COMPARISON BETWEEN NUMERIC SOLUTIONS OF DIRECT METHOD AND STAGGERED METHOD APPLIED TO ANALYSE POROELASTIC BONES

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Abstract. *This article has the objective to present the computational differences between the numerical direct method of staggered method, with a plane poroelastic model of the head of femur. Aspects had been compared as time of processing, memory allocation and results precision, being that this comparison did not have objective to indicate, in general, that method is better, and yes, to make a study of a specific case. The solver was confectioned in Fortran90 language for the numerical solution through finite elements method on the basis of the theory of the poroelasticity. This theory aims at the modeling of a system formed for two distinct phases, a solid and other fluid, connected in an interdependent system. The model presented two regions of different properties, one represented for the cortical region of the bone and to another one for cancellous region. Using the formularization (u,p), the direct method presented an memory allocation and time processing very high. The staggered method, that presents more stages of solution, but solving symmetrical matrixes, expended a lesser time of processing and minor memory allocation. The precision, analyzed with the analytical model of the porous column, the direct model behaved better. Concludes that, for this specific case, the direct method was more precision, but the computational behavior of staggered method was better.*

Keywords: *Poroelasticity, Computational Efficience, Staggered Method, Bioengineering*

1. INTRODUCTION

The ample use of the resources and capacity of computational processing had favored in the diversification of the numerical methods. Many authors had studied methods for the solution of not linear, static, dynamic and transient systems. When comparison them self, these methods do not have to be collated of specific form, therefore each one presents some advantage on another one or on the influence of determined factor. This work considers to make a scientific study, on the computational parameters between the numerical solution methods, of the poroelastic finite element. In the literature two methods detach for this type of element: the direct method and staggered method (Park, 1982). The iterative methods normally are applied when it exists variation of a reply in elapsing of the time and when the systems are formed by different material phases, for example, in the poroelasticity and fluid-structure problems. These problems are call of couple problems, they are connected systems and have the characteristic to solve a different system with stiffness matrices, damping matrices and mass matrices for each material phase, with different largenesses, like scale and vectorial. An example of the application of the direct method as form of solution of a porous media system was presented by Xikui, in this work it showed a numerical formalization applied the dynamic problems (Xikui, 1990), another important work that used the direct method was of Silva Jr., in this it applied the direct method for solution of problems of acoustics with absorption poroelastic materials (Silva Jr., 2003). For the staggered method it can be cited the works published for Zienkiewicz (1988) and Park (1983), they had studied a solution for the unconditionally staggered method analysis on stabilization analyse. The poroelasticity is a theory developed for the modeling of a structure composed for two phases, an elastic and another fluid. This model represents the interaction between the deformation and the flow of fluid in a porous environment. The precursor of these studies, in its classic form, was Biot (1935, 1941, 1972), Belgian researcher that developed one technique of adensament three-dimensional. Structures contend poroelastic materials widely are used in engineering due its properties of sound absorption and characteristics of damping. Can be cited the materials used as isolating sonorous in vehicles, theaters and musical houses of spectacles, industries and studios (Silva Jr., 2003); in damping of vibrations, automobiles, eletroeletronics and toys; and aerospace structures (Lamary, 2001). The soil also is a poroelástico material, much studied in geology in the oil sheet formation, in the project of foundations of structures the general civil engineering (Lewis, 1987). Beyond these classic areas, the applications in bioengineering, mainly on the bone structures modeling (Cowin, 2001). This work is based on the same procedure used by Ferreira (1996), in his work he made a revision of the concepts of poroelasticity, implemented the staggered method for the column poroelastic case and of the infinite well case, it makes one it analyzes of the performance of the model and compares the method implemented with the direct method. The main difference of this work with the Ferreira is on application of the method, in this case, the mesh is bigger mesh then another and the application was a bioengineering model. Other different aspects are the physical characteristics in the problem. Despite the similarity, this work it aims at to present the difference enters the methods applied to each in case that, therefore the involved computational aspects in the problems of geosciences or mechanics of ground are different those applied on the medicine. The model simulated in this work the same was applied by Beaupré (1990) for cyclical loads, but with transient characteristics. The 2D model is constituted of an part femur with known conditions of shipment and in strategical positions. In the work of Beaupré the different distributions of densities in the bone profile had been tested as answers to imposed loads, this would give to an indicative of growth bone and its physical behavior. The part of femur simulated, in this work, is constituted two-piece main: the cortical bone part, where the bone is close and denser and; to cancellous bone part, where is the marrow bone and the density is lesser. Many approaches had been used for this model, therefore the bone is reality a heterogeneous materials and has nonlinear behavior.

