ADAPTIVE CONTROL OF A 3 DOF HELICOPTER MODEL USING NEURAL NETWORKS

Rafael Purcell Breckenfeld, rafaelpb@ita.br
Elder Moreira Hemerly, hemerly@ita.br
Technological Institute of Aeronautic, Mal. Eduardo Gomes, 50, Vila das Acácias, 12228-900, São José dos Campos, São Paulo, Brazil

Abstract. *A Neural Network (NN) combined with PD (proportional-derivative) controller is proposed in this paper for application in underactuated nonlinear systems. The main goal of this work is to solve the reference trajectory tracking problem of a three degrees of freedom (DOF) helicopter platform model with two control inputs obtained by Euler-Lagrange method. This control technique is derived from the estimate of the helicopter nonlinear function performed by the NN, combined with an outer PD tracking loop and an auxiliary signal that provides robustness in the face of unmodeled bounded disturbances, as well as unstructured unmodeled dynamics. The PD controller design is based on a LQR controller, which is designed by using the helicopter linearized model. Lyapunov second method is employed to establish stable weights adaptation laws, which are tuned on-line, and the control system stability, thereby guaranteeing small tracking errors and bounded control signals. The Lyapunov stability of the control system are presented and realistic simulation results are discussed. These results improve two features concerning the literature: 1) the NNPD can deal with parametric uncertainty and variation and unmodeled dynamics, and 2) it considers control of the 3 DOF helicopter model.*

Keywords: *Adaptive Control, Neural Networks, Underactuated Systems, Autonomous Helicopter, Lyapunov Stability.*

1. INTRODUCTION

An important aspect in any control design is the effect of parametric uncertainty and variation and unmodeled dynamics. Intensive research efforts have been devoted to adaptive control of uncertain nonlinear systems. Universal approximation properties of Neural Networks (NNs) have been widely employed to model complex nonlinear physical characteristics. The focus in this paper is on control design of underactuated nonlinear systems, in others words, mechanics systems with fewer actuators (i.e. controls) than degrees of freedom (DOF) of the systems, for example underwater and aerospace vehicles. A Tandem Fan (TF) in a 3 DOF platform is considered in this work, which is obtained by Euler-Lagrange method. This system is a laboratory model helicopter produced by Quanser (2005) that emulates some of the dynamics of a tilt-rotor in helicopter configuration (Rysdyk 2005) and allows the evaluation of multivariable control techniques.

The main goal of this work is to solve the reference trajectory tracking problem. An approach for this issue is presented in (Calise 2001), which is an adaptive output feedback design procedure, employing feedback linearization coupled with an NN. Nevertheless, the control design was tested only by controlling the pitch axis of the helicopter (Kutay 2005). The control technique proposed is based in (Lewis 1999) and belong in the large class of approximation-based controllers, which provide robustness even in the presence of unknown dynamics and disturbances.

Therefore, the contributions of this paper include: $1)$ a control framework to deal with the three degrees of freedom of the helicopter model, that is not performed either in (Calise 2001) or in (Kutay 2005); $2)$ an adaptation law to the NN weights and the formal stability proof of the closed-loop control scheme by employing Lyapunov's second method, and $3)$ a performance comparison of this controller with a PID controller, aiming at showing the improvements, as well as robustness and adaptation capability to handle parametric uncertainty and variation and unstructured unmodeled dynamics.

2. HELICOPTER DYNAMICS AND PROPERTIES

Consider the helicopter model shown in Fig. 1 with active disturbance mass, which can provide a parametric variation to the system defined by the designer. The dynamic model of the system was obtained by x-convention for the Euler angles, using the rotational kinematic of a rigid body (Galindo 2000). The Fig. 2 presents the coordinate axes of the helicopter in a right-hand inertial frame $\{x,\ y,\ z\}$, where the point C is the gravity center and guidance point, $\{x',\ y',\ z'\}$ denotes the right-hand body frame of the TF, M_h is the helicopter mass and M_{cw} is the counterweight mass. The generalized coordinates of the system ($\mathbf{q}$) are depicted by the roll (ϕ), pitch (θ) and yaw (ψ) angles, the length d_θ is the distance between the O and C points and d_ϕ is the half distance between the fans. The moments of inertia of the TF with respect to each axis in the (ϕ, θ, ψ) frame is I_ϕ, I_θ and I_ψ, respectively. The forces produced by the fans are F_f, to the front motor, and F_r to the rear.

The helicopter dynamics can be described by the Euler-Lagrange model conform

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_o(\dot{\mathbf{q}}, \mathbf{q})\dot{\mathbf{q}} + \mathbf{F}(\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \tau_d = \tau \tag{1}$$

where $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{3\times 3}$ is the inertia matrix, which is positive definite, $\mathbf{C}_o(\dot{\mathbf{q}}, \mathbf{q}) \in \mathbb{R}^{3\times 3}$ is the Coriolis/centripetal matrix,

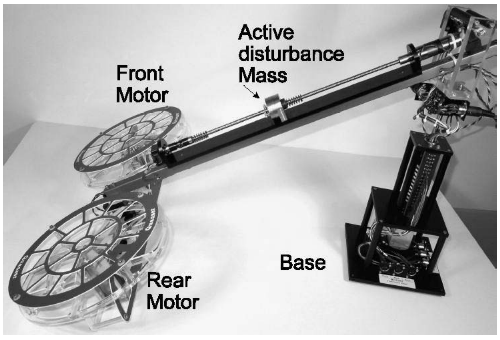

Figure 1. Helicopter with Active Disturbance Mass.

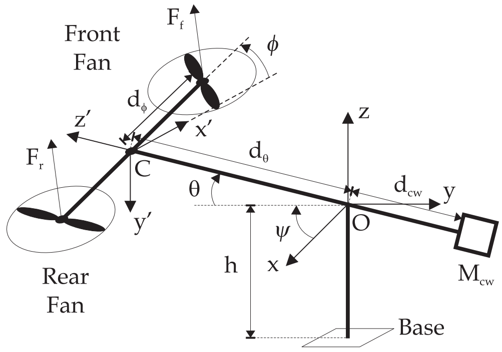

Figure 2. Tandem Fan in a 3 DOF Platform.

