close agreement with experimental data (De Paula *et al.*, 2006). In order to show this agreement, it is presented numerical and experimental chaotic strange attractors in Figure 2 assuming the parameters: $a = 1.6\times10^{-1}$ m; $b = 6.0\times10^{-2}$ m; $d = 4.8\times10^{-2}$ m; $D = 9.5\times10^{-2}$ m; $m = 1.47\times10^{-2}$ kg; I = 1.738×10^{-4} kg m^2; k = 2.47 N/m; $\zeta = 2.368\times10^{-5}\ \mathrm{kg\,m^2 s^{-1}}$; $\mu = 1.272\times10^{-4}\ \mathrm{N\,m}$; $\omega = 5.61\,\mathrm{rad/s}$.

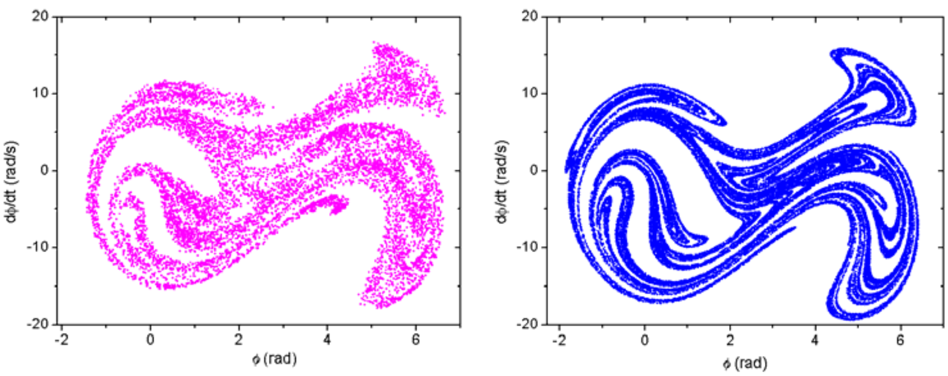


Figure 2. Chaotic strange attractors for $\omega = 5.61\,\mathrm{rad/s}$. Experimental, left side, and numerical, right side.

4. NUMERICAL SIMULATIONS

The first stage of the control strategy is the identification of UPOs embedded in the chaotic attractor. The close return method (Auerbach *et al.*, 1987) is employed with this aim. Figure 3 shows some UPOs identified during this stage. After the UPOs identification, the local dynamics expressed by the Jacobian matrix and the sensitivity matrix of the transition maps in a neighborhood of the fixed points are determined using the least–square fit method (Pereira-Pinto, 2004, 2005; Auerbach *et al.*, 1987; Otani & Jones, 1997). After that, the SVD technique is employed for determining the stable and unstable directions near the next fixed point. The sensitivity matrices are evaluated allowing the trajectories to come close to a fixed point and then one perturbs the parameters by five times the maximum permissible value. Once in multiparameter control the maximum parameters actuation is limited to smaller values than when only one parameter actuates, it is not considered the maximum values in the perturbations to evaluate the sensitive matrix as usual. In this case, it is assumed: $\left|\Delta l_{1\,m\acute{a}x}\right| = 5\,\mathrm{mm}$ and $\left|\Delta l_{2\,m\acute{a}x}\right| = 10\,\mathrm{mm}$.

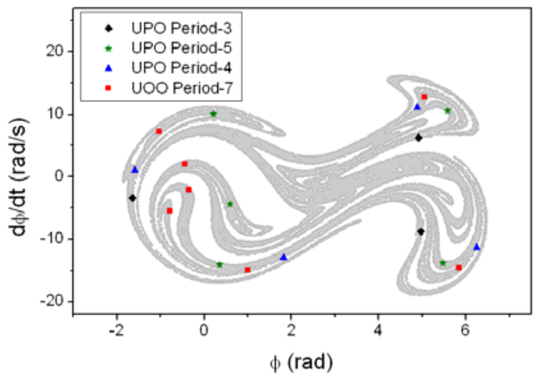


Figure 3. Identified UPOs evaluated by the close return method.

In order to verify the capacity of the proposed chaos control method, it is assumed the uncoupled procedure to follow a control rule that stabilize UPOs in the following sequence: a period-5 orbit during the first 500 periods, a period-4 from period 500 to 1000, a period-7 from 1000 to 1500 and, finally a period-1, from period 1500 to 2000. Figures 4 and 5 present the system evolution in two different control sections. It is presented the controlled displacement and also the control parameters. It should be highlighted that this procedure stabilized the desired orbits with small perturbations in both parameters.

