# A MULTI-OBJECTIVE OPTIMIZATION OF A COGENERATION SYSTEM

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***Abstract.*** *The design and operation of energy systems nowadays must consider efficient utilization of energy resources, reduced environmental harms, and sustainable development. Many techniques for energy systems analysis and optimization have been developed worldwide. To evaluate different methodologies, the benchmark CGAM problem was proposed, which consisted in the optimization of a cogeneration system with given physical, thermodynamic, and economic models. The original CGAM problem was formulated as a single objective optimization problem, where the objective function was the sum of the maintenance and operation, purchased-equipment, and fuel consumption costs. However, in real-life applications, costs must be analyzed individually; for example, one might increase equipment costs, but save in fuel consumption for the entire system life. In this paper, a multi-objective hybrid optimization of the CGAM system is performed. A hybrid optimization algorithm combines and takes advantage of deterministic and heuristic methods. Usually, it employs a heuristic method to locate a region where the global extreme point lies, and then switches to a deterministic method to get to the exact point faster. The objective functions are the fuel consumption cost rate and the total capital investment. Thus, a Pareto front is obtained for all non-dominated solutions, where the final decision can be made considering appropriate scenarios.*

***Keywords****: Multi-objective Optimization, Cogeneration, Hybrid Optimization, Thermoeconomics*

## 1. INTRODUCTION

The design and operation of energy systems nowadays must consider efficient utilization of natural energy resources, reduced harms to the environment, and sustainable development (Rosen and Dincer, 2001). The multidisciplinary field of exergoeconomics (Bejan *et al.*, 1996) can address environmental issues, reveal the cost formation process of system products, and aid system design optimization. A large number of techniques for energy systems analysis and optimization have been developed worldwide in the past two decades (Bejan *et al.*, 1996; Frangopoulos, 2003; Lazzaretto and Tsatsaronis, 2006; Valero, 2006). In Brazil, research in exergoeconomics has focused on the evaluation and interpretation of different cost partition methodologies (Cerqueira and Nebra, 1999; Júnior and Arriola, 2003), and on exergoeconomic optimization and improvement techniques (Vieira *et al.*, 2004; Vieira *et al.*, 2006). To evaluate and compare different exergoeconomic methodologies, *C*. Frangopoulos, *G*. Tsatsaronis, *A*. Valero, and *M*. von Spakovsky have proposed the optimization of the CGAM five-component cogeneration system as a benchmark problem (Tsatsaronis, 1994), which gained wide acceptance thereafter.

While exergoeconomics provides insights to system improvement (Bejan *et al.*, 1996), the actual optimization requires the application of a mathematical method. The optimization problem for an energy system can be formulated with the thermodynamic property and balance equations, and the component model equations as constraints (Jaluria, 1998). In the case of a complex, many-component system, optimization is a large-scale problem. Recently (Vieira *et al.*, 2004; Vieira *et al.*, 2006), optimization and improvement algorithms have been integrated with a professional process simulator, such that the thermodynamic constraints are dealt with competently by the program. The selection of the optimization method is still important, such that the whole optimization or improvement task is accomplished efficiently. As a matter of fact, to optimize even the relatively simple CGAM cogeneration system, one has to deal with $O(10^2)$ variables; note that the number of variables rapidly increases as the system becomes more complex, as in real energy production systems.