$\mathbf{F}(\dot{\mathbf{q}}) \in \mathbb{R}^3$ represents the friction terms, $\mathbf{G}(\mathbf{q}) \in \mathbb{R}^3$ is the gravity vector and $\tau_d(t) \in \mathbb{R}^3$ denotes the bounded unknown disturbances, including unstructured unmodeled dynamics. The control input vector $\tau \in \mathbb{R}^3$ is the torque in the generalized coordinate frame. The matrices $\mathbf{M}(\mathbf{q})$, $\mathbf{C}_o(\dot{\mathbf{q}}, \mathbf{q})$ and $\mathbf{G}(\mathbf{q})$ can be found in (Galindo 2000) and the vector $\mathbf{F}(\dot{\mathbf{q}})$ in (Lewis 1999). The following properties present some physical features of the helicopter that simplify the solution of the control problem.

Property 1 *The Coriolis/centripetal vector can always be selected so that the matrix* $\boldsymbol{S}(\dot{\boldsymbol{q}}, \boldsymbol{q}) \equiv \dot{\boldsymbol{M}}(\boldsymbol{q}) - 2\boldsymbol{C}_0(\dot{\boldsymbol{q}}, \boldsymbol{q})$ *is skew-symmetric. Therefore,* $\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{x} = 0$ *for all vector* $\boldsymbol{x}$*.*

Property 2 *The disturbances term are bounded so that* $\|\tau_d(t)\| \leq d_B$*.*

In Fig. 2, F_f and F_r can be equivalently represented by a force $F_s \triangleq F_f + F_t$ applied to the C point plus a torque $F_d d_\phi$ around the z'-axis, where $F_d \triangleq F_f - F_t$. Assume that $F_f = K_f V_f$ and $F_r = K_f V_r$, with K_f being a motor force constant and V_f and V_r the voltages applied in the front and rear motors, respectively. From (Galindo 2000), the torque in the inertial frame is given by $\tau = \mathbf{T}(\mathbf{q})\mathbf{V}$, where $\mathbf{T}(\mathbf{q}) \in \mathbb{R}^{3\times 2}$ is a control transformation matrix and $\mathbf{V} = [V_f \; V_r]^T$.

Thus, it can be seen that the helicopter model is a nonlinear, coupled and underactuated system, in other words, the yaw (ψ) motion occurs due to the combined motion of the roll (ϕ) and pitch (θ) angles. Furthermore, θ and ψ are not be controllable for $\theta = 0,5k\pi$, with $k = 1,\ 3,\ \ldots$ and for $\phi = k\pi$, with $k = 0,\ 1,\ \ldots$, respectively.

3. ADAPTIVE CONTROL VIA RNA

3.1 Approximation-based control framework

From Equation (1), the helicopter dynamics can be rewritten by

$$I_\phi \ddot{\phi} + F_\phi + \tau_{d\phi} = d_\phi K_f V_d \tag{2}$$

$$\bar{\mathbf{M}}(\bar{\mathbf{q}})\ddot{\bar{\mathbf{q}}} + \bar{\mathbf{C}}_o(\dot{\bar{\mathbf{q}}}, \bar{\mathbf{q}})\dot{\bar{\mathbf{q}}} + \bar{\mathbf{F}}(\dot{\bar{\mathbf{q}}}) + \bar{\mathbf{G}}(\bar{\mathbf{q}}) + \bar{\tau}_d = \bar{\tau} \tag{3}$$

where $\bar{\mathbf{q}} = [\theta \; \psi]^T$ and $\bar{\tau} = \bar{\mathbf{T}}(\mathbf{q})V_s$, and the Property 1-2 continue valid. In control design the objective is generally to make the system follow a prescribed and bounded reference trajectory, $\bar{\mathbf{q}}_r = [\theta_r \; \psi_r]^T$, which take the point C as a guidance point. Finding a control signals $V_s = V_f + V_r$ and $V_d = V_f - V_r$ that causes this to occur is called the tracking design problem. Thus, given the reference trajectory $\bar{\mathbf{q}}_r$, the tracking error $\mathbf{e}(t) = [e_\theta(t) \; e_\psi(t)]^T$ and filtered tracking error $\mathbf{r}(t) = [r_\theta(t) \; r_\psi(t)]^T$ are defined by

$$\mathbf{e} = \bar{\mathbf{q}}_r - \bar{\mathbf{q}} \tag{4}$$

$$\mathbf{r} = \dot{\mathbf{e}} + \Lambda\mathbf{e} \tag{5}$$

with $\Lambda = diag\{\Lambda_\theta, \Lambda_\psi\}$ a positive definite design matrix. Then, the Eq.(5) is a stable system so that $\mathbf{e}(t)$ is bounded as long as the controller guarantees that the filtered error $\mathbf{r}(t)$ is bounded. From (Lewis 1999), it is possible to shown that

$$\|\mathbf{e}\| \leq \frac{\|\mathbf{r}\|}{\sigma_{\min}(\Lambda)} \quad \text{and} \quad \|\dot{\mathbf{e}}\| \leq \|\mathbf{r}\| \tag{6}$$

where $\sigma_{\min}(\Lambda)$ is the minimum singular value de Λ and $\|\cdot\|$ is the 2-norm. In practical situations, the reference trajectory always satisfies the following boundedness assumption.

Assumption 1 *The reference trajectory is bounded so that* $\|[\boldsymbol{q}_r^T \; \dot{\boldsymbol{q}}_r^T \; \ddot{\boldsymbol{q}}_r^T]^T\| \leq q_B$*, with* q_B *a known scalar bound.*

Differentiating the Equation (5) and invoking the Eq.(3), it can be seen that the helicopter dynamics are expressed in terms of the $\mathbf{r}(t)$ as

$$\bar{\mathbf{M}}\dot{\mathbf{r}} = -\bar{\mathbf{C}}_o\mathbf{r} + f(\mathbf{x}) + \bar{\tau}_d - \bar{\tau} \tag{7}$$

where the nonlinear helicopter function is defined as

$$f(\mathbf{x}) = \bar{\mathbf{M}}(\bar{\mathbf{q}})(\ddot{\bar{\mathbf{q}}}_r + \Lambda\dot{\mathbf{e}}) + \bar{\mathbf{C}}_o(\dot{\bar{\mathbf{q}}}, \bar{\mathbf{q}})(\dot{\bar{\mathbf{q}}}_r + \Lambda\mathbf{e}) + \mathbf{F}(\dot{\bar{\mathbf{q}}}) + \mathbf{G}(\bar{\mathbf{q}}) \tag{8}$$

Vector $\mathbf{x}$ contains all the time signals needed to compute $f(\cdot)$ and may be defined as $\mathbf{x} = [\mathbf{e}^T\ \dot{\mathbf{e}}^T\ \bar{\mathbf{q}}_r^T\ \dot{\bar{\mathbf{q}}}_r^T\ \ddot{\bar{\mathbf{q}}}_r^T]^T$. It is important to note that $f(\mathbf{x})$ contains all potentially unknown helicopter parameters, or that hardly can be modeled with accuracy, like mass, moments of inertia and coefficients friction, expect for the $\bar{\mathbf{C}}_o\mathbf{r}$ term in Eq.(7), which cancels out in controller stability Lyapunov proofs.

