COMPARISON OF NUMERICAL METHODS OF FINITE DIFFERENCES AND ORTHOGONAL COLLOCATION FOR THE DYNAMIC PROBLEMS' RESOLUTION OF TEMPERATURE'S MEASURES

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Abstract. *The use of numerical methods for the simulation of heat transfer problems has proven to be an appropriate tool for the evaluation of several engineering problems. Among these methods, we have the method of finite differences and orthogonal collocation, both with different characteristics. Due to the necessity of a larger discretization, the first demands time to process, while the other produces results with a small discretization. In this work, these two methods are used in the simulation of the temperature increase in a thermocouple covered with plastic material. The simulation results, achieved trough finite differences, presented a better behaviour tthan the orthogonal collocation method.*

***Keywords**: Finite Differences, Orthogonal Collocation, temperature measurement.*

1. INTRODUCTION

The use of numerical methods for the simulation of heat transfer problems has being useful in resolving engineering problems. Among these methods, we can call the attention to the method of the finite differences and the orthogonal collocation. This last one is the start point to the development of the finite elements method. The finite differences method, which is popular among the researchers in the heat transfer area, uses a big number of discretization points to obtain good results. This technique solves, according to Pinto J.C. (2001), contour or initial value problems, involving differential ordinary equations or partial equations, which is common in the heat transfer area.

The orthogonal collocation method uses collocation points where the residue of the differential equation must be zero. In this method application it is used the Jacobi Polynomial, whose roots define the collocation points. The polynomial solution can be expressed in the Lagrange Polynomial form. Through the utilization of collocation points and numerical resolution with polynomials, the method provides a quick problem solution; with lower discretization.

Ma, Z., Guiochon, G. (1991) studied the simulation of the chromatography that's not linear, comparing to orthogonal collocation models and finite elements. These models were compared with results that were obtained through the methodology of the finite differences. This last one got better results, though, with a bigger computational time to resolve.

Bieniasz, L.K., Britz, D. (1994) compared the simulation of the electrochemical kinetic efficiency, using finite differences and orthogonal collocation.

Griffing, G.J., Wood, D.G. (2000) used a combined model of finite differences and orthogonal collocation to solve a catalytic combustion problem. Also Yun J.X. et al. (2005) developed a protein absorption prediction model through a numerical solution of the problem, using a combined model of finite differences and orthogonal collocation.

Dulhoste, J.F. et al. (2004) studied the dynamism of the water draining out in opening channels, through the orthogonal collocation method, comparing this resolution with the ones, which were made with the finite differences and finite elements methods.

The objective of this work is to make a comparison between these two methods in the resolution of a problem with dynamic reading of a thermocouple. The line's method was implanted for both methodologies and the computational development of the model was elaborated using the Matlab platform.

2. PROBLEM'S PRESENTATION

The problem that was analyzed in this work was presented by Rice and Do (1995). This problem describes the behavior of a thermocouple, which extremity is admitted as a perfect sphere with a plastic film in contact with external fluid of T_f temperature.

The problem's model is given by:

$$\rho C_P \frac{\partial T}{\partial t} = k \frac{1}{r^{*2}} \frac{\partial}{\partial r} \left(r^{*2} \frac{\partial T}{\partial r^*} \right) \tag{1}$$

Where p is the specific weight, Cp is the specific heat, k is the thermal conductivity, t is time, T is the thermocouple temperature, and r is the variable ray of the thermocouple.
The conditions of the problem's boundary are given by:

$$-k\frac{\partial T}{\partial r}=h\left(T-T_f\right),\quad for\ r=R \tag{2}$$

and

$$\frac{\partial T}{\partial r}=0,\quad for\ r=0 \tag{3}$$

The used properties to the plastic (expanded polystyrene) were taken from Incropera (1998), (Cp = 1210 J/kgK, ρ = 55 kg/m^3, k = 0,027 W/mK) and h, coefficient for heat convection of the fluid to measure the temperature, is equal to 50 W/ m^2 K.

