A THEORETICAL AND EXPERIMENTAL EVALUATION OF THE NATURAL CONVECTION INSIDE A 3-D RECTANGULAR ENCLOSURE

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Abstract. The objective of this work is to study the motion and the heat transfer for a fluid confined in a 3-D rectangular enclosure. In one of the enclosure sides a prescribed heat flux is imposed, in order to simulate the heat dissipation of electronic components, and the opposite side is kept at a fixed lower temperature. All the other surfaces that form the enclosure are considered adiabatic. The mass, momentum and energy conservation equations, for steady and laminar flows, associated with the Boussinesq approximation for modeling buoyancy effects are used to describe the flow fields and heat transfer under different conditions. The resultant mathematical model was solved using a finite volume technique associated with a CGSTAB (Conjugate Gradient Squared Stabilized) Algorithm for the nonsymmetrical linear system obtained by the discretization method. The simulations were done using a computer program developed specially for the proposed configuration. Solutions for specific cases were obtained and compared with the available literature data, to verify the quality of the numerical results. This program was already used to generate some data published in previous works. An experimental apparatus was constructed in order to obtain some data about the working temperature range for the electronic components assembled in a enclosure as well as the temperature at the enclosure walls. The same three tested cases were simulated using the computational program covering a Rayleigh number range from 10^4 to 10^6 and the results were compared. The main differences observed are point out and justified and some improvements to both the computational program and experimental apparatus are proposed.

Keywords: enclosures with heat sources, natural convection, 3-D modeling

1. INTRODUCTION

Studies related to the fluid transport phenomena in enclosures can be found in the open literature since a long time ago (Elenbaas, 1942a, 1942b). The brand of applications and the challenges in the solution like great diversity of geometries encountered in actual situations, different fluids, solution of the proposed mathematical models and use of more suitable and representative boundary conditions are some of the motivations for the increasing number of the studies in natural convection. A brief literature revision is presented, ordered by the numerical method used in the solution and chronological appearance.

1.1. Finite differences

Mallinson *et al.* (1977) solved the problem of the transient flow and heat transfer for a rectangular enclosure using the 3D Navier-Stokes equations. Two opposite walls at different temperatures were considered and the others were kept adiabatic. They used non-slip conditions in the enclosure walls. For a Rayleigh number of 10^4 the three-dimensional motion was described as a combination of the rotational inertial flow for stationary walls with the contributions of the buoyancy force resultant from the longitudinal temperature gradient. The inertial effect was inversely proportional to the Prandtl number and thermal effects were approximately constant. For higher values of Rayleigh numbers, multiple

longitudinal flows appear and the dependence with the Rayleigh and Prandtl numbers and the enclosure aspect ratio showed to be very complex.

Karyakin *et al.* (1988) performed a transient analysis of heat transfer by natural convection for a triangular enclosure. The study employed a bi-dimensional model to study the laminar flow and the velocity components, pressure and temperature as variables. The Grashof number was varied in a range of $10^3 < \text{Gr} < 10^8$ and the aspect ratio, defined as the ratio between the height and the base length of the enclosure, from 0.25 to 2. The results showed the maximum values for the stream function and Nusselt number as damped oscillations close to the steady state solution. For higher values of Grashof number the flow near the walls was a boundary layer type and the temperature field was close to a stratified flow, at the central region of the enclosure.

Moreira Filho et al (2001) obtained a numerical solution for transient flow and temperature distributions inside an axisymmetric spherical enclosure. The study of the interference effects during the combustion of a infinite linear array of spherical droplets, with the absence of peripheral convective effects was the motivation for this work, in order too evaluate the importance of the use of symmetrical conditions during the combustion analysis. The flow field was obtained trough the use of axially approximated stream function-vorticity formulation. The solution evidenced the formation of a toroidal vortex surrounded by a viscous boundary layer close to the enclosure's external radial surface and by internal wake inside the symmetrical region near the axis. The temperature field was also analyzed taking into account the effects of the Reynolds number in the temperature field development. Analytical transformations of the coordinate system were applied and a great number of points close to the enclosure surfaces were considered. The results showed that the transient effects were significant.