Let the nonlinear dynamic of the helicopter, a sort of approximation-based controller is depicted by

$$\bar{\tau} = \hat{f}(\mathbf{x}) + \mathbf{K}_d\mathbf{r} - \tau_r(t) \tag{9}$$

with $\hat{f}$ as an estimate of $f(\mathbf{x})$, $\mathbf{K}_d\mathbf{r} = \mathbf{K}_d\dot{\mathbf{e}} + \mathbf{K}_d\Lambda\mathbf{e}$ two outer PD tracking loops, and $\tau_r(t)$ an auxiliary signal to provide robustness in the face of disturbances and modeling errors. Since $\bar{\tau} = \bar{\mathbf{T}}(\mathbf{q})V_s$, the motors voltages sum is given by

$$V_s = \bar{\mathbf{T}}(\mathbf{q})^{\dagger}[\hat{f}(\mathbf{x}) + \mathbf{K}_d\mathbf{r} - \tau_r(t)] \tag{10}$$

where $\bar{\mathbf{T}}(\mathbf{q})^{\dagger}$ is the pseudoinverse of $\bar{\mathbf{T}}(\mathbf{q})$.

Hence, the controller design and the stability Lyapunov proof of the system are based on the closed-loop dynamic error (Lewis 1999), which is found by the substitution of the Eq.(9) into Eq.(3), resulting

$$\bar{\mathbf{M}}\dot{\mathbf{r}} = -\bar{\mathbf{C}}_o\mathbf{r} - \mathbf{K}_d\mathbf{r} + \tilde{f} + \bar{\tau}_d + \tau_r(t) \tag{11}$$

where the function approximation error is given by $\tilde{f} = f - \hat{f}$.

The controller design problem is to select the estimate $\hat{f}$ and to define a robust term $\tau_r(t)$ in the control law of Eq.(9) so that this dynamic error is stable. Solving this problem ensures that the filtered tracking error $\mathbf{r}(t)$ is bounded and the Eq.(6) guarantee that the tracking error $\mathbf{e}(t)$ is bounded. Then, the helicopter follows the prescribes trajectory $\bar{\mathbf{q}}_r(t)$.

The closed-loop control structure to the ϕ angle is done employing a PD controller in the ϕ dynamic, given in Eq.(2), which is expressed by

$$V_d = K_{p\phi}(-\phi_r - \phi) - K_{d\phi}\dot{\phi} \tag{12}$$

In the controller design, it will be supposed, for convenience, that the friction torque F_ϕ and the disturbances term $\tau_{d\phi}(t)$ are null. So, the transfer function that describe the ϕ motion is given by

$$\frac{\Phi(s)}{\Phi_r(s)} = -\frac{d_\phi K_f K_{p\phi}}{I_\phi s^2 + d_\phi K_f K_{d\phi} s + d_\phi K_f K_{p\phi}} \tag{13}$$

with $\Phi(s) = \mathfrak{L}\{\phi(t)\}$, where $\mathfrak{L}$ is the Laplace transform operator.

If the gains $K_{p\phi}$ and $K_{d\phi}$ are positive constants, the inner closed-loop system Eq.(13) is said stable. Defining

$$\phi_r(t) \triangleq K_{d\psi}r_\psi(t) = K_{d\psi}\dot{e}_\psi(t) + K_{d\psi}\Lambda_\psi e(t) \tag{14}$$

is possible to note a framework shaped by cascade PD controllers, where the PD controller given in Eq.(14) provides a reference to the controller described in Eq.(12). Consequently, the roll angle $\phi(t)$ will be bounded only if the filtered tracking error $\mathbf{r}(t)$ is bounded.

Therefore, the solution of the reference trajectory tracking problem, consequently the control of the helicopter model with three degrees of freedom, is given by Eq.(10), Eq.(12) and Eq.(14). The Figure 3 illustrate the approximation-based control structure of the helicopter. Note that only the angular position of the helicopter is used for feedback, not requiring those states concerning the angular velocity.

3.2 Approximation of the helicopter nonlinear function using neural networks

The feedforward NN used in this work is shaped for three layers and has two layers of adjustable weights. The net output $\mathbf{y}$ ia a vector with m components that are determined in terms of the n components of the input vector $\mathbf{x}$ by

$$y_i = \sum_{j=1}^{N_h}\left[w_{ij}\,\sigma\left(\sum_{k=1}^{n} v_{jk}x_k + \theta_{vj}\right) + \theta_{wi}\right] \qquad i = 1, \ldots, m \tag{15}$$

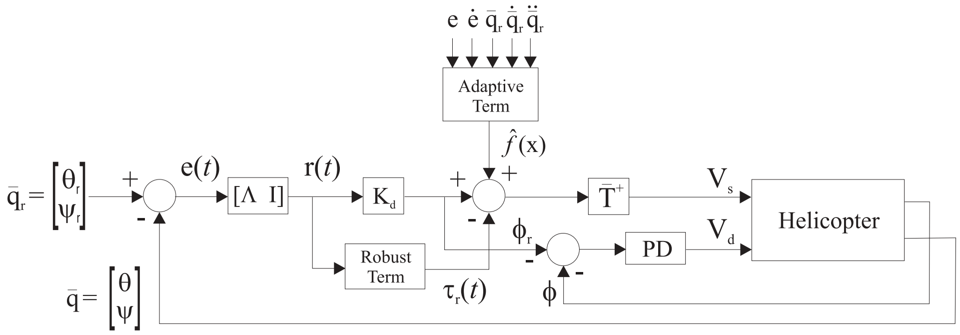

Figure 3. Approximation-based control framework to the helicopter.

where $\sigma(\cdot)$ are the activation functions and N_h is the number of hidden-layer units (neurons). The inputs-to-hidden-layer interconnection weights are denoted by v_{jk} and the hidden-layer-to-outputs interconnection weights by w_{ij}. The threshold offsets are denoted by θ_{vj} and θ_{wi} (Lewis 1999).