The motivation to pursue more efficient optimization strategies applicable to energy systems is thus clear. In this paper, a multi-objective optimization approach is employed to optimize thermoeconomically the CGAM reference system. For the single-objective optimization, two different hybrid optimization schemes were previously tested

(Padilha *et al*., 2007), based on a genetic algorithm and the quasi-Newton method BFGS (Broyden-Fletcher-Goldfarb-Shanno). Genetic algorithms are easy to code and robust, i.e., will less likely stop at local optima, but they tend to be computationally expensive. Gradient, Newton and quasi-Newton methods are efficient, but at the cost of calculating derivatives and Hessians, which is not always possible in energy systems problems. Also, gradient-based methods are strongly dependent on the initial guess, when the problem has many local optima. Typically, hybrid algorithms attempt to combine the efficiency of gradient-based methods with the robustness of evolutionary algorithms (Colaço *et al*., 2006; Colaço *et al*., 2005). Since in real-life applications costs must be analyzed individually, a multi-objective hybrid optimization of the CGAM system is here performed. The objective functions are the fuel consumption cost rate and the total capital investment. Thus, a new result for the CGAM system is obtained: a Pareto front for all non-dominated solutions, from which the final decision can be made considering appropriate scenarios.

## 2. THE CGAM PROBLEM

The benchmark CGAM problem (Tsatsaronis, 1994) consists in the optimization of a cogeneration system, for which the thermodynamic, physical, and economic models are given explicitly. The equations of the first two models, together with the system physical limits, represent the equality and inequality constraints of the optimization problem. The CGAM problem, though small-scale, is typical of energy systems optimization, in that it is nonlinear, and has an objective function which does not behave smoothly over the entire design domain.

The CGAM system, shown in Fig. 1, is a cogeneration system that produces fixed amounts of electrical power and saturated steam. The electricity production is 30 MW, and the saturated steam mass flow rate at 20 bar is 14 kg/s. The CGAM system consists of the following five components: air compressor, air preheater, combustor, gas turbine, and heat recovery steam generator (HRSG). The combustor fuel is natural gas with a lower heating value of 50000 kJ/kg.

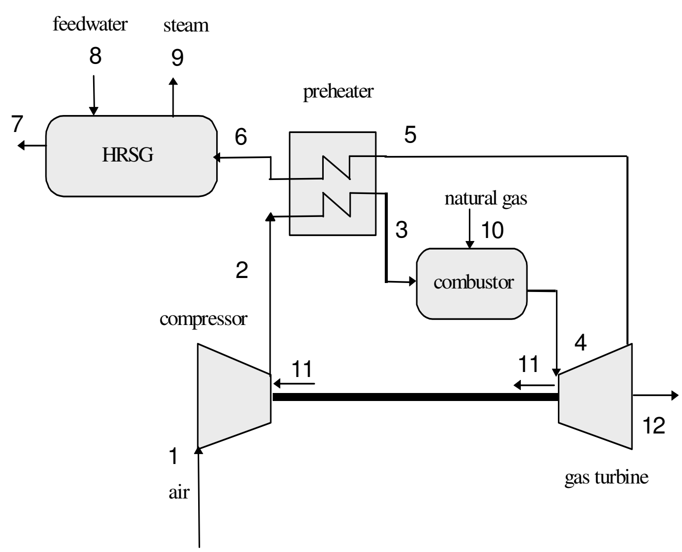


Figure 1. The benchmark CGAM cogeneration system.

The selected decision variables for the optimization problem are the air compressor pressure ratio, $R_c$, the compressor and gas turbine isentropic efficiencies, respectively $\eta_{AC}$ and $\eta_{GT}$, the temperature of the air at the inlet to the combustion chamber, $T_3$, and the temperature of the combustion gases at the inlet to the gas turbine, $T_4$. The restrictions on (i.e., the ranges which establish the limiting values for) the decision variables are (Padilha, 2006): $7 \le R_c \le 27$; $0.7 \le \eta_{AC} \le 0.9$; $0.7 \le \eta_{GT} \le 0.9$; 700 K $\le T_3 \le$ 1100 K; 1100 K $\le T_4 \le$ 1500 K.

## 3. PHYSICAL, THERMODYNAMIC AND ECONOMIC MODELS

The equations of the physical and thermodynamic models are standard and well-known, and are given in detail in (Tsatsaronis, 1994); therefore, they will not be repeated here.