2. MECHANIC POROELASTIC MODEL

The model showed for Biot has two distinct phases, each phase is couple on another phase in a the same geometric system. One solid phase and another with interstitial fluid. The behaviour of solid phase is represented with displacement variables (u, v, w) and stress variable [σ]. The fluid phase is represented with pressure variable (p) and flux (q). On the Biot formulation consider the octaedric infinit element citizen to the stresses, analogous to the three-dimensional elasticity theory.
Using this theory and applying the boundary conditions of balance for the plain problem, disrespecting the volume forces, it is:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0 \tag{1}$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0 \tag{2}$$

where σ_{xx} is the stress in the direction of axle x, σ_{yy} is the stress in the direction of the axle y, σ_{xy} is the shear stress with relation to the plan xy, $\frac{\partial}{\partial x}$ is the partial derivative with regard to x, $\frac{\partial}{\partial y}$ is the partial derivative with regard to y, thus for ahead.

Considering a consisting structural phase of a isotropic material, linear homogeneous and elastic, the Law of Hook can be applied. Adding the effect of the hydrostatic pressure acting in the interstices, or pores, of the structural phase, it is had:

$$\left\{ \begin{array}{c} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{array} \right\} = \left[\begin{array}{ccc} \frac{1}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{2G} \end{array} \right] \left\{ \begin{array}{c} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{array} \right\} + \left\{ \begin{array}{c} p/3H \\ p/3H \\ 0 \end{array} \right\} \tag{3}$$

where E is the Young modulus, G is the shear modulus, ν is Poisson coeficient, ε_{xx} is the strain on the axle x direction, p is the hydrostatic pressure and H represent the constant of ponderaction of pressure correlated with the constants of quantity fluid variation.

The Eq. (3) can be rewrite in term of the stresses, of form that pressure equation is reorganized, thus defining, new poroelastics constants.

$$\left\{ \begin{array}{c} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{array} \right\} = \left[\begin{array}{ccc} 2G + \frac{\nu}{1-2\nu} & \frac{\nu}{1-2\nu} & 0 \\ \frac{\nu}{1-2\nu} & 2G + \frac{\nu}{1-2\nu} & 0 \\ 0 & 0 & 2G \end{array} \right] \left\{ \begin{array}{c} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{array} \right\} - \left\{ \begin{array}{c} \alpha p \\ \alpha p \\ 0 \end{array} \right\} \tag{4}$$

The new poroelastic variable α is called coefficient of Biot. It relates the states solid and fluid through the coupling coefficients and, with the pressure, she gives the stress provoked for the fluid in the porous system. It has adimensional unit and can be represented in the following ways:

$$\alpha = \frac{2(1+\nu)}{(1-2\nu)} \frac{G}{3H} \tag{5}$$

$$\alpha^2 = \frac{2G(\nu_u - \nu)}{Q(1-2\nu_u)(1-2\nu)} \tag{6}$$

being Q the poroelastic constant that represents the variation of the shear and longitudinal properties of a system subject to shipment and the coupling of these with the solid and fluid portions . From the definition of stress and strain, using a model of incompressible fluid, Biot (1941) joined the concepts of elasticity and mechanic of fluid and defined a new system, that express in function of the displacements and the pressure, is given by:

$$\begin{aligned} G\nabla^2 u + \frac{G}{1-2\nu}\frac{\partial e}{\partial x} - \alpha\frac{\partial p}{\partial x} = 0 \\ G\nabla^2 v + \frac{G}{1-2\nu}\frac{\partial e}{\partial y} - \alpha\frac{\partial p}{\partial y} = 0 \end{aligned} \tag{7}$$

where e is the volume variation give by $\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$.

Analyzing it Eq. (7) perceives that the system has two equations and three incognito. The solution of these equations becomes viable considering a fluid flow in the porous, according to Law of Darcy (1856). As Darcy there are variables that define the variation of the volume of fluid of a system, related with the pressure. As the volume variation is directly related the variation of the displacements, the relation between the pressure, time and displacement can be gotten through the Eq. (8).

$$\kappa\nabla^2 p = \alpha\frac{\partial e}{\partial t} + \frac{1}{Q}\frac{\partial p}{\partial t} \tag{8}$$

wher κ is the dynamic permeability coefficient of fluid.