Many different activation functions $\sigma(\cdot)$ are commonly used, including sigmoid, hyperbolic tangent and gaussian. In this paper will be used the sigmoid activation function

$$\sigma(x) = \frac{1}{1+e^{-x}} \tag{16}$$

whose gradient, for instance, is given by $\sigma'(x) = \sigma(x)[1-\sigma(x)]$. By collecting all the NN weights v_{jk} and w_{ij} into matrices of weights $\mathbf{W}^T$ and $\mathbf{V}^T$, the Eq.(15) can be written in matrix notation as

$$\mathbf{y} = \mathbf{W}^T\sigma(\mathbf{V}^T\mathbf{x}) \tag{17}$$

with the vector of activation functions defined by $\sigma(z) = [\sigma(z_1)\ \sigma(z_2)\ \cdots\ \sigma(z_{N_h})]^T$ for a vector $\mathbf{z} \in \mathbb{R}^{N_h}$. The threshold are included as the first columns of the weight matrices. To adapt this feature, the vectors $\sigma(\cdot)$ and $\mathbf{x}$ need to augmented like $\mathbf{x} \equiv [1\ x_1\ x_2\ \cdots\ x_n]^T$, for example. So, any tuning of $\mathbf{W}$ and $\mathbf{V}$ also includes tuning of the thresholds.

For control purposes, it will be used a important feature of the NN, the universal approximation property (Barron 1993) where, for every smooth function $f(\mathbf{x})$ from $\mathbb{R}^n \rightarrow \mathbb{R}^m$, there exists a neural net works such that

$$f(\mathbf{x}) = \mathbf{W}^T\sigma(\mathbf{V}^T\mathbf{x}) + \varepsilon \tag{18}$$

for some weights $\mathbf{W}$ and $\mathbf{V}$. This approximation holds for all $\mathbf{x}$ in a compact set $\mathcal{S}$ and the functional estimation error ε is bounded so that $\|\varepsilon\| < \varepsilon_M$, with ε_M a known bound dependent of $\mathcal{S}$. Then, an estimate of $f(\mathbf{x})$ is given by

$$\hat{f}(\mathbf{x}) = \hat{\mathbf{W}}^T\sigma(\hat{\mathbf{V}}^T\mathbf{x}) \tag{19}$$

with $\hat{\mathbf{W}}$ e $\hat{\mathbf{V}}$ the actual values of the NN weights given by a tuning algorithm and $\hat{f}(\mathbf{x})$ is the NN output. Now, some definitions and assumptions are required to proceed.

Definition 1 *Given $\boldsymbol{A} = [a_{ij}]$ and $\boldsymbol{B} \in \mathbb{R}^{m\times n}$, the Frobenius norm is defined by*

$$\|\boldsymbol{A}\|_F^2 = tr\{\boldsymbol{A}^T\boldsymbol{A}\} = \sum_{i,j} a_{ij}^2 \tag{20}$$

with tr{} the trace. The associated inner product is $\langle \boldsymbol{A},\boldsymbol{B}\rangle_F = tr\{\boldsymbol{A}^T\boldsymbol{B}\}$. The Frobenius norm is compatible with the 2-norm so that $\|\boldsymbol{A}\boldsymbol{x}\| \leq \|\boldsymbol{A}\|_F\|\boldsymbol{x}\|$, with $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ and $\boldsymbol{x} \in \mathbb{R}^n$ (Lewis 1999).

Definition 2 *For notational convenience, all the NN weights are defined as $\hat{\boldsymbol{Z}} = diag\{\hat{\boldsymbol{W}}, \hat{\boldsymbol{V}}\}$ and the weight estimation errors are defined as $\tilde{\boldsymbol{V}} = \boldsymbol{V} - \hat{\boldsymbol{V}}$, $\tilde{\boldsymbol{W}} = \boldsymbol{W} - \hat{\boldsymbol{W}}$ and $\tilde{\boldsymbol{Z}} = \boldsymbol{Z} - \hat{\boldsymbol{Z}}$.*

Definition 3 *The hidden-layer output error for a given $\boldsymbol{x}$ is defined as*

$$\tilde{\sigma} = \sigma - \hat{\sigma} = \sigma(\boldsymbol{V}^T\boldsymbol{x}) - \sigma(\hat{\boldsymbol{V}}^T\boldsymbol{x}) \tag{21}$$

The Taylor series expansion of $\sigma(\cdot)$ for a given $\boldsymbol{x}$ may be written as

$$\sigma(\boldsymbol{V}^T\boldsymbol{x}) = \sigma(\hat{\boldsymbol{V}}^T\boldsymbol{x}) + \hat{\sigma}'(\hat{\boldsymbol{V}}^T\boldsymbol{x}).\tilde{\boldsymbol{V}}^T\boldsymbol{x} + O_\sigma(\tilde{\boldsymbol{V}}^T\boldsymbol{x}) \tag{22}$$

with

$$\sigma'(\hat{z}) \equiv \left[\frac{d\sigma(z)}{dz}\right]_{z=\hat{z}} = diag\{\sigma(\hat{z})\}[\boldsymbol{I} - diag\{\sigma(\hat{z})\}] \tag{23}$$

the Jacobian matrix an $O_\sigma(\tilde{\boldsymbol{V}}^T\boldsymbol{x})$ denoting the higher-order terms in the Taylor series. Denoting $\hat{\sigma}' = \sigma'(\hat{\boldsymbol{V}}^T\boldsymbol{x})$, result

$$\tilde{\sigma} = \sigma'(\hat{\boldsymbol{V}}^T\boldsymbol{x})\tilde{\boldsymbol{V}}^T\boldsymbol{x} + O_\sigma(\tilde{\boldsymbol{V}}^T\boldsymbol{x}) = \hat{\sigma}'\tilde{\boldsymbol{V}}^T\boldsymbol{x} + O_\sigma(\tilde{\boldsymbol{V}}^T\boldsymbol{x}) \tag{24}$$

The importance of the Eq.(24) is that it replaces $\tilde{\sigma}$, which is nonlinear in $\tilde{\mathbf{V}}$, by an expression linear in $\tilde{\mathbf{V}}$ plus higher-order terms. This allows to determination of tuning algorithms for the matrices of the weights.