Wakashima et al (2004) solved the three-dimensional problem of natural convection inside a cubical enclosure. The assumed boundary conditions were opposite isothermal walls, differentially heated. The other walls were considered adiabatic. Computations were performed for 10^4 , 10^5 and 10^6 Rayleigh numbers, with air as working fluid. A space-time method, presenting a high efficiency and low computational effort for solving fluid flow and heat transfer problems developed by one of the authors was employed. This methodology was compared with other numerical procedures.

1.2 Finite volumes

Peng et al (1999) applied the LRN (Low Reynolds Number) $\kappa - \omega$ model to calculate thermally stratified turbulent flows in enclosures. They investigated the influence of thermal stratification on the turbulence evolution with emphasis to the transition to turbulence inside the vertical walls boundary layers (cold and hot walls). Through the use of both, a damping function for the fluctuation in the κ equation and a standard gradient diffusion hypothesis(SGDH), the dependence with the grid spacing was eliminated and the solution could be found asymptotically by successive grid refinement. This formulation was applied to a problem of natural convection in rectangular enclosure with Rayleigh number of 5 10¹⁰ and also to a square enclosure in presence of mixed convection with instable thermal stratification. The results were compared with the literature and experimental data.

Arcidiacono et al (2001) performed a numerical two-dimensional simulation of fluid flow and heat transfer processes inside square enclosures at low Prandt number fluids with internal energy generation. The vertical walls were considered isothermal and with the same temperature and the horizontals were adiabatic and the modified Grashof number, based on the volumetric heat rate and enclosure width, was taken between 10^5 and 10^9 . For Grashof up to 10^7 the flow was stable and symmetric in relation to the vertical central plane. When this value reached $3x10^7$ the spatial symmetry disappeared and for Grashof higher than 10^8 the flow became totally instable. Finally, for Grashof 10^9 and higher the bilateral symmetry was recovered and the development of a bi dimensional turbulence was also observed.

Santos et al (2001) examined the cooling of a stainless steel tank filled with water at constant temperature. Two different boundary conditions were taken into account: a time dependent temperature profile, extracted from experimental data found in the literature, was prescribed along the enclosure wall; and a third kind condition, in which the Nusselt number for the external flow was imposed to prescribe the external heat transfer coefficient. The results were presented using streamlines and isothermal lines and the effects of the external parameters over the cooling rate were discussed.

Zanardi et al (2001) studied the laminar natural convection inside triangular enclosures. The bi dimensional mass, momentum and energy conservation equations with the Boussinesq approximation were solved. Adiabatic base and a time dependent heat flux at the lateral walls were adopted as boundary conditions. Results for a large range of Grashof number and aspect ratio were commented.

Ferreira Filho et al (2003) used steady, two-dimensional form of mass, momentum and energy conservation equations, assuming laminar flow and applying the Boussinesq approximation to study the flow inside a prismatic enclosure. The main objective was represent the geometry as close as possible of the actual shape, in order to avoid the use of blocking techniques as described by Patankar (1980) through a quadrilateral grid.

1.3 Finite elements

Del Campo et al (1988) used the Galerkin method along a stream function-vorticity formulation to solve the governing equation of a steady laminar two-dimensional natural convection inside a isosceles triangular enclosure. All the possible boundary condition combinations (adiabatic and isothermal) were explored for different Grashof numbers and aspect ratios. The flow patterns, isothermal lines and Nusselt numbers were presented and compared with experimental data found in the literature.