Therefore, the mechanical poroelastic model, can be express through the Eq. (9), that it represents the transient behavior, of a incompressible saturated fluid system.

$$\begin{aligned} G\nabla^2 u + \frac{G}{1-2\nu}\frac{\partial e}{\partial x} - \alpha\frac{\partial p}{\partial x} = 0 \\ G\nabla^2 v + \frac{G}{1-2\nu}\frac{\partial e}{\partial y} - \alpha\frac{\partial p}{\partial y} = 0 \\ \kappa\nabla^2 p = \alpha\frac{\partial e}{\partial t} + \frac{1}{Q}\frac{\partial p}{\partial t} \end{aligned} \tag{9}$$

The model express in the Eq. (9) is bidimensional and is written in the displacement variable (u, v), stress (σ), pressure (p) and flow (q). Rewriting these equations in function of the stress and pressure, has been the equations of conservation of the moment and mass, given respectively for Eq. (10) e (11).

$$\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \\ \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \end{aligned} \tag{10}$$

$$\kappa\nabla^2 p - \alpha\frac{\partial e}{\partial t} - \frac{1}{Q}\frac{\partial p}{\partial t} = 0 \tag{11}$$

Applying the weighed residues method they are had:

$$\begin{aligned} \int_\Omega \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y}\right) W d\Omega = 0 \\ \int_\Omega \left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}\right) W d\Omega = 0 \\ \int_\Omega \left(\kappa\nabla^2 p - \alpha\frac{\partial e}{\partial t} - \frac{1}{Q}\frac{\partial p}{\partial t}\right) W d\Omega = 0 \end{aligned} \tag{12}$$

being W the fuction weight and Ω the dominius of poroelastic problem.

Breaking up the terms of direction x, y and the component of pressure, are had the equations for each one these terms. Applying the Galerkin method and rewrite the terms derived on the time for finite differences, it is give following matrical expression:

$$[K_e]\{\vec{u}_i\} = [L_e]\left\{\dot{\vec{u}}_i\right\} + \left\{\vec{F}\right\} \tag{13}$$

where $\{\vec{u}_i\}^T = \{u_i\ v_i\ p_i\}$ are the displacements and pressure in instant i, $\left\{\vec{F}_i\right\}^T = \{f_x\ f_y\ q\}$ is the external loads applied and $\left\{\dot{\vec{u}}_i\right\}^T = \{\dot{u}_i\ \dot{v}_i\ \dot{p}_i\ \}$ the derivatives in the time of the displacements and pressure.

The couple matrices $[K_e]$ and $[L_e]$ are given by:

$$[K_e] = \begin{bmatrix} K_{xx} & K_{xx} & -Q_x \\ K_{xx} & K_{xx} & -Q_y \\ 0 & 0 & H \end{bmatrix} = \begin{bmatrix} K & -Q \\ 0 & H \end{bmatrix} \tag{14}$$

$$[L_e] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ Q_x^T & Q_y^T & G \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ Q^T & G \end{bmatrix} \tag{15}$$

being [K] the elastic stiffness matrix, [H] the fluid stiffness matrix, [Q] the coupling matrix and [G] the fluid damping matrix.

2.1 Matrices of Direct Method

The solution through direct method is based in average weighed for the approach of the derivative time . Introduced the variable θ dependent of the two intervals of consecutive times, Reddy (1984). Such method approach of time independ of number spaces variable. For dealing of a general formularization, its implementation could be used for an approach of the type of the Eq.(13) or in problems of any dimensions. Of this form:

$$\theta \left\{ \dot{\vec{u}}_{i+1} \right\} + (1-\theta) \left\{ \dot{\vec{u}}_i \right\} = \frac{\{u\}_{i+1} - \{u\}_i}{\Delta t} \tag{16}$$

This type of approach, Eq.(16), characterizes parabolic transient problems, for intervals of time varying 0>t>t0. A factor θ, determinative of the time approach method, was chosen as being equal the 2/3, or either, the method chosen was the Galerkin method.