Assumption 2 *On any compact subset of $\mathbb{R}^n$, the ideal NN weights are bounded by known positive values so that $\|\boldsymbol{V}\|_F \leq V_B$, $\|\boldsymbol{W}\|_F \leq W_B$ or $\|\boldsymbol{Z}\|_F \leq Z_B$ with Z_B known.*

Lemma 1 (Bound on NN Input x) *For each t, the vector $\boldsymbol{x}(t)$ is bounded by*

$$\|\boldsymbol{x}\| \leq c_1 + c_2\|\boldsymbol{r}\| \leq q_B + c_0\|\boldsymbol{r}(0)\| + c_2\|\boldsymbol{r}\| \tag{25}$$

for computable positive constants c_i.

Lemma 2 (Bounds on Taylor Series Higher-Order Terms) *For sigmoid activation function, the higher-order terms in the Taylor series, in Eq.(24), are bounded by*

$$\|O_\sigma(\tilde{\boldsymbol{V}}^T\boldsymbol{x})\| \leq c_3 + c_4\|\tilde{\boldsymbol{V}}\|_F + c_5\|\tilde{\boldsymbol{V}}\|_F\|\boldsymbol{r}\| \tag{26}$$

for computable positive constants c_i.

The function estimate error of the helicopter, $\tilde{f} = f - \hat{f}$, is obtained adding and subtracting $\mathbf{W}^T\hat{\sigma}$ from $\tilde{f}$, resulting

$$\tilde{f} = \mathbf{W}^T\sigma + \varepsilon - \hat{\mathbf{W}}^T\sigma = \mathbf{W}^T\tilde{\sigma} + \tilde{\mathbf{W}}^T\hat{\sigma} + \varepsilon \tag{27}$$

Adding and subtracting $\hat{\mathbf{W}}^T\tilde{\sigma}$ in Eq.(27), yields

$$\tilde{f} = \tilde{\mathbf{W}}^T\tilde{\sigma} + \tilde{\mathbf{W}}^T\hat{\sigma} + \hat{\mathbf{W}}^T\tilde{\sigma} + \varepsilon \tag{28}$$

Using Eq.(24) from the Def. 3, $\tilde{f}$ may be written as

$$\tilde{f} = \hat{\mathbf{W}}^T\hat{\sigma}'\tilde{\mathbf{V}}^T\mathbf{x} + \tilde{\mathbf{W}}^T(\hat{\sigma} - \hat{\sigma}'\hat{\mathbf{V}}^T\mathbf{x}) + \delta \tag{29}$$

where disturbances terms are defined as

$$\delta(t) = \tilde{\mathbf{W}}^T\hat{\sigma}'\mathbf{V}^T\mathbf{x} + \mathbf{W}^T O_\sigma(\tilde{\mathbf{V}}^T\mathbf{x}) + \varepsilon \tag{30}$$

It is important to note that the NN reconstruction error $\varepsilon(\mathbf{x})$ and the higher-order terms in the Taylor series expansion of $\tilde{\sigma}$ have exactly the same influence as disturbances in the error system.

Lemma 3 (Bounds on the Disturbance Term) *The disturbance term in Eq.(30) is bounded according to*

$$\|\delta(t)\| \leq (\varepsilon_M + c_3 Z_M) + c_6 Z_M\|\tilde{\boldsymbol{Z}}\|_F + c_7 Z_M\|\tilde{\boldsymbol{Z}}\|_F\|\boldsymbol{r}\| \quad \textit{or} \quad \|\delta(t)\| \leq C_0 + C_1\|\tilde{\boldsymbol{Z}}\|_F + C_2\|\tilde{\boldsymbol{Z}}\|_F\|\boldsymbol{r}\| \tag{31}$$

with C_i known positive constants.

Note that C_0 becomes larger as increases the NN estimation error ε. The Proofs of Lemmas 1-3 are omitted here due to the lack of space. The importance of these lemmas lies on the definition of an upper bound to the $\|\mathbf{x}(t)\|$, $\|O_\sigma(\cdot)\|$ and $\|\delta(t)\|$ by known computable function $\|\tilde{\mathbf{Z}}\|_F$ and $\|\mathbf{r}(t)\|$. In this case, if the controller guarantees that $\|\tilde{\mathbf{Z}}\|_F$ and $\|\mathbf{r}(t)\|$ are bounded, thus the norms given by Eq.(25), Eq.(26) and Eq.(31) will be bounded as well.

3.3 Neural networks weight tuning for tracking stability

Substituting the Equation (29) into Eq.(11), the dynamic error of the system is expressed by

$$\bar{\mathbf{M}}\dot{\mathbf{r}} = -(\mathbf{K}_d + \bar{\mathbf{C}}_o)\mathbf{r} + \hat{\mathbf{W}}^T\hat{\sigma}'\tilde{\mathbf{V}}^T\mathbf{x} + \tilde{\mathbf{W}}^T(\hat{\sigma} - \hat{\sigma}'\hat{\mathbf{V}}^T\mathbf{x}) + \delta + \bar{\tau}_d + \tau_r \tag{32}$$

The next theorem shows how to tune the weights in the NN so that tracking and internal stability are guaranteed.

Theorem 1 *Consider the control structure of the Fig. 3 to the 3 DOF helicopter described by Eq.(1), with the control laws given by Eq.(10), Eq.(12) and Eq.(14), with robustness term*

$$\tau_r(t) = -k_z(\|\hat{\boldsymbol{Z}}\|_F + Z_M)\boldsymbol{r} \tag{33}$$

where $k_z > C_2 > 0$. Let the NN weight update laws be provided by

$$\dot{\hat{\boldsymbol{W}}} = \boldsymbol{F}(\hat{\sigma} - \hat{\sigma}'\hat{\boldsymbol{V}}^T\boldsymbol{x})\boldsymbol{r}^T - k\|\boldsymbol{r}\|\boldsymbol{F}\hat{\boldsymbol{W}} \tag{34}$$

$$\dot{\hat{\boldsymbol{V}}} = \boldsymbol{G}\boldsymbol{x}\boldsymbol{r}^T\hat{\boldsymbol{W}}^T\hat{\sigma}' - k\|\boldsymbol{r}\|\boldsymbol{G}\hat{\boldsymbol{V}} \tag{35}$$

where $\boldsymbol{F}$ and $\boldsymbol{G}$ are positive definite matrices and k a positive constant defined in the design. Then, for a large enough control gain $\boldsymbol{K}_d$, the filtered tracking error $\boldsymbol{r}(t)$ and the NN weight error $\|\tilde{\boldsymbol{Z}}\|_F$ are Uniformly Ultimately Bounded (UUB).