3. MODEL RESOLUTION

The first step for the numerical solution was the normalization of T and r, given by the following equations:

$$T^*=T-T_f/T_\infty-T_f \tag{4}$$

$$\frac{\partial T^*}{\partial r}=\frac{\partial T}{\partial r}\frac{1}{T_\infty-T_f} \tag{5}$$

$$\frac{\partial T}{\partial r}=\left(T\infty-T_f\right)\frac{\partial T^*}{\partial r} \tag{6}$$

and, r* = r/R δr* = δr/R δr = R δr*

T∞ is the ambient's temperature, T* and r* are the temperature and the ray dimensionless, and R is the thermocouple's ray. Substituting in the problem equation, we have:

$$\frac{\partial T^*}{\partial t}=\frac{\alpha}{R^2}\frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial T^*}{\partial r^*}\right) \tag{7}$$

where α is the thermal diffusivity.

Developing this equation, we have:

$$\frac{\partial T^*}{\partial t}=\frac{\alpha}{R^2}\frac{1}{r^{*2}}\left(r^{*2}\frac{\partial^2 T^*}{\partial r^{*2}}+2r^*\frac{\partial T^*}{\partial r^*}\right) \tag{8}$$

Simplifying,

$$\frac{\partial T^*}{\partial t}=\frac{\alpha}{R^2}\left(\frac{\partial^2 T^*}{\partial r^{*2}}+\frac{2}{r^*}\frac{\partial T^*}{\partial r^*}\right) \tag{9}$$

Now redefining the boundary conditions:

To r* = 1,

$$-k\frac{\partial T^*}{\partial r^*}=RhT^*\qquad \frac{\partial T^*}{\partial r^*}=-\left(\frac{h}{k}\right)RT^*=-BiT^* \tag{10}$$

Bi is the Biot number.

To r* = 0, we have $\frac{\partial T^*}{\partial r^*} = 0$

To t = 0, we have T* = 1

Using the technique of finite differences, the equations and boundary conditions comprise the model of the differences using n discretization points, with step H:

To j=0 (applying the second boundary condition)

$$\frac{\partial T}{\partial t} = 3\left(\frac{\alpha}{R^2}\right)\left(2T(1) - 2T(0) / H^2\right)$$

To j=1: (n-1)

$$\frac{\partial T}{\partial t} = \left(\frac{\alpha}{R^2}\right)\left(\left(T(2) - 2T(1) + T(0)\right) / H^2 + \frac{2}{r^*}\left(T(2) - T(0)\frac{1}{2H}\right)\right)$$

To j=n

$$\frac{\partial T}{\partial t} = \left(\frac{\alpha}{R^2}\right)\left(\left(T(n+1) - 2T(n) + T(n-1)\right) / H^2 + \frac{2}{r^*}\left(T(n+1) - T(n-1)\frac{1}{2H}\right)\right)$$

$$\frac{\partial T}{\partial t} = \left(\frac{\alpha}{R^2}\right)\left(\left(T(n-1) - 2\left(BiH + 1\right)T(n)\right) / H^2 + \frac{2}{r^*}\left(BiT(n)\right)\right)$$ (with the utilization of the first boundary condition)

The formulation of using the orthogonal collocation technique is showing below, as a function of the Lagrange polynomial coefficient:

$$\frac{\partial T}{\partial t}(0) = \frac{1}{R^2} 3\alpha \sum\left(B(1,:)T_j\right) \qquad (11)$$

To i=1:n-1

Adding $1 = \sum(A(i,:)T_j)$

Adding $2 = \sum(B(i,:)T_j)$

Problem Equation:

$$\frac{\partial T}{\partial t}(i) = \frac{1}{R^2}\left(2\alpha \frac{1}{r(i)} Adding\ \ 1 + \alpha\ \ Adding\ \ 2\right) \qquad (12)$$

Where the matrixes A and B are coefficients of the Lagrange Polynomial.

4. RESULTS AND DISCUSSIONS

Figure 1 shows the results of the simulation using finite differences. 5, 10, 15 and 20 points were considered. The results show that the increase of the discretization provided a greater stability in the numerical evaluation, through settling time reduction.