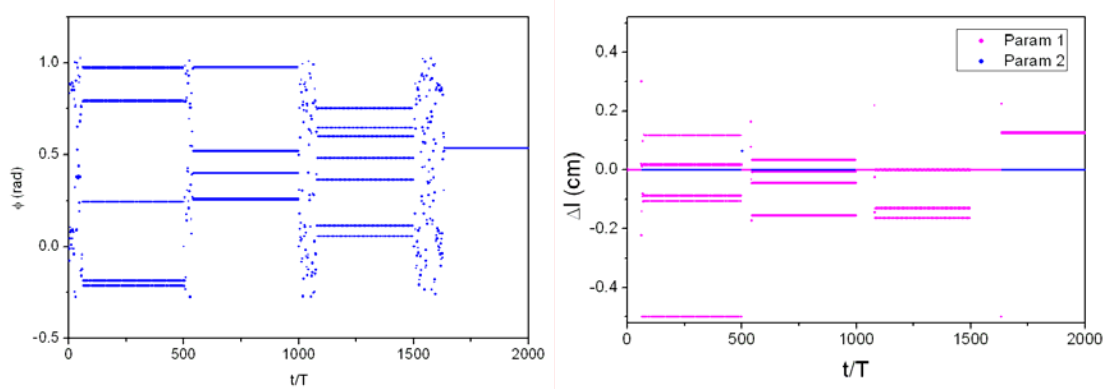


Figure 4. System controlled using uncoupled approach at the control station #1: (a) Displacement; (b) Perturbation.

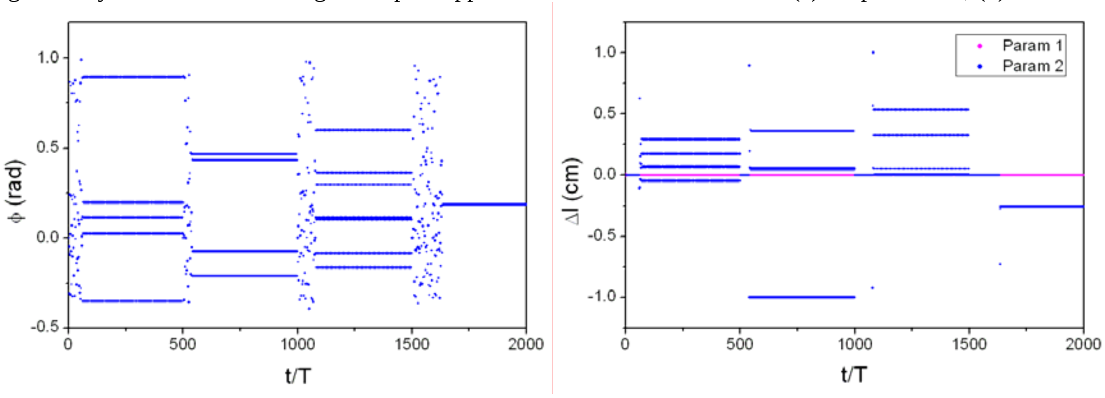
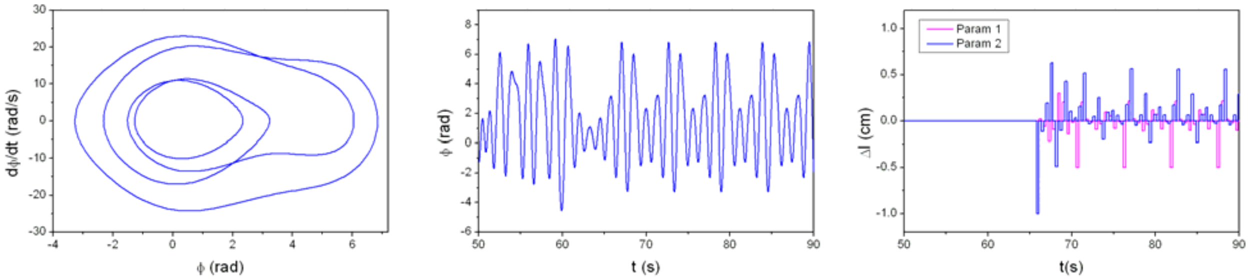