Proof: Let the NN approximation property Eq.(18) hold for the function $f(\mathbf{x})$ given in Eq.(8) with a given accuracy ε_N for all $\mathbf{x}$ in the compact set $S_{\mathbf{x}} \equiv \{\mathbf{x} \mid \|\mathbf{x}\| < b_x\}$ with $b_x > q_B$. Defining $S_r \equiv \{\mathbf{r} \mid \|\mathbf{r}\| < (b_x - q_M)/(c_0 + c_2)\}$, using Eq.(25), and considering $\mathbf{r}(0) \in S_r$, then the approximation property holds.

Consider the Lyapunov function candidate

$$V(\mathbf{r}, \tilde{\mathbf{W}}, \tilde{\mathbf{V}}) = \frac{1}{2}\mathbf{r}^T\bar{\mathbf{M}}(\bar{\mathbf{q}})\mathbf{r} + \frac{1}{2}tr\{\tilde{\mathbf{W}}^T\mathbf{F}^{-1}\tilde{\mathbf{W}}\} + \frac{1}{2}tr\{\tilde{\mathbf{V}}^T\mathbf{G}^{-1}\tilde{\mathbf{V}}\} \quad (36)$$

Note that V is definite positive, $V > 0$. Differentiating V and substituting the Eq.(32), it is obtained

$$\begin{aligned}\dot{V} \;=\; & -\mathbf{r}^T\mathbf{K}_d\mathbf{r} + \frac{1}{2}\mathbf{r}^T(\dot{\bar{\mathbf{M}}} - 2\bar{\mathbf{C}}_o)\mathbf{r} + tr\{\tilde{\mathbf{W}}^T(\mathbf{F}^{-1}\dot{\tilde{\mathbf{W}}} + (\hat{\sigma} - \hat{\sigma}'\hat{\mathbf{V}}^T\mathbf{x})\mathbf{r}^T)\} + \\ & + tr\{\tilde{\mathbf{V}}^T(\mathbf{G}^{-1}\dot{\tilde{\mathbf{V}}} + \mathbf{x}\mathbf{r}^T\hat{\mathbf{W}}^T\hat{\sigma}')\} + \mathbf{r}^T(\delta + \bar{\tau}_d + \tau_r) \quad (37)\end{aligned}$$

The skew symmetric property (Property 1) makes the second term zero, and since $\dot{\tilde{\mathbf{W}}} = -\dot{\hat{\mathbf{W}}}$ and $\dot{\tilde{\mathbf{V}}} = -\dot{\hat{\mathbf{V}}}$, the tuning rules yields

$$\begin{aligned}\dot{V} \;&=\; -\mathbf{r}^T\mathbf{K}_d\mathbf{r} + k\|\mathbf{r}\|(tr\{\tilde{\mathbf{W}}^T\hat{\mathbf{W}}\} + tr\{\tilde{\mathbf{V}}^T\hat{\mathbf{V}}\}) + \mathbf{r}^T(\delta + \bar{\tau}_d + \tau_r) \quad (38)\\ &=\; -\mathbf{r}^T\mathbf{K}_d\mathbf{r} + k\|\mathbf{r}\|tr\{\tilde{\mathbf{Z}}^T(\mathbf{Z} - \tilde{\mathbf{Z}})\} + \mathbf{r}^T(\delta + \bar{\tau}_d + \tau_r) \quad (39)\end{aligned}$$

Since $tr\{\tilde{\mathbf{Z}}^T(\mathbf{Z} - \tilde{\mathbf{Z}})\} = \langle\tilde{\mathbf{Z}}, \mathbf{Z}\rangle_F - \|\tilde{\mathbf{Z}}\|_F^2 \leq \|\tilde{\mathbf{Z}}\|_F\|\mathbf{Z}\|_F - \|\tilde{\mathbf{Z}}\|_F^2$ and substituting the Eq.(33), results

$$\dot{V} \leq -k_{d_{\min}}\|\mathbf{r}\|^2 - k\|\mathbf{r}\|\|\tilde{\mathbf{Z}}\|_F(\|\tilde{\mathbf{Z}}\|_F - Z_M) - k_z(\|\hat{\mathbf{Z}}\|_F + Z_M)\|\mathbf{r}\|^2 + \|\mathbf{r}\|(\|\delta\| + \|\bar{\tau}_d\|) \quad (40)$$

with $k_{d_{\min}}$ the minimum singular value de $\mathbf{K}_d$. From Property 2 and substituting Eq.(31), obtain

$$\begin{aligned}\dot{V} \;&\leq\; -k_{d_{\min}}\|\mathbf{r}\|^2 - k\|\mathbf{r}\|\|\tilde{\mathbf{Z}}\|_F(\|\tilde{\mathbf{Z}}\|_F - Z_M) - (k_z - C_2)\|\tilde{\mathbf{Z}}\|_F\|\mathbf{r}\|^2 + \|\mathbf{r}\|(d_M + C_0 + C_1\|\tilde{\mathbf{Z}}\|_F) \quad (41)\\ &\leq\; -\|\mathbf{r}\|[k_{d_{\min}}\|\mathbf{r}\| + k\|\tilde{\mathbf{Z}}\|_F(\|\tilde{\mathbf{Z}}\|_F - Z_M) - (d_M + C_0) - C_1\|\tilde{\mathbf{Z}}\|_F] \quad (42)\end{aligned}$$

where the last inequality holds due to $k_z > C_2 > 0$. Thus, $\dot{V}$ is guaranteed negative as long as the term in brackets in Eq.(41) is positive. Defining $C_3 = (1/2)(Z_M + C_1/k)$ and completing the square yields

$$\dot{V} \leq -\|\mathbf{r}\|\{k_{d_{\min}}\|\mathbf{r}\| + k(\|\tilde{\mathbf{Z}}\|_F - C_3)^2 - (d_M + C_0) - kC_3^2\} \quad (43)$$

and to $\dot{V} < 0$, require

$$\|\mathbf{r}\| > \frac{d_M + C_0 + kC_3^2}{k_{d_{\min}}} \equiv b_r \qquad \text{or} \qquad \|\tilde{\mathbf{Z}}\|_F > C_3 + \sqrt{C_3^2 + \frac{d_M + C_0}{k}} \equiv b_Z \quad (44)$$

Therefore, $\dot{V}$ is negative outside a compact set. According to a standard Lyapunov theory, this demonstrates the UUB of both $\|\mathbf{r}\|$ and $\|\tilde{\mathbf{Z}}\|_F$ as long as the control remains valid within this set. □

Note that $\|\mathbf{r}(t)\|$ can be kept arbitrarily small by increasing the gain $k_{d_{\min}}$ in Eq.(44). The right-hand sides of Eq.(44) can be taken as practical bounds on $\mathbf{r}(t)$ and the NN weight estimation errors. The tuning parameter k offers a design trade off between the relative magnitudes of $\|\mathbf{r}(t)\|$ and $\|\tilde{\mathbf{Z}}\|_F$. Moreover, the Equation (44) represents the worst case one can have. In fact, the actual convergence region is a subset of the set given by Eq.(44).