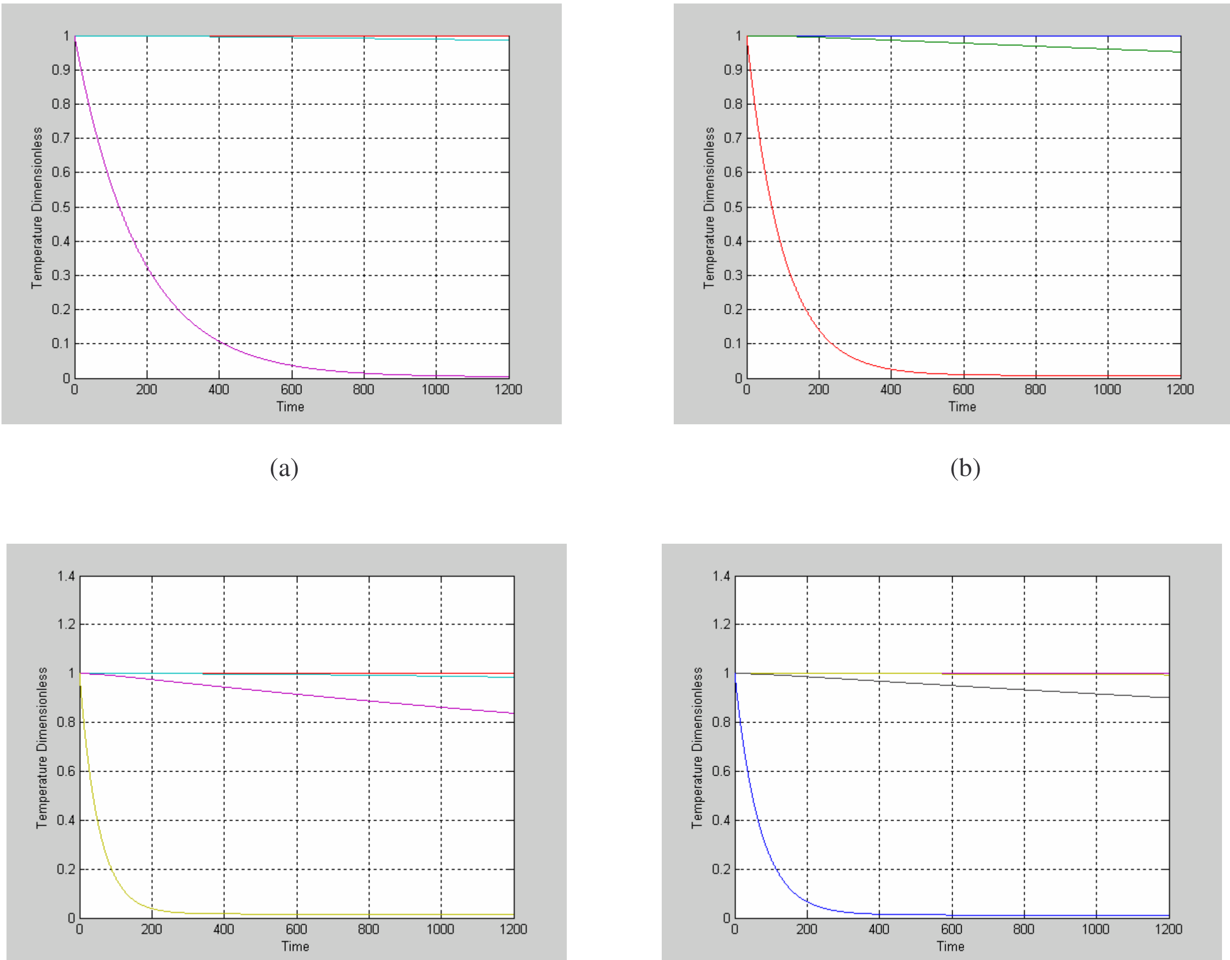
The settling time of this system is long due to the plastic material of the thermocouple's film in the internal thickness of the thermocouple. The stabilization of the temperature's value occurs, effectively, in the external surface of the thermocouple ($r^*=1$), which is in contact with the fluid.

Figure 2 illustrates the simulation's results using the orthogonal collocation method.

The increase in the discretization level causes instability in the numerical solution, which is associated to the increase of the Lagrange Polynomial grade.

It is clear that the stability of the numerical solution in the orthogonal collocation follows an opposite way compared to the finite differences, which became more stable through the increase of the analyzed points number.

The orthogonal collocation method shows coherent results when there is a few level of discretization and it was considered a drop in the temperature with the fluid interaction, where the thermocouple is inserted only inside in the more external position of the thermocouple. The others internal points of the thermocouple were not influenced by the reading too much, due to the low plastic conductivity. On the other hand, the finite differences solution were coherent in all analyzed cases.