Applying the Eq. (refeq16) in (13) and rework the terms, it is had:

$$\left(\begin{bmatrix} 0 & 0 \\ Q^T & G \end{bmatrix} + \theta\Delta t \begin{bmatrix} K & -Q \\ 0 & H \end{bmatrix} \right) \left\{ \begin{array}{c} u_{i+1} \\ p_{i+1} \end{array} \right\} = \left(\begin{bmatrix} 0 & 0 \\ Q^T & G \end{bmatrix} - (1-\theta)\Delta t \begin{bmatrix} K & -Q \\ 0 & H \end{bmatrix} \right) \left\{ \begin{array}{c} u_i \\ p_i \end{array} \right\} + \Delta t \left\{ \begin{array}{c} f \\ p \end{array} \right\} \tag{17}$$

The Eq. (17) can be reorganized in more compact way:

$$([L_e] + \theta\Delta t[K_e]) \{\vec{u}_{i+1}\} = ([L_e] - (1-\theta)\Delta t[K_e]) \{\vec{u}_i\} + \Delta t \left\{ \vec{F} \right\} \tag{18}$$

2.2 Matrices of Staggered Method

The used numerical mechanical model for the staggered method is identical to the used on the direct method. One of the differences is in integration method used, therefore while in the direct the Galerkin method is used, as Eq. (16), in the staggered method uses the discretization method for finite differences (Euler formulation), as presented in the Eq. (19).

$$\frac{\partial x}{\partial t} = \frac{x_n - x_{n-1}}{\Delta t} \tag{19}$$

Applying the Eq. (19) in (13) it is had:

$$\begin{bmatrix} K & -Q \\ 0 & H \end{bmatrix} \left\{ \begin{array}{c} u_i \\ p_i \end{array} \right\} + \begin{bmatrix} 0 & 0 \\ Q^T & G \end{bmatrix} \left\{ \begin{array}{c} \frac{u_i - u_{i-1}}{\Delta t} \\ \frac{p_i - p_{i-1}}{\Delta t} \end{array} \right\} = \left\{ \begin{array}{c} f \\ q \end{array} \right\} \tag{20}$$

being f load vector of the elastic part and q the vector of flow of the fluid.

Another difference between the methods is the attainment of symmetrical matrices, that are solve in the solution of the staggered method, from the Eq. (20). Of this form, reorganizing the Eq. (20) is had:

$$\begin{bmatrix} \Delta tK & -\Delta tQ \\ Q^T & G + \Delta tH \end{bmatrix} \left\{ \begin{array}{c} u_i \\ p_i \end{array} \right\} = \begin{bmatrix} 0 & 0 \\ Q^T & G \end{bmatrix} \left\{ \begin{array}{c} u_{i-1} \\ p_{i-1} \end{array} \right\} + \left\{ \begin{array}{c} \Delta tf \\ \Delta tq \end{array} \right\} \tag{21}$$

The Eq. (21), of the form with that is presented, can be partitioned in elastic equations and fluid equations, connected for a matrix of coupling to both equations.

$$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \left\{ \begin{array}{c} u_i \\ v_i \end{array} \right\} = \left\{ \begin{array}{c} f_x \\ f_y \end{array} \right\} + \begin{bmatrix} Q_x \\ Q_y \end{bmatrix} \{p_i\} \tag{22}$$

$$[G + \Delta tH] \{p_i\} = \begin{bmatrix} Q_x & Q_y \end{bmatrix} \left\{ \begin{array}{c} u_{i-1} - u_i \\ v_{i-1} - v_i \end{array} \right\} + [G] \{p_{i-1}\} + \{\Delta tq\} \tag{23}$$

Using a compact notation for Eq. (22) and (23), it is had:

$$[K]\{u_i\} = \{f\} + [Q]\{p_i\} \tag{24}$$

$$[Kp\} = \left[Q^T\right]\{\Delta u\} + [G]\{p_{i-1}\} + \{\Delta tq\} \tag{25}$$

being $[K_p]$ the stiffness matrix of staggered fluid, form by [G + Δt.H].

Eq. (24) and (25) represents the equations of the elastic and fluid part, for solution of the staggered method, respectively.

3. COMPUTATIONAL IMPLEMENTATION

The programming was carried through using language FORTRAN on solver developed for the research group of Computational Mechanics Department at Unicamp, that constitutes a didactic tool and easy computational implementation. The element implemented, for this work, was the quadrilateral linear for quasi-static systems (transient). A assembly of sub-routines was implemented forming the base of all programming. Many available routines in the half academic had been used, proving the simplicity on the programation, that a strategy based on FORTRAN allows.