There is in this scheme no required preliminary off-line tuning phase. In fact, selecting the initial weights $\mathbf{W}(0)$ and $\mathbf{V}(0)$ as zero takes the NN out of the circuit and leaves only the outer PD tracking loop in Fig. 3. The PD term $\mathbf{K}_d\mathbf{r}$ in Eq.(10) can then stabilize the system until the NN updates its weights. A formal proof reveals that $\mathbf{K}_d$ should be large enough and the update laws have shown that the NN weights are tuned on-line in real time; as the NN estimates the helicopter nonlinear function, the tracking error decreases.

The first term of the NN tuning laws are modified versions of the standard backpropagation algorithm. The last terms correspond to the e-modification (Lewis 1999) in standard use in adaptive control to guarantee bounded parameter estimates; they form a special sort of forgetting term in the weight updates. Their function is to add to $\dot{V}$ a quadratic term in $\|\tilde{\mathbf{Z}}\|_F$ so it can be shown that $\dot{V}$ is negative outside a compact set in the ($\|\mathbf{r}(t)\|$,$\|\tilde{\mathbf{Z}}\|_F$) plane. The second term is a bequest from the function approximation error $\tilde{f}$, when purposed to write the Eq.(30) by $\|\mathbf{r}(t)\|$ and $\|\tilde{\mathbf{Z}}\|_F$. A robustness control term is needed to overcome higher-order modeling error terms.

4. SIMULATION RESULTS

To illustrate the NN control scheme presented in Fig. 3 and to compare its performance, two controllers were proposed: 1) NNPD: a NN combined with PD controller and 2) PID: a proportional-integral-derivative controller. The PD and PID controllers design are based on a LQR controller, using a 3 DOF helicopter linearized model obtained from Eq.(1). Such design procedure can be seen in (Quanser 2005).

The simulations were carried out in MATLAB and used the 3 DOF helicopter parameters produced by Quanser (2005). The designed gains to the controllers were: $Q = diag\{[100\ 500\ 2000\ 100\ 60\ 1500\ 50\ 10]\}$ and $R = diag\{[5\ 5]\}$ to the LQR controller design, and consequently, to the linear controller design. To the NNPD was considered $k = 2$, $k_z = 0.05$, $\mathbf{F} = k_c.\mathbf{I}_{N_h \times N_h}$, $\mathbf{G} = k_c.\mathbf{I}_{n \times n}$, with $k_c = 20$, $N_h = 10$ and $Z_M = 10$. It was considered a sampling time equal to $T = 0.01$s and a helicopter motors saturation voltage equal to 5V. The initial posture vector $(\mathbf{q}_i)$ to the reference trajectory and to the both controllers was defined as $\mathbf{q}_i = [0° - 27°\ 0°]^T$. It was also considered on the simulations $\|\mathbf{F}(\dot{\mathbf{q}})\| \leq 3.5$ N.m and $\tau_d \sim N(0; 0.01)$.

The performance of the controllers is quantified through the Mean Absolute Error (MAE) expressed by

$$\bar{e}_\theta = \frac{1}{t_{final}} \int_0^{t_{final}} |\theta_r(t) - \theta(t)| dt \quad \text{and} \quad \bar{e}_\psi = \frac{1}{t_{final}} \int_0^{t_{final}} |\psi_r(t) - \psi(t)| dt \tag{45}$$

where $\bar{e}_\theta$ and $\bar{e}_\psi$ denote the mean absolute errors referring to pitch angle $\theta(t)$ and to yaw angle $\psi(t)$, respectively. 500 Monte Carlo simulations were done and the mean and the variance were calculated from the errors set.

The Figure 4 shows the C point motion of the helicopter, described in Fig. 2, in a circular trajectory where the PID controller presents a tracking error larger than NNPD controller, and where it has not been able to track the prescribed trajectory. In $t = 15$ s, the mass disturbance step with mass nominal variation of 17.8% was applied in the system to analyze the robustness in the presence of parametric variation. In the Figures 4-5, it is possible to see the NN capability in to approach the helicopter nonlinear function and cope with parametric uncertainty, and likewise reduce the unknown and bounded disturbances effects. The NNPD behavior in the beginning of the trajectory is due to large weights estimation errors, and consequently, to large NN approximation error.

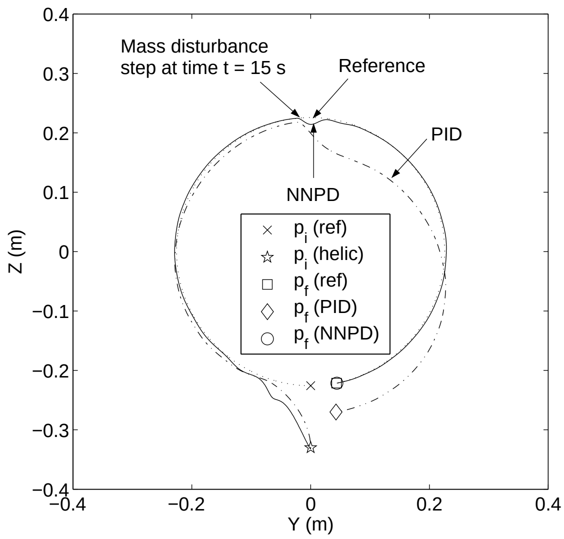

Figure 4. Reference and helicopter trajectories.