De Marco et al (2001) analyzed the natural convection for a square enclosure through a penalty formulation based on the mass conservation. The discretization equations were established by the Galerkin method applied for an adaptive unstructured grid with triangular elements and the algebraic equation system was solved by interactive methods. The penalty method make the numerical process stable and increased the convergence ratio since the formulation imposes the mass balance and then it is expected that the penalty parameter increases as the divergence of the velocity vector tends to zero. The natural convection inside the enclosure was studied with Bousinesq and non-Bousinesq approximations, with a variation of the specified temperature at the vertical walls. Thus, in the first case the density variation in the mass conservation equation was neglected resulting that the velocity divergent must be null and the density variation was considered only at the buoyancy term. In the second case, the density was considered varying according to the perfect gas model. They concluded that the mean Nusselt number was slightly affected in the usual temperature range.

Sampaio et al (2002) proposed an insulation system with beehive geometry filled with water with the objective of the reduction of heat transfer between the pressurizer, located at the pressure vessel top and the primary system of the IRIS nuclear reactor, achieving good results. The CFD (Computational Fluid Dynamics) technique was chosen to simulate the coupled heat transfer problem, including conduction in the solid parts and convection inside the cells. This paper intends to continue the investigation started by Nzamba et al (2006) where the theoretical model for a rectangular enclosure with a heat source was compared with experimental results. The only case reported then was for a Rayleigh number of 4 x 10^3 (almost pure conduction) and the results pointed for the necessity of corrections in both, numerical model and experimental apparatus. Here some modifications in the numerical procedure are presented, in order to solve convergence problems for Rayleigh numbers higher than 10^4 in original work.

2. MATHEMATICAL MODEL

The traditional model for an enclosure, that includes the mass, momentum and energy conservation equations, was considered for the geometry schematized in Fig.1.



Figure 1. Geometry and boundary conditions.

The following simplifying hypothesis for the conservation equations were applied: laminar steady state flow, Newtonian fluid, constant thermophysical properties (except the density in the body force term), Bousinesq approximation, no contact resistance and negligible viscous dissipation. Equations are written in a dimensionless form using the new variables:

$$x = \frac{X}{L_y}; \qquad \qquad y = \frac{Y}{L_y}; \qquad \qquad z = \frac{Z}{L_y}; \qquad \qquad (1)$$

$$u = \frac{UL_y}{\alpha_f}; \qquad v = \frac{VL_y}{\alpha_f}; \qquad w = \frac{WL_y}{\alpha_f}; \qquad (2)$$

$$p = \frac{P}{\rho(\alpha_f/L_y)^2}; \qquad \theta = \frac{T - T_c}{\left(q''L_y/k_f\right)}; \qquad Ra = \frac{g\beta q''L_y^4}{k_f\alpha v};$$
(3)

were x, y and z are the dimensionless space coordinates, u, v and w the dimensionless velocity components, L_y is the heat source height, α_f is the thermal diffusivity, p is dimensionless pressure, ρ is the density, θ is the non dimensional temperature, T_c is the temperature of the wall opposite to the one where the heat source is installed, q" is the source heat flux, k_f is the fluid thermal conductivity, Ra is the modified Rayleigh number, g is gravity vector, β is thermal expansion coefficient, υ is the cinematic viscosity, R_h is the ratio between the thermal conductivities of the heat source and the fluid, R_s is the ratio between the thermal conductivities of the vall and the fluid and Pr is the Prandtl number.

2.1 Mass conservation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

2.2 Momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + Pr\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$
(5)

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + Pr\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + Ra Pr \theta$$
(6)

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + Pr\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$
(7)

2.3 Energy equation

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = \frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}$$
(8)

2.4 Energy conservation for the heat source and walls

$$\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) \lambda = 0$$
(9)

where

$$\lambda = \begin{cases} R_h(heat source) \\ R_s(wall) \end{cases}$$

Due to the non-homogeneity of the wall material the coupled analysis imposes the necessity of the heat flux and temperature continuity at the interfaces. The adopted procedure (Patankar, 1980) consists in evaluate the thermal conductivities at the interfaces by the harmonic average between the material conductivities. Boundary conditions are shown in Tab. 1.