3.1 Direct Method

As already presented previously, the direct method has this name due the solution of the problem to be decided directly by the Eq. (18). The matrices that compose these equations are not-symmetrical and the strategy of construction is sparce. But for the initial time, of solution in the time, that the symmetrical matrices are sparces. This because to have an initial parameter of resolution, applyed an initial condition of not-drained system, or either, does not have variation of mass of the fluid of the system, ζ = 0. Of this form:

$$\zeta = \alpha e + \frac{1}{Q}p = 0 \tag{26}$$

where ζ is the variation of the amount of fluid of the system. Isolating changeable of pressure, it is had:

$$p = \alpha Q e \tag{27}$$

In this way the field of pressures can be evaluated through volumetric dilatation caused by an initial field of deformations.

Becoming assembly of the constituent equations and applying the same previous conditions, has construction of a new matrix of initial rigidity, being able to be called matrix of not-drained stiffness [Kei]. This matrix is basically the same one for the plain state of strain of the elastic case, its difference is in the constituent matrix that is increased of the poroelastics terms $\alpha^2 q$ (Ferreira, 1995).

The solver used was the MA32, of the free package of routines in FORTRAN, NAG. This solver solves not-symmetrical sparces matrices with verification of the positioning of values of elementary matrices.

In summary, the direct method of resolution of the parabolic transient poroelastic problems can be decided as:

1. Determinate poroelastic constants of the material (minimum of 5);
2. Matrical Coupling of the equations of solid and fluid phases;
3. Assembly of the matrices of finite elements - characteristic of stiffness, compressibility and coupling of the system. Assembly used sparce matrices through the method of compressed column;
4. Application of the initial conditions in t =0;
5. Discretização on time, solution for Gauss Elimination
6. Assembly of the vectors of the right side of the Eq. (18);
7. Solve the Eq. (18) with solving MA32 of package NAG;
8. Increment the time (δt) and to repeat the process ties to reach the final time.

3.2 Staggered Method

The staggered method is an iterative method of solution of symmetrical sparces matrices, that for the case of the poroelastic model, solves elastic and fluid equations, connected for matrices of correlation between them. Each equation represents a phase of the system, a solid and to another fluid.

The solver of solution was the MA27, of free subrotines called NAG. This solver is specialized in the solution of symmetrical sparces matrices without verification of position values of the matrix.

Of this form the procedure used for the solution of the staggered method is based on the convergence of the pressure, where the displacements depend on it to be stabilized on time. To follow they are presents the steps for that solution:

1. Determination of poroelastics constants material;
2. Assembly of the global matrices of the system: [K], [Q], [Kp]...;
3. Solve the initial system formed by [Kei]u_i=f;
4. Look for initials strains and to calculate the initial pressures through the Eq. (27);
5. For each instant of time t_i becomes:
   (a) Solve the problem of the solid part through the Eq. (24), from the pressures in the time t_i;
   (b) Calculate the increment of the displacement and change the pressure using the fluid partition, Eq. (25);
   (c) Repeat again the steps 5(a) and 5(b) until the pressures convergence, using a stop critery;
   (d) From the field of pressures calculated, restart the new iteration for the time t_{i+1};
   (e) The solution is finished in the instant of maximum time tax.

4. RESULTS

Two cases, in this work, had been tested. The first one, is a classic example of poroelastic theory, that was solve by Biot in 1941 for the case of unidimensional adensamento, known as poroelastic column. The second example, is formed by a bioengineering model, where were simulated the loads on the distal part of one femur bone.

4.1 Poroelastic Column

In this problem the three-dimensional equations are simplified for the unidimensional case, considering that its geometry presents height bigger than its width. For this simulation used the poroelastic bidimensional element of four nodes with three degrees of freedom (u, v, p). The boundary of conditions and the geometry of problem are presented in the Fig. 1.

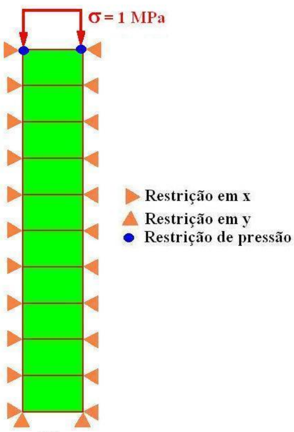

Figure 1. Model and mesh of poroelastic column

It is noticed existence of a distributed load of 1 MPa in the top of the column, together with a restriction of null pressure. Restrictions in the displacement on x direction for all nodes and restriction of the displacement in y for node of the base. The properties and variable used for this example had been based on the work of Siqueira (1995) and are presented in the Tab. (1).

The Fig. 2 and 3 show the results of gotten displacement and pressure in nodes on the top and the base of column, respectively, of the answers analytical, for the direct and staggered method.