The physical model equations comprise the mass and energy balances applied to the control volumes (i.e., components) of the CGAM system. The compression and expansion processes have prescribed isentropic efficiencies. Pressure losses are prescribed as percent fractions of inlet pressures in the combustion chamber, air preheater, and heat recovery steam generator. A positive temperature difference is imposed at the HRSG pinch point.

The thermodynamic model specifies the reference environment: $T_0$ = 25 °C, $P_0$ = 1.013 bar, relative humidity is 60%. The mole fractions of the gases which compose the atmospheric air are specified. The fuel is pure methane gas, and the combustion is complete in the combustion chamber. The air and the combustion gases behave as ideal gases with constant specific heats. The thermophysical properties of all fluids are given.

To evaluate costs associated with an energy system, one should consider the capital investment cost, the operation and maintenance costs, and the fuel cost. For the CGAM problem, because it serves as a reference for comparison of different optimization methodologies, a simplified economic model is assumed, based on the capital recovery factor, *CRF* (Bejan *et al*., 1994). In this model, the total capital investment, *TCI* ($), of a system is given by the sum of all the purchased-equipment costs, *PEC* ($), of the components of the system multiplied by a factor $\beta$, as given by

$$TCI = \sum_k TCI_k = \sum_k \beta\, PEC_k = \beta \sum_k PEC_k = \beta\, PEC \tag{1}$$

where $k = 1,\ldots,NK$ denotes the k$^{th}$ component, and *NK* is the total number of system components; here, *NK* = 5. The purchased-equipment costs of the air compressor (AC), combustion chamber (CC), gas turbine (GT), air preheater (APH), and heat recovery steam generator (HRSG) of the CGAM system are respectively given by (Tsatsaronis, 1994):

$$PEC_{\mathrm{AC}} = \left(\frac{39.5\dot{m}_{\mathrm{a}}}{0.90-\eta_{\mathrm{AC}}}\right)\left(\frac{P_2}{P_1}\right)\ln\left(\frac{P_2}{P_1}\right) \tag{2}$$

$$PEC_{\mathrm{CC}} = \left(\frac{25.6\dot{m}_{\mathrm{a}}}{0.995-\left(P_4/P_3\right)}\right)\left[1+\exp\left(0.018T_4-26.4\right)\right] \tag{3}$$

$$PEC_{\mathrm{GT}} = \left(\frac{266.3\dot{m}_{\mathrm{g}}}{0.92-\eta_{\mathrm{GT}}}\right)\ln\left(\frac{P_4}{P_5}\right)\left[1+\exp\left(0.036T_4-54.4\right)\right] \tag{4}$$

$$PEC_{\mathrm{APH}} = 2290\left(\frac{\dot{m}_{\mathrm{g}}\left(h_5-h_6\right)}{U\left(LMTD\right)}\right)^{0.6} \tag{5}$$

$$U = 0.018\ \mathrm{kW}/\left(\mathrm{m}^2.\mathrm{K}\right) \tag{6}$$

$$PEC_{\mathrm{HRSG}} = 3650\left[\left(\frac{\dot{Q}_{\mathrm{PH}}}{\left(LMTD\right)_{\mathrm{PH}}}\right)^{0.8}+\left(\frac{\dot{Q}_{\mathrm{EV}}}{\left(LMTD\right)_{\mathrm{EV}}}\right)^{0.8}\right]+11820\dot{m}_{\mathrm{s}}+658\dot{m}_{\mathrm{g}}^{1.2} \tag{7}$$

where PH and EV stand for water preheater and evaporator, respectively.

The capital recovery factor, *CRF*, is given by

$$CRF = \frac{i\left(1+i\right)^l}{\left(1+i\right)^l-1} \tag{8}$$

The fuel cost rate, $\dot{C}_{\mathrm{f}}$ ($/h), is given by

$$\dot{C}_{\mathrm{f}} = c_{\mathrm{f}}\,\dot{m}_{\mathrm{f}}\,LHV \tag{9}$$

where the fuel mass flow rate in (9) must be given in kg/h.