Coordinate	Boundary conditions				
<i>x</i> = 0	u = v = w = 0	Heat source: $\frac{\partial \theta}{\partial x} = -\frac{l}{R_h}$	Wall: $\frac{\partial \theta}{\partial x} = 0$		
<i>x</i> = 1.5	u = v = w = 0	$\theta = 0$			
<i>y</i> = 0	u = v = w = 0	$\frac{\partial \theta}{\partial y} = 0$			
<i>y</i> = 5.88	u = v = w = 0	$\frac{\partial \theta}{\partial y} = 0$			
<i>z</i> = 0	$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = w = 0$	$\frac{\partial \theta}{\partial z} = 0$			
<i>z</i> = 2.25	u = v = w = 0	$\frac{\partial \theta}{\partial z}$	- = 0		

Table 1. Boundary conditions.

3. SOLUTION METHOD

The conservation equations were discretized using the Finite Volume Method described by Patankar (1980) with the calculus domain divided in a number of finite control volumes in which equations were integrated. This procedure generated algebraic equation systems, one for each variable, and they were solved using the CGSTAB (Conjugate Gradient Squared Stabilized) method as described by Van den Vorst et al (1990). This method is suitable for the solution of convection-diffusion problems with seven diagonals non-symmetrical coefficient matrix. The coupling between the pressure and the velocity components was established by the use of the SIMPLE (Semi-Implicit Method for Pressure Linked Equations) where a pressure correction equation is written from the mass conservation equation.

The computational procedure was written in FORTRAN Language and the details could be found in Nzamba (2006). The residual term named SORCE, which is formed using the arithmetical mean of the maximum residuals from each algebraic system, was used as convergence parameter. The limit value used was 10⁻⁵. The maximum variation for the variables between interactions was also verified using the equation:

$$ERROMAX = \left| \frac{\phi_{n+1} - \phi_n}{\phi_{max} - \phi_{min}} \right|$$
(10)

where ϕ assumes the value of the dependent variable of the considered system and n is the interaction number.

4. EXPERIMENTAL DATA

In order to obtain the experimental data, an apparatus was built as described in Nzamba (2006). The testing procedure was also the same as there and not included here because the lack of space. The experimental results obtained and utilized here are detailed in table 2.

Case	Heat flux [W]	Cold plate temperature [ºC]	Rayleigh number	
1	0.08	0.7	4.5×10^{4}	
2	0.20	1.1	1.3×10^{5}	
3	2.00	2.1	1.1×10^{6}	

Table 2. Experimental results.

According to Figs 2 to 4, steady state was reached in all tests. Thermocouple position the wall where the heat source was attached is shown in Tab. 3. It is important to point out that temperature measurements at the cold wall (cooper plate) were performed using a thermocouple located at the center of the wall and these values were considered uniform over the whole wall in the comparisons to theory, taking into account the high thermal conductivity of the wall material. The temperature values for the heat source wall used for the comparisons were considered the average value of the last five measurements of each thermocouple. A mean value of a row was taken, and the same procedure was

followed for the numerical results, what made it possible to represent the results in a bi-dimensional form, allowing a straight comparison between the temperature distributions.

Row	Thermocouple coordinates (x ; y ; z)				
1	(0.41; 5.29; -2.06)	(0.41;5.29;0)	(0.41; 5.29; 1.88)		
2	(0.41; 4.41; -2.06)	(0.41;4.41;0)	(0.41; 4.41; 1.88)		
3	(0.41; 3.53; -1.47)	(0.41;3.53;0)	(0.41; 3.53; 1.41)		
4	(0.41; 2.35; -1.47)	(0.41;2.35;0)	(0.41; 2.35; 1.41)		
5	(0.41; 1.47; -2.06)	(0.41; 1.47; 0)	(0.41; 1.47; 1.88)		
6	(0.41; 0.59; -2.06)	(0.41; 0.59; 0)	(0.41; 0.59; 1.88)		

Table 3. Dimensionless position of thermocouples.







Figure 3. Temperature distribution for case 2.



Figure 4. Temperature distribution for case 3.