Table 1. Properties for a poroelastic column

Constants	Nomenclature (Units)	Values
Shear Modulus	G [Pa]	$6,0 \times 10^9$
Drained Poisson coeffition	ν	0,2
Non-Drained Poisson coeffition	ν_u	0,33
Skempton coeffition	B	0,62
Permeability coeffition	κ [m^2/Pa.s]	$2,0 \times 10^{-11}$
Time Step	Δt [s]	1,0
Height of column	h [m]	6,0

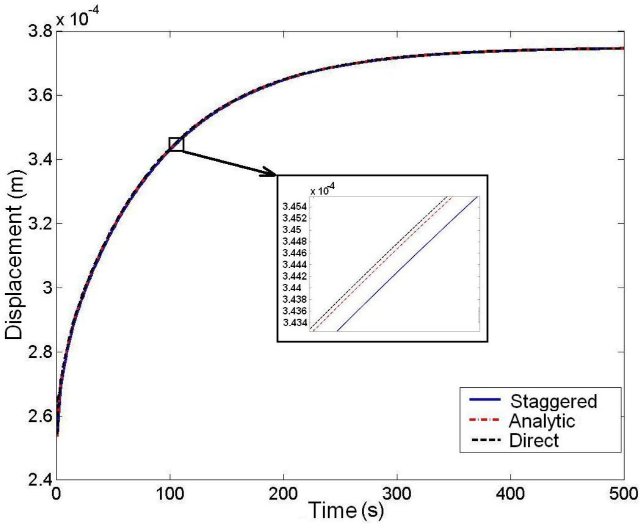

Figure 2. Displacement on the top of column

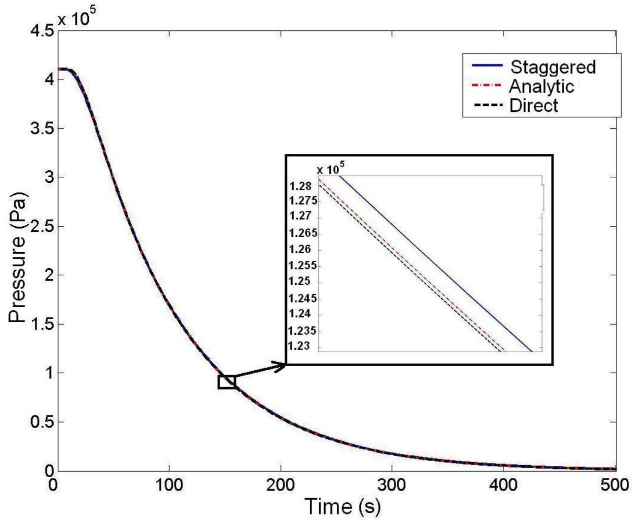

Figure 3. Pressure on the base of column

The parameters that take the methods to possess a greater or minor precision, in comparison to the analytical result, are the variable θ and δt for direct method e, ε (variable of convergence) and δt for the staggered method. Of this form these parameters can be modified to get a greater or minor precision. Fixed values gifts in literature and compared the results. Thus for θ of 2/3, δt of 1 second and ε of 0.001 was gotten the results presented in the Tab. 2.

Table 2. Compare results between the methods for poroelastic column

Analyse type		Method	
		Direct	Staggered
Processing time		1,48 s	0,38 s
Allocate memory		2MB	1MB
Precision	Displacement	0,02%	0,075%
	Pressure	0,24%	1,55%

These results would be different in case that other values of precision were used, for example, when ε for 0.00001 is diminished verifies an increase in the processing time on staggered case and one better precision of the results. The displacement and pressures, with the change in the precision variable, are next to the analytical result.

4.2 Distal Part of Femur

The bones can be simulated through the poroelastic model due its functions and geometry, had view that in it some porous portions in the satured environment fluid, depending local, it can be the blood, marrow bone, cells and fluid bone. The bones are classified in accordance with its form: long, short and flat bones. The long bones have two extremities or epifises, the body of the bone is diafise and between diafise and each epifise is metafise. The short bones have the three extremities practically equivalents and are found in the hands and the feet. The flat bones are formed by two layers of compact bone tissue, having between them a spongy layer of bone tissue and marrow bone (Vilela, 2005). The cortical bone is very dense and normally it is presented in the external surface of the bone. Less dense and very porous the cancellous bone, or spongy bone, is in central region of the bone. It has this name, therefore is formed by truss bone that branch off (Junqueira, 2004). For having different characteristics, the simulation of the distal part of femur, it took in account the average properties of the cortical bone (mat1) and cancellous (mat2). These properties had been removed of different literatures as of Müller (1996), Smit (2002), Turner (1999), Gilbert (2000), Cowin (1989; 2001), Neuman (1999) and Bayraktar (2004). One knows that the properties of bones are dependents of diverse factors, between them the density. Its anisotropic characteristic confers the same multiple conditional variable for analysis point (Gómez, 2005), therefore admitted characteristic averages and, for the calculation of the poroelastic constant the equations had been used proposals for Biot and Detournay (1993). The adopted properties are presented in the Tab. 3.