(a) (b) (c) (d)

Figure 1: Temperature's Dimensionless. n = 5 (a), n = 10 (b), n = 15 (c), n = 20 (d) (finite differences method). The analysed points change between r^* to zero at 1, generating by discretization.

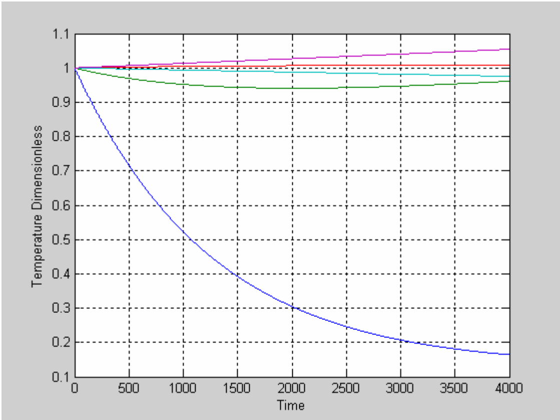
5. CONCLUSIONS

The present work compares the numerical solution using finite differences and orthogonal collocation in a dynamic problem of temperature´s measurement.

The problem's evaluation identified that a higher discretization in the finite differences's method brought more stability to the numerical evaluation, that the system's time to enter in a regimen decreased with a higher discretization in the finite differences method.

The evaluation of the orthogonal collocation method showed instability in the solution through the increase of the discretization. This method just showed coherent results with a low level of analyzed discretization.

The finite differences method was revealed as the best solution to obtain stable and coherent solutions to the problem that was proposed. The orthogonal collocation method was not disclosed as the best solution for the problem, which was proposed, because this method produced unstable solutions with the increase of the dicretization.



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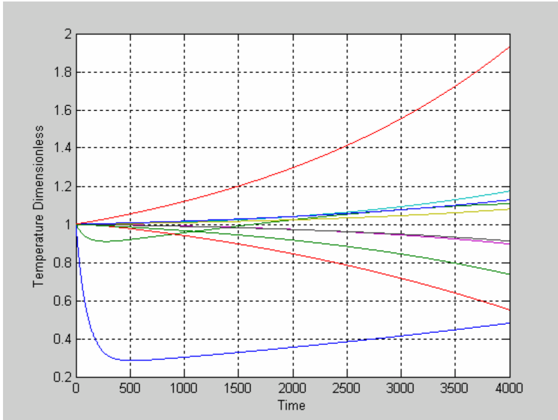

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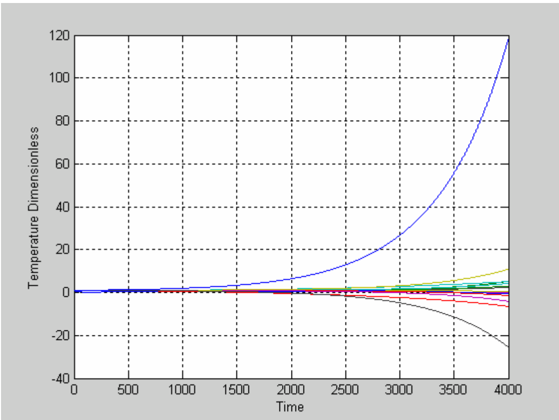

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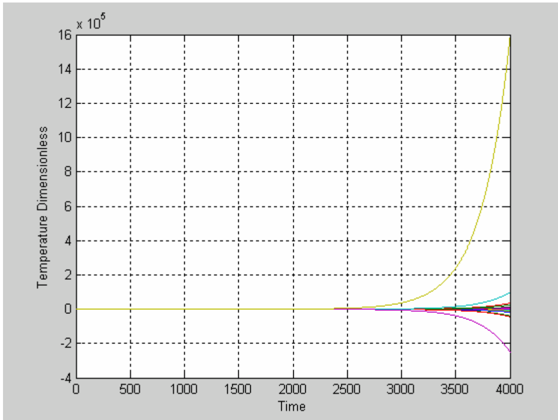

(d)

Figure 2: Temperature Dimensionless. n = 5 (a), n = 10 (b), n = 15 (c), n = 20 (d) (orthogonal collocation). The analyzed points change between r^* to zero at 1, generating by discretization.

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