## 4. MULTI-OBJECTIVE OPTIMIZATION OF THE CGAM SYSTEM

Using single-objective optimization methods, it is possible to deal with problems with only one objective function. However, real life problems usually demand more than one objective function. As an example, one might want to maximize the power of a gas turbine, while at the same time minimizing the fuel consumption in the combustion chamber.

It is thus necessary to analyze simultaneously various objective functions, according to their relative priorities. In order to deal with such situations, there are several approaches. One technique is to optimize a multi-objective problem by means of scalar methods. These methods are based on a combination of the individual objective functions into a single function. In other words, the original objective functions $f_1, f_2, \ldots, f_k$ are combined into a generic function $f$ as $f = G(f_1, f_2, \ldots, f_k)$, where $G$ is a linear or non-linear combination of the original functions. The resultant function $f$ can be minimized using the traditional single-objective optimization techniques. Thus, the final result is a single value, corresponding to the optimum values of the variables of the generic function $f$. The technique of combining different objective functions allows one to define individual weights to each one of the original functions, according to their relative importance.

Although easy to implement, the technique described above has some disadvantages. The combination of all individual objective functions into a single function does not allow the designer to evaluate globally the relative importance of the functions. In order to offer to the designer a wider knowledge of the problem at hand, the new paradigm for multi-objective optimization no longer combines all functions into a single one. A better solution is to deal with the original functions, leading to a set of all possible solutions, where the optimum values are located. Such set of solutions is called a 'Pareto front' or 'Pareto set' (Deb, 2001), and will be briefly discussed next.

To better understand the role of the Pareto front, it is necessary to define the concepts of dominated and non-dominated solutions. A solution $S_1$ is defined as dominated, when there is another solution $S_2$ such that, for all objective functions, the solution $S_1$ is worse than the solution $S_2$. On the other hand, a solution $S_1$ is defined as non-dominated, when there is no such solution $S_2$ for which all its objective function values are better than $S_1$. An alternative definition for a non-dominated solution is the following: a non-dominated solution is one, for which an improvement in one of the objective functions cannot be performed without deterioration in one or more of the other objective function values.

The set of all non-dominated solutions is called Pareto front. The so-called best solution, taken from the Pareto front, is obtained through additional decision criteria (economic, technical, political, or other, based on some specific scenario for the application). The choice of the appropriate decision criteria is left to the designer.

### 4.1. Mathematical formulation

The general multi-objective optimization problem is proposed as a vector of objective functions $\vec{F}(\vec{x})$, defined as

$$\vec{F}(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{10}$$

where $f_i(\vec{x}),\ i = 1, 2, 3, \ldots, k$, are the objective functions, and $\vec{x} = \{x_1, x_2, x_3, \ldots, x_n\}$ is an n-dimensional vector composed of the optimization parameters. The vector $\vec{F}$ must have its individual components minimized or maximized, according to the problem being analyzed.

In the same way as in the single-objective optimization problem, the set of variables $\vec{x}$ might be subjected to constraints, which can be written as inequalities in the form $g_j(\vec{x}) \le 0,\ j = 1, 2, 3, \ldots, m$. The constraint expressions can also be written in vector form, $\vec{G}(\vec{x}) = [g_1(\vec{x}), g_2(\vec{x}), g_3(\vec{x}), \ldots, g_m(\vec{x})]$, such that the inequality constraint for the problem can be written as

$$\vec{G}(\vec{x}) \le \vec{0} \tag{11}$$

The set of constraints in Eq. (11) define the domain of solutions, $\Gamma = \{\vec{x} \mid \vec{G}(\vec{x}) \le 0\}$.