Table 3. Bone properties

Constants	Nomenclature (Units)	Values	
		Cortical Bone	Cancellous Bone
Shear Modulus	G [Pa]	20×10^9	18×10^9
Dreined Poisson Coeffition	ν	0,325	0,0,242
Non-Dreined Poisson Coeffition	ν_u	0,333	0,399
Skempton Coeffition	B	0,344	0,94
Permeabilidade Coeffition	κ [m^2/Pa.s]	$1,5 \times 10^{-17}$	$4,7 \times 10^{-4}$
Step Time	Δt [s]	1,0	

For the boundary conditions were applied the same contained in the work of Kuhl (2003) differing itself for loads applied , therefore on work of Kuhl were done three kind of loads, already for this work only one was used, being applied the load of 2317 N in the region of head, 703 N in trocanter, due the reaction provoked for the hamstring of balance body. A prescribed pressure of zero in the base of geometry was imposed as condition of transference of fluid there is between the bone and the external environment. The Fig. (4) presents the mesh of 5281 nodes and 5076 elements for the simulation of the distal part and its boundary conditions. The transient problem was simulated for a maximum time of 500 seconds, with a step time of 1 seconds and an variable of convergence, for the staggered method of 0.001 and θ = 2/3 for the direct method. Geometry was gotten through the Italian center of research BEL - Biomechanical European Laboratory (Viceconti, 1996) and the mesh was constructed in software GID, for pre-processing and pos-processing.

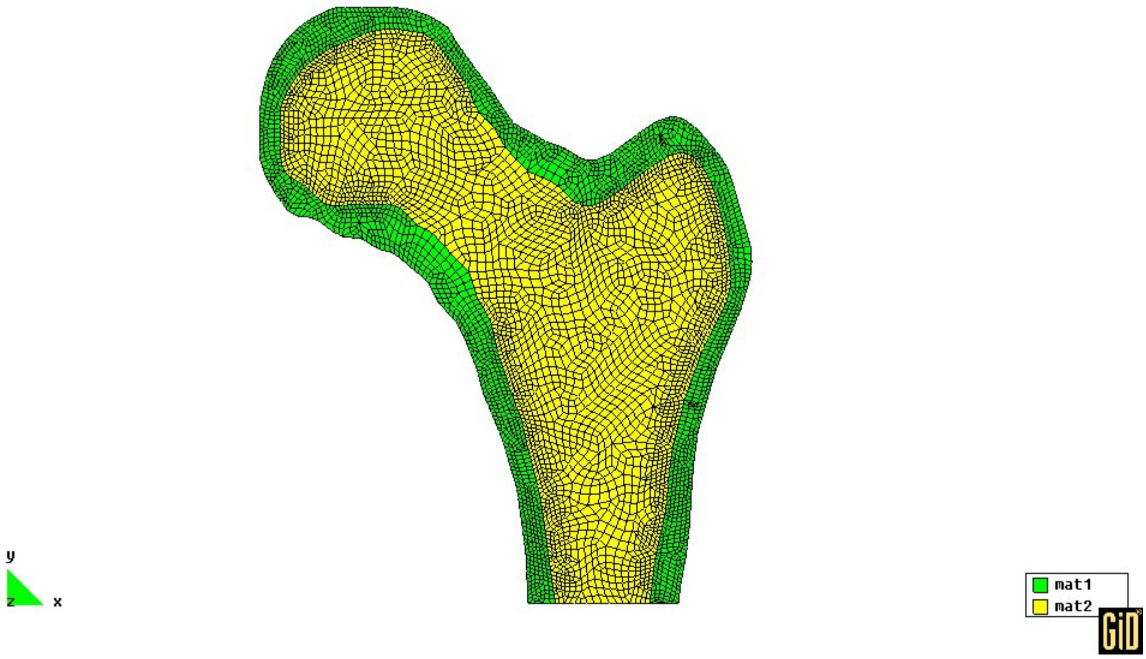

Figure 4. Model and Mesh of Femur Portion

On Tab. 4 show the compare results between staggered method and direct method for that matter to computational conditions.