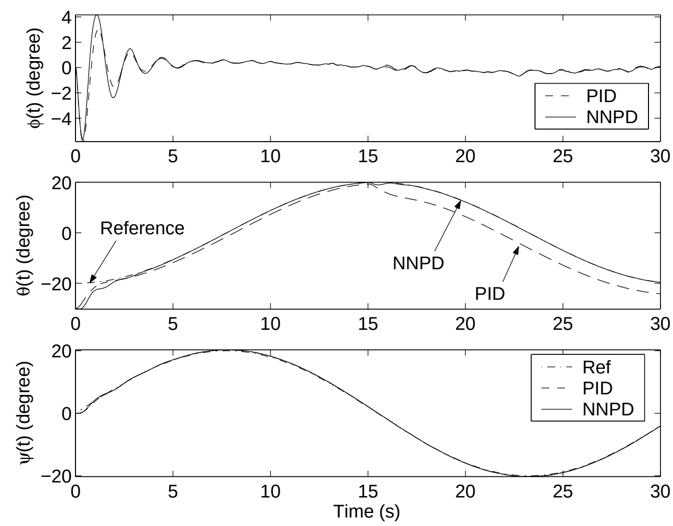

Figure 5. Generalized coordinates of the helicopter.

The tracking error of both controllers remain bounded, however in the moment that the mass disturbance step was applied only the NNPD could deal with parametric variation, conform show the Fig. 6. The PID controller requires exact knowledge of the helicopter dynamics in order to work properly, but it does not happen with the NNPD that estimate the dynamic parameters of the TF in real time. The Figure 7 illustrates a fundamental difference between the control input signals of the controllers. In $t = 15$ s, the NN increases its contribution on the control signal when it was capable to adapt to the mass variation provided by the disturbance step. The mean of the MAE, using the NNPD and considering the pitch angle $\theta(t)$, was 3.5 times lesser than PID. This result illustrates that the NNPD can handle with perturbation effect and determine variables, which are hard to estimate, like friction force. Both controllers allow the helicopter to follow the reference $\psi_r(t)$ in an accurate and efficient way, as show the mean and standard deviation of the yaw angle in the Table 1.

Table 1. Performance of the controllers: Mean ± Standard Deviation.

Controller	$\theta(t)$ **(degree)**	$\psi(t)$ **(degree)**
PID	2.6283 ± 0.0187	0.5832 ± 0.0590
NNPD	0.7428 ± 0.0227	0.6149 ± 0.0639

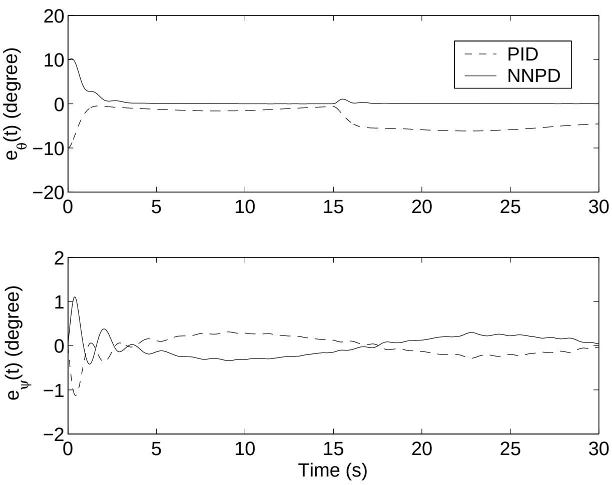

Figure 6. Tracking error of the pitch and yaw angles.

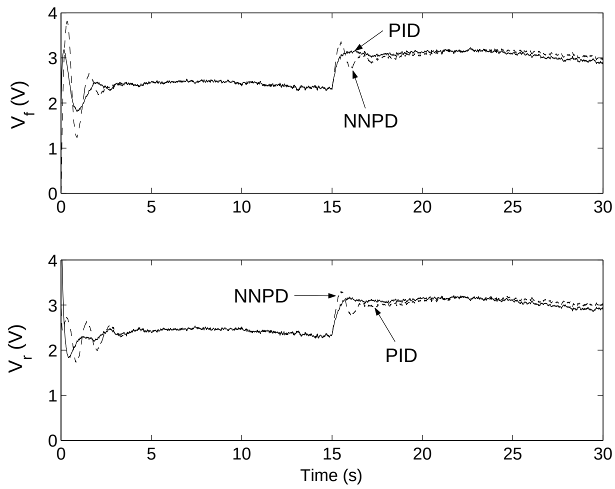

Figure 7. Control input signals of the helicopter.

5. CONCLUSIONS

A stable control framework capable of dealing with the three degrees of freedom of a helicopter, without requiring knowledge of the helicopter dynamics, was derived in this work by using an NN approach to solve the reference trajectory tracking problem. This feedback servo-control scheme provides better performance than that provided by a PID controller when there are bounded disturbances and mass disturbance step. Real parameters of the helicopter produced by Quanser were used in realistic simulations to compare the proposed controller with the PID.

In summary, a neural network combined with a PD controller was capable of estimating the helicopter nonlinear function through the NN weights on-line tuning and to follow the prescribed trajectory. With this controller it was possible to model the uncertainties effects as the friction surface, parametric variation and unknown and bounded disturbances.

6. ACKNOWLEDGEMENTS

The authors thank to CAPES and CNPq for the financial support.

7. REFERENCES

References

Calise, A. J., Hovakimyan N. and Idan M., 2001, Adaptive Output Feedback Control of Nonlinear Systems Using Neural Networks, Automatica, Vol. 37, pp. 1201-1211.

Barron, A. R., 1993, Universal approximation bounds for superpositions of a sigmoidal function, IEEE Transactions on Information Theory, Vol. 39, pp. 930-945.

Galindo, R. and Lozaro R., 2000, "Control of Under-Actuated Systems. Application to a Tandem Fan in 3DOF Platform", Proceedings of the IEEE International Conference on Control Applications, pp. 686-691.

Kutay T. A., Calise, A. J., Idan M. and Hovakimyan N., 2005, Experimental Results on Adaptive Output Feedback Control Using a Laboratory Model Helicopter, IEEE Transactions on Control Systems Technology, Vol. 13, No. 2, pp. 196-202.

Lewis, F. L., Jagannathan S. and Yesildirek A., 1999, Neural Networks Control of Robot Manipulators and Nonlinear Systems, Taylor & Francis, New Jersey.

Quanser, 2005, 3D Helicopter System (with active disturbance), Quanser Consulting.

Rysdyk R. and Calise A. J., 2005, Robust Nonlinear Adaptive Flight Control for Consistent Handling Qualities, IEEE Transactions on Control Systems Technology, Vol. 13, No. 6, pp. 896- 910.

8. Responsibility notice

The authors are the only responsible for the printed material included in this paper.