Since each solution vector $\vec{x} \in \Gamma$ gives only one vector of functions $\vec{F}(\vec{x})$, there is a set $\Phi$ of feasible functions $\vec{F}(\vec{x})$ defined as $\Phi = \{\vec{F}(\vec{x}) \mid \vec{x} \in \Gamma\}$. Thus, $\vec{F} : \Gamma \mapsto \Phi$ or, in words, the domain $\Gamma$ is mapped by $\vec{F}$ through the image $\Phi \subset \Re^k$. The boundary of $\Phi$ is represented by $\partial\Phi$, and it can contain the extreme values of the image. Thus, the Pareto front will be contained in $\partial\Phi$.

Let us now define rigorously the concept of dominancy. In a minimization problem, given two solution vectors $\vec{x}$ and $\vec{y} \in \Gamma$, the solution vector $\vec{x}$ is dominant with respect to the solution vector $\vec{y}$, or $\vec{x} \succ \vec{y}$, if

$$\forall i \in \{1, 2, 3, \ldots, k\} : f_i(\vec{x}) \le f_i(\vec{y}), \ \text{ and } \ \exists j \in \{1, 2, 3, \ldots, k\} : f_j(\vec{x}) < f_j(\vec{y}) \tag{12}$$

Thus, the solution vector $\vec{x}$ is dominant with respect to the solution vector $\vec{y}$, if $\vec{x}$ gives better results for all individual objective functions. Conversely, if $\vec{x}$ is dominant with respect to $\vec{y}$, the solution vector $\vec{y}$ is dominated. Furthermore, if the solution $\vec{x}$ is not dominated by any other solution, it is defined as a non-dominated solution. From the concept of dominancy, it is possible to define rigorously the Pareto set.

The feasible solution vector $\vec{x}^*$ may represent a Pareto set, only if there is no other feasible solution vector $\vec{x}$ such that, for $i \in \{1, 2, 3, \ldots, k\}$,

$$f_i(\vec{x}) \leq f_i(\vec{x}^*) \tag{13}$$

and, for at least one $j \in \{1, 2, 3, \ldots, k\}$,

$$f_j(\vec{x}) < f_j(\vec{x}^*) \tag{14}$$

The Pareto front is a subset of $\partial\Phi$, which contains all non-dominated solutions.

**4.2. Minimization technique**

There are several types of techniques for solving a multi-objective optimization problem, such as the Min-Max technique (Hwang *et al*., 1980), the weighted sum technique and the Goal Programming method (Steuer, 1986), the approach based on a non-linear combination of the functions (Andersson *et al*., 1998), fuzzy logic (Chiampi *et al*., 1998), the method of the utility function and the lexicographic method (Haimes *et al*., 1975), the step method (Benayoun *et al*., 1971), the particle swarm method (Parsopoulos and Vrahatis, 2002), and the genetic algorithm itself.

In this work, similarly to the treatment of the single-objective optimization problem presented previously (Padilha *et al*., 2007), a hybrid method is employed. For this hybrid method, an initial population is generated, where the variables are defined as

$$\vec{X} = (\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N) \tag{15}$$

The values of each one of the objective functions are stored, corresponding to each member of the population. Then, the non-dominated individuals are found, considering that the individual $\vec{x}_i$ is dominated if there is any $\vec{x}_j$ such that

$$f_k(\vec{x}_j) < f_k(\vec{x}_i) \tag{16}$$

for every $k$. The search for local minima is effected by comparing the value of the objective function of each individual with the value of its neighborhoods. The local minima are those that have lower values of the objective function than the neighborhoods. Computationally, the search can be done by calculating the distance between the individual $\vec{x}_i$ and all the other individuals $\vec{x}_j$. Then, the individual $\vec{x}_{j0}$ which is closest to $\vec{x}_i$, and is dominated by it, is sought. The distance between the individuals $\vec{x}_{j0}$ and $\vec{x}_i$ will be called maximum radius, $r_{max}$, and is defined as

$$r_{max,i} = \min\{dist(x_i, x_j)\}, \quad 0 \leq i, j \leq N \tag{17}$$

Figure 2 illustrates the determination of the maximum radius.