One notices that the computational results gotten using the direct method are not advantageous. The processing time is almost four times slower and the placed memory is almost 1,5 times more than staggered. The high consumption of memory, in the direct method is justified by the use of not-symmetrical matrices, or either, in this method the matrices are bigger and with more numerical values. The slow processing can be related library NAG chosen for that solution.

Table 4. Compare results between of methos for a femur portion

Analyse Type	Method	
	Direct	Staggered
Processing Time	3h. 26min.	24min. 16s.
Allocate Memory	267MB	173MB

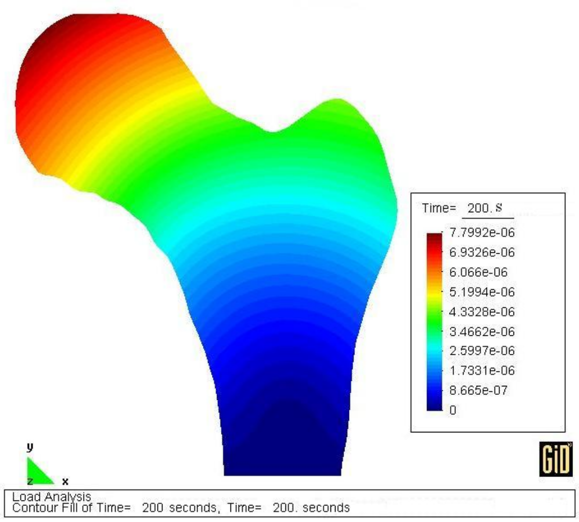

Figure 5. Displacement in 200 s (m)

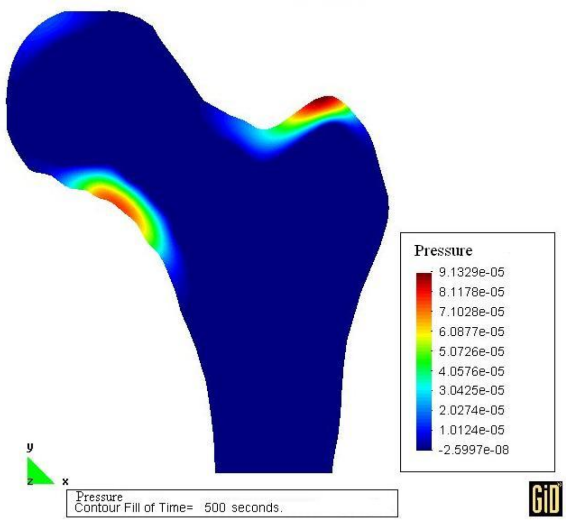

Figure 6. Pressure in 500 s (Pa)

The Figures 5 and 6 present the answers of the distribution of displacement and pressure of the staggered model. The distributions of the direct model also are similar for the analyzed times of iteration.

The distribution of the displacements presents uniform and its stabilization time also can be perceived soon in the first iterations. The pressure, different of the displacement, is stabilized close to the time of 600 s and has pressures of compression and traction acting in the cortical part, as the displacement that the structure is submitted. The results of the poroelastic bone, although could not be analyzed of quantitative form, for being an approach 2D of a problem of three dimensions, severals phenomena can be observed as be presented by Moura (2007).

The computer that processed the results was a AMD Duron 1,8 GHz with 1.0 GB of memory, the used Windows XP 32 bits operational system.

5. CONCLUSION

The methods presented in this work had showed a similar behavior how much final responses of the systems. The numerical comparison for the bidimensional case, with few elements and nodes, is very relative to be carried through, therefore the variations are very small, there is not resulted clear. The presented values only serve as largeness reference and not as absolute value. Already for the case simulating the distal part of one femur evidently notices the differences between the methods, with significant values, mainly when the total time of processing is analyzed. For this specific case the staggered method behaved better than direct method, but as already described, factors as time step and coefficient of integration, can modify the results. In this work it are used based sub-routines of solution in libraries NAG, but they exist diverse that they could be applied. It is very probable that the result is modified well with the application of other libraries. In the comparison of these methods it also exists, beyond the computational factors, the human factor, that has as changeable the ability of the programmer, good programmers tend to make routines more enxutas, providing to times of shorter processings and lesser computational cost. It is evident that this work did not take such factors in account, but this always must be pointed out.

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7. Responsibility notice

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