5. RESULTS

Details of geometry and enclosure characteristics supplied to the computational program are listed in Table 4. The testing of the grid and the validation of the computational program was performed using the data available in the literature (Nzamba, 2006).

	Aspect ratio	Grid (x; y; z)	R _s	R _h	Pr	Fluid	ERROMAX	SORCE
Case 1	4.4	$80 \times 80 \times 31$	4.6	9011	0.707	Air	2.1×10^{-5}	9.7×10^{-6}
Case 2	4.4	$80 \times 80 \times 31$	4.6	9011	0.707	Air	7.9×10^{-6}	9.9×10^{-6}
Case 3	4.4	$80 \times 80 \times 31$	4.6	9011	0.707	Air	3.0×10^{-11}	9.9×10^{-6}

Table 4. Geometric aspects used in the computational program.

Figure 5 represents the comparison between numerical and experimental wall temperature profiles with the heat source. The Rayleigh number is 4.5×10^4 and the Grashof number is 6.4×10^4 . In this case the buoyancy effects already affect the flow and an important convective influence in the heat transfer process shall be noted.

Performing the calculus of the maximum difference between the arithmetical average of the last five values for the three points located at the same row, the results are 1.07°C for row six in case 1, 1.99°C for row 3 in case two and 20.14°C for row 3 in case 3. For cases 1 (Fig. 5) and 2 (Fig. 6) the small differences do not invalidate the proposed methodology. However, for the case 3 (Fig. 7) the difference may affect the analysis by changing the true local of the maximum temperature that is particularly important in the case of electronic devices cooling. The averaged values were used for comparison, in order to improve the response quality of the computational program.

Better agreement is obtained from the case 1. Cases 2 and 3 presented experimental values far from those predicted by the numerical program and, the higher the Rayleigh numbers, the worse the results. A characteristic noted in all three cases from the experiments was the tendency of higher temperatures in the superior region of the source, what was not captured by the theoretical model. This behavior has being investigated and it may be caused by the thermocouple positioning near the heat source. Also, the temperatures reached in the experimental tests are smaller than the predicted by the model. The computational model must be altered to permit the inclusion of heat transfer from the walls to the environment. The different shape of numerical and experimental curves can be explained by the different number of points used for plotting, between theoretical and experimental temperatures.

A three-dimensional temperature distribution for the solid fluid interface in case 1, obtained using the theoretical model, is shown in Fig. 8, and the three-dimensional effects could also be noted. The behavior of such a distribution is coherent with the expectations for a warmed region near the source.

Finally, Fig. 9 presents the isotherms for the same case in the vertical plane at the mean transversal section, where the presence of convective effects can be seen clearly. Also the influence of combined conduction-convection heat transfer at the solid-fluid boundary is evidenced with high values of temperature gradients.



Figure 5. Comparison between numerical and experimental results for case 1.



Figure 6. Comparison between numerical and experimental results for case 2.



Figure 7. Comparison between numerical and experimental results for case 3.



Figure 8. Numerical result at solid-fluid interface for case 1.



Figure 9. Isotherms in the xy plane for z=0 – case 1.

6. CONCLUSION

A three-dimensional study for the steady state laminar natural convection heat transfer in a rectangular enclosure with a heat source at the center of one of the vertical walls was performed. The effects of the conduction inside the heat source and wall were also included. Experiments were conducted with the objective of measuring the temperature distribution at wall with the heater. Experiments were compared with the theoretical results, obtained from a mathematical model based on the conservation equations. This model was solved using a finite volume method and a CGSTAB linear equation system solver.

Theoretical results presented reasonable agreement when compared to experimental data ,however it indicates some directions to to improve the model quality. The first modification is to admit heat losses to the ambient, since the temperatures inside the enclosure were lower than the numerical forecast, and the differences increase as the heating power rises. These losses have to be evaluated during the experimental tests and radiation shields must be provided to the thermocouples near the heat source, to improve the measurement reliability. Finally temperature dependence of properties should be considered, as reported in Andrade and Zaparoli (2003).

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