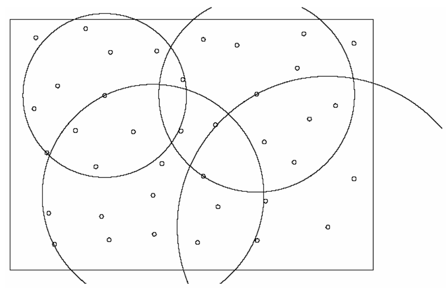

Figure 2. Determination of the maximum radius, according to Eq. (17).

After having determined the maximum radius for each individual, the local minima can be obtained. Two approaches are considered. In the first approach, it is verified how many points are contained within a distance $r_{max,i}$ of the individual $x_i$. If the number of points is greater than a prescribed constant, the $x_i$ individual is considered as a local minimum, and kept for the next step. In the second approach, the maximum radius is compared with the mean distance among the individuals. If the radius $r_{max,i}$ is sufficiently greater than the mean distance, the $x_i$ individual is considered as a local minimum, and kept for the next step. The result of the local-minima search process is shown in Fig. 3, for the points appearing in Fig. 2.

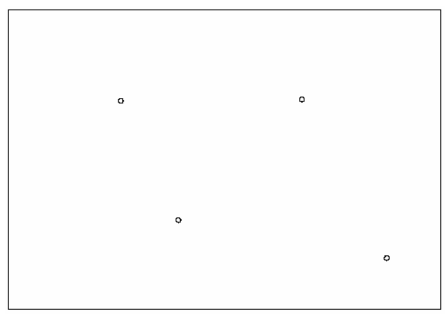

Figure 3. Local minima obtained with the search process applied to the points in Figure 2.

After selecting some local minima, according to the search process just described, the algorithm proceeds to optimize the regions close to each local minimum.

For each point $\vec{x}_i$ obtained, a cluster is generated around the individual. This cluster consists of new individuals randomly chosen, according to an exponential distribution around $\vec{x}_i$,

$$\vec{x}_{i,j} = \vec{x}_i + r_{min,i} \cdot \log(y_j) \cdot \vec{v}_j \tag{18}$$

where $\vec{v}_j$ is a random vector with unitary modulus, $r_{min,k}$ is the distance to the closest individual, and $y_j$ is a random vector with uniform distribution. The clustering scheme is illustrated in Fig. 4.

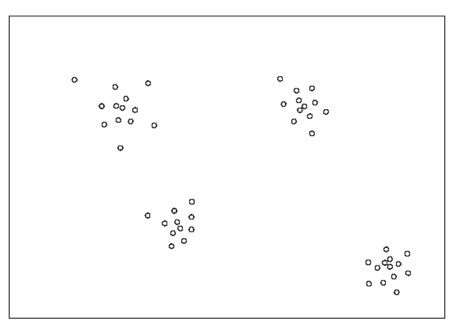

Figure 4. Clustering around the local minima.

The new non-dominated individuals are again obtained by using Eq. (16). In order to produce a uniformly distributed Pareto front, and not to increase excessively the computational cost, the closest points are discarded. For each non-dominated individual $\vec{x}_k$, a new function is defined as

$$F_{\vec{w}}(\vec{x}) = \vec{w} \cdot \vec{f}(\vec{x}) \tag{19}$$

The vector $\vec{w}$ is chosen in such a way, that the $\vec{x}_k$ individual becomes the one, among the non-dominated individuals, which minimizes the function. After this, the steepest descent method is applied to the function $F_{\vec{w}}(\vec{x})$, having the individual $\vec{x}_k$ as the initial guess.