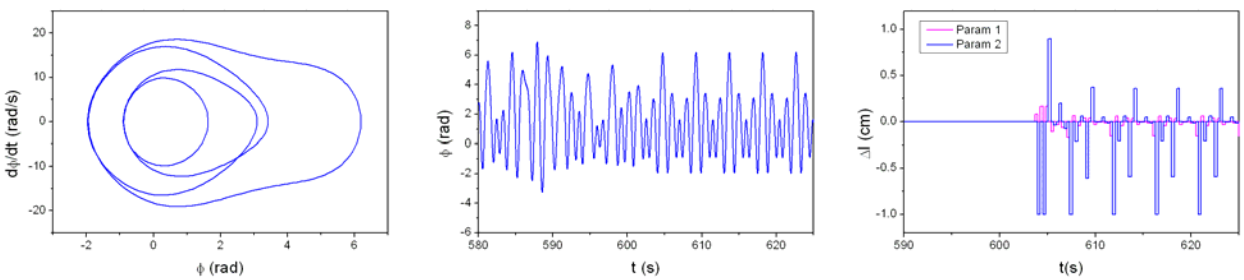
Figure 5. System controlled using uncoupled approach at the control station #2: (a) Displacement; (b) Perturbation.

The stabilized UPOs are shown in Figures 6-9 together with the control signal. Notice that phase space, displacement and control signal time history are presented.



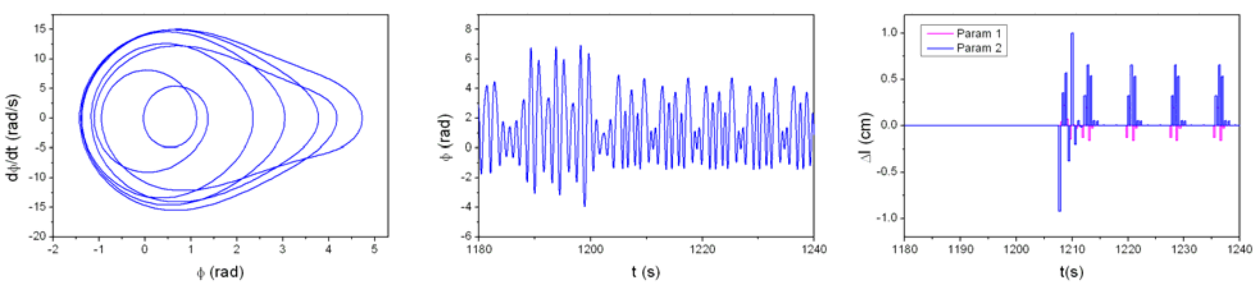
(a) (b) (c)

Figure 6. UPO period-5 stabilized using uncoupled approach: (a) Phase space; (b) $\phi(t)$; (c) $\Delta l_1(t)$ and $\Delta l_2(t)$.



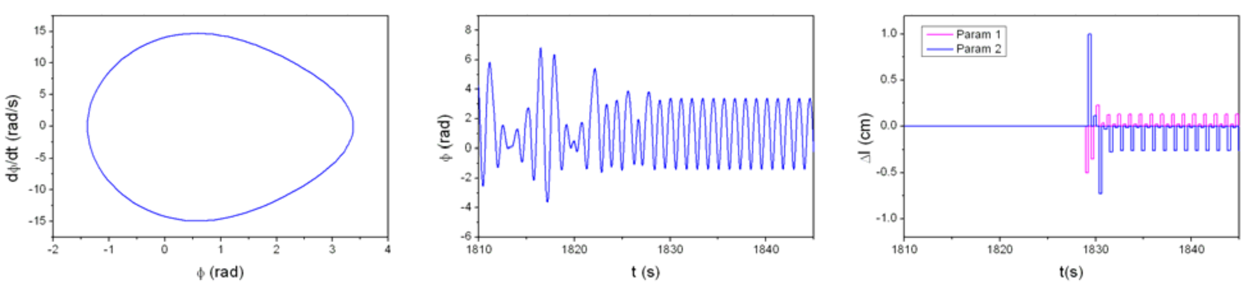
(a) (b) (c)

Figure 7. UPO period-4 stabilized using uncoupled approach: (a) Phase space; (b) $\phi(t)$; (c) $\Delta l_1(t)$ and $\Delta l_2(t)$.



(a) (b) (c)

Figure 8. UPO period-7 stabilized using uncoupled approach: (a) Phase space; (b) $\phi(t)$; (c) $\Delta l_1(t)$ and $\Delta l_2(t)$.



(a) (b) (c)

Figure 9. UPO period-1 stabilized using uncoupled approach: (a) Phase space; (b) $\phi(t)$; (c) $\Delta l_1(t)$ and $\Delta l_2(t)$.