### 4.3. Objective function for the CGAM problem

The original CGAM problem formulation defines the total cost rate as the sum, on a rate basis, of the capital investment cost, the operation and maintenance costs, and the fuel cost; in fact, the total cost rate is the objective function, *OF* (\$/h), to be minimized when solving the CGAM problem, and is written as (Bejan *et al.*, 1996; Tsatsaronis, 1994; Padilha, 2006)

$$OF = \sum_{k=1}^{5} \dot{Z}_k + \dot{C}_{\mathrm{f}} = \dot{Z} + \dot{C}_{\mathrm{f}} = \frac{\left( \sum_{k=1}^{5} CRF\,(1+\gamma)\,TCI_k \right)}{\tau} + c_{\mathrm{f}}\,\dot{m}_{\mathrm{f}}\,LHV \tag{20}$$

The values prescribed for the parameters of the economic model are (Tsatsaronis, 1994; Padilha, 2006): $\beta = 1$, $i$ = 12.7%, $l$ = 10 years, $\tau = 8000$ hours, and $\gamma = 0.06$. In Table 1, the optimal values for the decision variables and objective function of the CGAM problem are shown (Tsatsaronis, 1994).

Table 1. Optimal values for the decision variables and objective function of the CGAM problem.

| Variable | Optimal value |
|---|---|
| $R_{\mathrm{c}}$ | 8.5234 |
| $\eta_{\mathrm{AC}}$ | 0.8468 |
| $T_3$ (K) | 914.28 |
| $\eta_{\mathrm{GT}}$ | 0.8786 |
| $T_4$ (K) | 1492.63 |
| $OF$ (\$/h) | 1303.23 |

The original CGAM problem is formulated in terms of one objective function only (Tsatsaronis, 1994). This objective function is defined as the sum of all system cost rates, Eq. (20). From a different perspective, a new multi-objective formulation can be proposed to the optimization of the CGAM system, which consists in the independent evaluation of the main costs of the system.

The objective function of the original CGAM problem, *OF*, is the total cost rate of the system, which is composed of the cost rate of fuel, $\dot{C}_{\mathrm{f}}$, the total purchased-equipment cost rate, $\dot{Z}_{\mathrm{PEC}}$, and the cost rate associated with the operation and maintenance of the plant, $\dot{Z}_{\mathrm{O\&M}}$, such that

$$OF = \dot{Z}_{\mathrm{PEC}} + \dot{Z}_{\mathrm{O\&M}} + \dot{C}_{\mathrm{f}} = \dot{Z} + \dot{C}_{\mathrm{f}} \tag{21}$$

Comparing Eq. (20) with Eq. (21), and using Eq. (1) with $\beta = 1$, we can verify that

$$\dot{Z}_{\mathrm{PEC}} = \frac{CRF}{\tau} \left( \sum_{k=1}^{5} TCI_k \right) = \frac{CRF}{\tau} TCI \tag{22}$$

$$\dot{Z}_{\mathrm{O\&M}} = \frac{CRF\ \gamma}{\tau}\left(\sum_{k=1}^{5} TCI_k\right) = \frac{CRF\ \gamma}{\tau} TCI \tag{23}$$

$$\dot{C}_{\mathrm{f}} = c_{\mathrm{f}}\, \dot{m}_{\mathrm{f}}\, LHV \tag{24}$$

Here, in order to perform a multi-objective optimization of the CGAM system, it is proposed to use separate costs as objective functions, instead of using the total cost rate of the system. Therefore, the fuel consumption cost rate, $\dot{C}_{\mathrm{f}}$, and the total capital investment, *TCI*, are used as the objective functions to be minimized. This choice for the objective functions is justified, since the total purchased-equipment cost rate, $\dot{Z}_{\mathrm{PEC}}$, and the cost rate associated with the operation and maintenance of the plant, $\dot{Z}_{\mathrm{O\&M}}$, can be obtained easily using Eqs. (22) and (23), respectively, with $\tau$ = 8000 hours, $\gamma$ = 0.06, and *CRF* = 18.2%, as discussed previously.

The main advantage of this new proposal consists in the non-dependency of the result with respect to the amortization rate (*CRF*). The designer is then able to evaluate the influence of the acquisition cost on the fuel consumption cost. In this way, the designer can make a better decision, according to the appropriate economical scenario at the time.

### 4.4. Multi-objective optimization results

Figure 5 shows the Pareto front obtained as a solution of the multi-objective optimization of the CGAM system, using the objective functions defined in the previous section. The standard CGAM system has its optimum located at the coordinates (5.4516 x $10^6$ \$, 0.325489 \$/s), where the first coordinate is the total capital investment, and the second coordinate is the fuel consumption cost rate, given in \$/s. As one can deduce from the analysis of the Pareto front in Fig. 5, the single-objective result is a particular solution of the multi-objective solution. In fact, the solution of the single-objective optimization is 1303.23 \$/h, which can be recovered promptly by replacing the appropriate values of $\dot{C}_{\mathrm{f}}$, *TCI*, $\gamma$, *CRF*, and $\tau$ into Eqs. (21)-(24).

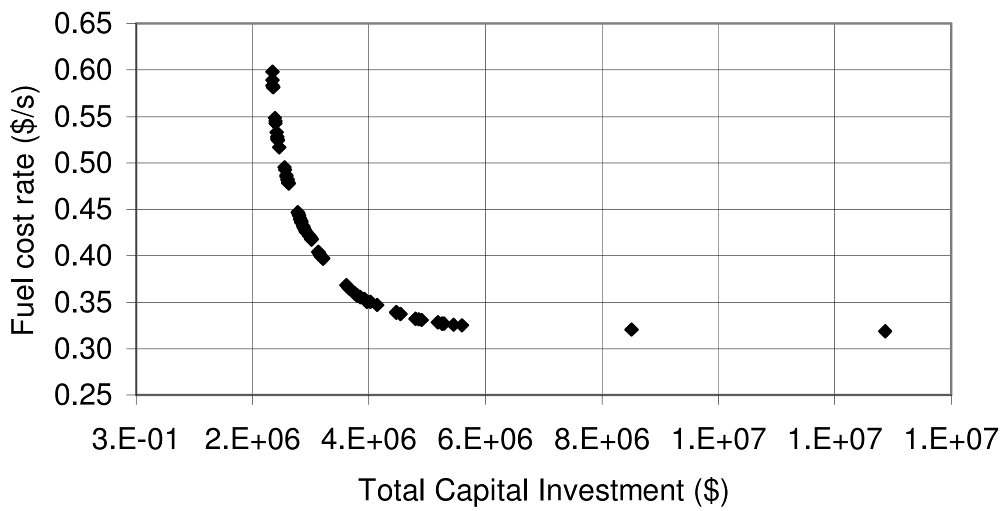


Figure 5. Pareto front for the CGAM system, corresponding to the objective functions $\dot{C}_{\mathrm{f}}$ and TCI.

## 5. CONCLUSIONS

Efficient design of energy systems is an integral part of the solution to meet current demands on minimal energy use and minimal environmental impacts. In the literature, it is common to perform single-objective design optimization of an energy system. In fact, the benchmark CGAM problem was formulated as a single-objective design optimization problem, where the objective function was the sum, on a rate basis, of the purchased-equipment, maintenance and operation, and fuel consumption costs. However, in real-life applications, costs must be analyzed individually. In this work, a new perspective is thus taken, in that multi-objective optimization of the CGAM system is effected using a hybrid algorithm, which attempts to combine the strengths of deterministic and heuristic methods. The objective functions in the multi-objective optimization are the fuel consumption cost rate and the total capital investment. The

Pareto front is obtained for all non-dominated solutions, which encompasses the original CGAM single-objective solution. Based on the Pareto front, the designer is better equipped to make decisions based on specific, appropriate scenarios.

## 6. ACKNOWLEDGEMENTS

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