In order to establish a comparison between the multi-parameter and the single-parameter method, the same control rule is applied to the single-parameter technique considering the actuation performed by the parameters Δl_1 and Δl_2. At first, Δl_1 actuation is of concern. Figure 10 show the stabilized UPOs and also the control parameter at section SC1. Figure 11, on the other hand, presents the same pictures assuming the actuation of parameter Δl_2. Notice that both procedures are not capable to follow the control rule and only three UPOs are stabilized. It should be also noticed that the maximum control parameter values: $\left|\Delta l_{1\,máx}\right| = 15\,\text{mm}$ for the first case and $\left|\Delta l_{2\,máx}\right| = 25\,\text{mm}$ for the second, are greater than the ones presented by the multi-parameter control. By observing the MPCC results it is noticeable that the maximum values are $\left|\Delta l_{1\,máx}\right| = 5\,\text{mm}$ and $\left|\Delta l_{2\,máx}\right| = 10\,\text{mm}$.

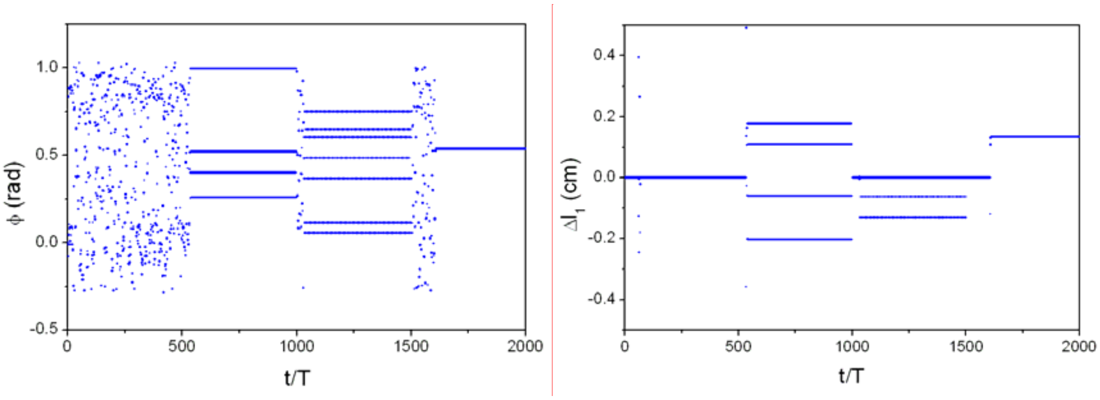


Figure 10. System controlled using parameter Δl_1 at the station #1: (a) Displacement; (b) Perturbation.

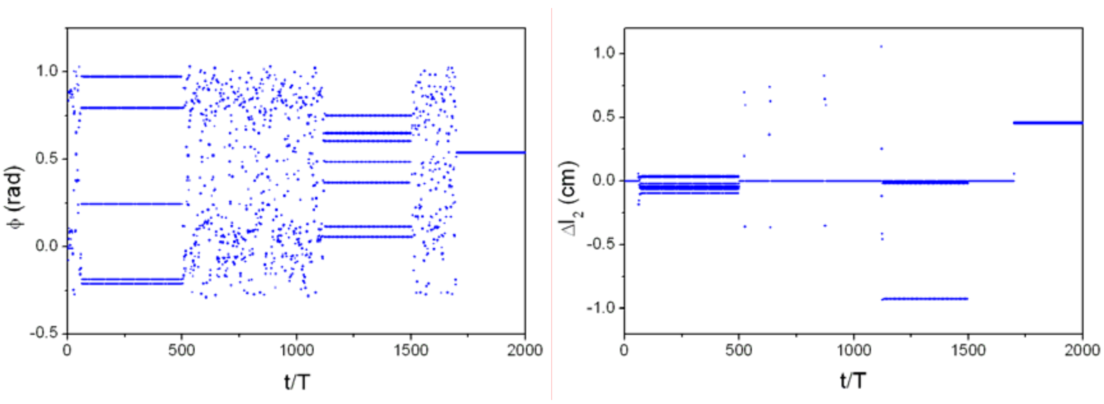


Figure 11. System controlled using parameter Δl_2 at the station #1: (a) Displacement; (b) Perturbation.

5. CONCLUSIONS

This contribution presents a multi-parameter semi-continuous chaos control method built upon the OGY technique. Two different situations may be adopted for the general formulation: coupled and uncoupled actuation. As an application of the general formulation, the uncoupled actuation is employed in the nonlinear pendulum chaos control. Results show that the multi-parameter procedure tends to be more effective in order to stabilized unstable periodic orbits embedded in the chaotic attractor. Moreover, it should be highlighted that the multi-parameter procedure is more effective using smaller values of the accessible control parameter than the ones provided by the single-parameter procedure.

6. ACKNOWLEDGEMENTS

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