THEORETICAL AND EXPERIMENTAL EVALUATION OF PARAMETERS FOR THE PI CONTROL OF A REFRIGERATION SYSTEM

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***Abstract**. The refrigeration power control of a conventional refrigeration system is generally done by on-off compressor. This type of control is one of the main causes of energetic efficiency losses, so alternatives have been sought for its substitution. The knowledge of the dynamic characteristics of the refrigeration system is necessary in the substitution of on-off control by modulated control. In this work, dynamic characteristics of a coaxial multiple tube evaporator, as gain and time constant, were determined regarding the compressor speed and the evaporation temperature. In order to determine these characteristics, the response of the degree of superheating to step increase perturbations in the mass flow of the refrigeration fluid at the inlet of the evaporator was analyzed. This dynamic characterization allowed the elaboration of a numerical model of the adaptive PI controller, which acts regulating the degree of superheating at the outlet of the evaporator. In all the simulations made, the controller was effective in leading the system to a stable behavior after a perturbation.*

***Keywords**. Refrigeration system, PI control, Experiment.*

1. Introduction

Air conditioning and refrigeration systems have a significant participation in the global consumption of electrical energy. In this context, some studies were made to identify the causes of the efficiency losses and the project options that can be used in the current refrigeration systems in order to improve its efficiency. In some of this works (Murphy, 1986; O'Neal, 1996; Radermncher, 1996; Pedersen, 1999; Binneberg, 1999) it was verified that the on-off power control is not the most appropriate since it reduces the efficiency of the system by virtue of the shut-on and shut-off dynamic of the compressor. At the instant that the compressor is turned off (under normal conditions of operation) a great part of the total mass of the refrigerant is in the liquid form, in the liquid line or in the condenser. The liquid, under high pressure and high temperature, tends to an equilibrium state with the refrigerant of the evaporator, which is under low pressure and low temperature (Murphy, 1986). This phenomenon causes increasing of the temperature in the evaporator and imposes the necessity of redistribution of the refrigerant during the shut-on of the compressor, which reduces the performance of the system (Coulter, 1997).

Theoretically, the most efficient method of controlling the refrigeration capacity is the variation of the speed of the compressor, resulting in a continuous adjustment of the capacity of the system to its load (Tassou, 1998). Knowledge of the dynamic behavior of each component of the refrigeration circuit, specifically the assemblage evaporator-expansion valve, is required for this method to be successful. During the dynamic functioning, the expansion valve must be able to feed the evaporator appropriately, causing a superheating of the refrigeration fluid at the outlet of the evaporator that is sufficient to avoid the aspiration of the two-phase fluid by the compressor. On the other hand, it is desirable that this superheating does not go beyond certain limits in order to guarantee that the area of thermal exchange of the evaporator is well used (Tamainot-Telto, 1996; Stoecker, 1976). The typical expansion devices like capillary tube and thermostatic expansion valve are not fully able to smoothen the functioning of the system operating with variable speed with the accurate mass flow of refrigerant fluid in the evaporator (Itoh, 1986). Because of this, the use of rapid response expansion valves and cycles of efficient control is needed (Schmit, 1999).

For the expansion valve to operate in an adequate manner, it is necessary to establish the command law that will be responsible for the control of the opening and closing of the throttling passage of the refrigeration fluid. By this command law, it will be possible to maintain the superheating at the outlet of the evaporator within the desirable limits of operation. Since there is an interaction between the expansion valve and the evaporator, knowledge of the dynamic behavior of the evaporator is necessary in order to determine the parameters of regulation for each functioning point of the refrigeration system.

The objective of this work is to present and to analyze the response of the degree of superheating at the outlet of the evaporator to step increase perturbations in the mass flow of the refrigeration fluid at the inlet of the evaporator. This study will permit the dynamic characterization of the evaporator, which enables the development of a controller to regulate the superheating degree of the refrigerant at the outlet of the evaporator.

2. Experimental apparatus

The experimental device used is shown in Fig. (1). This device consists of a heating and refrigerating vapor compression system, which has R134a as the refrigeration fluid and, as secondary fluid, pure water in the evaporator and in the condenser. The system is basically composed by a reciprocating compressor, a condenser, a sub-cooler, an evaporator, two expansion valves (one manual and one thermostatic) and systems to do measures and data acquisition. The open alternative compressor has a piston displacement of 50 cm^3. The three-phase asynchronous electrical motor used in the drive of the compressor has nominal revolution of 1700 rpm and 0.5 hp of power, with maximum operational frequency of 100 Hz. The electrical motor is fed by a frequency inverter that enables the variation of the revolution speed of the motor-compressor assemblage, in a band of 0 to 100 Hz. The shell and tube condenser used has a capacity of 6 kW. The temperature of the secondary fluid is adjusted by mixing warm water that comes from the condenser itself with room temperature water, coming from the feeding system. The sub-cooler is of the coaxial type, made of an envelope tube and of an internal tube in "U". The electrical heating system of the sub-cooler still needs some alterations for an efficient functioning. In this work it was decided not to use the sub-cooler. All the water from the tank and of the sub-cooler was drained to avoid any kind of interference in the temperature of the refrigeration fluid. The multiple tubes coaxial evaporator is composed by an envelope tube of flexible PVC and of three inner cooper tubes through which the refrigeration fluid flows. The water flows in counter flow in the annular space between the PVC and cooper tubes. The evaporator was projected to furnish a maximum refrigeration power of 3 kW. The set of tubes was assembled so that the evaporator becomes more compact and, at the same time, promotes a greater turbulence in the flow of the fluids. The water temperature of the evaporator is controlled by a PID controller that acts continuously on the power supplied by the electrical resistance, maintaining the temperature of the water within the desirable limits. The assemblage has two valves of expansion placed in parallel. One is thermostatic with an outside pressure equalization and the other is manually regulated. A blockage valve permits the isolated operation of each component of expansion. In this work, only a manual expansion valve was used. The testing bench has 11 thermocouples of the T type (cooper-constantan) with 1.5 mm of diameter, response time of 0.3 s in the air and $\pm$ 0.5ºC of uncertainty. The thermocouples were implanted in the tubes at the inlets and outlets of each components of the system. Two Bourdon manometers were used (uncertainty of $\pm$ 0.25% over the band of measurements) for measuring pressure. They were installed at the inlet and at the outlet of the compressor.

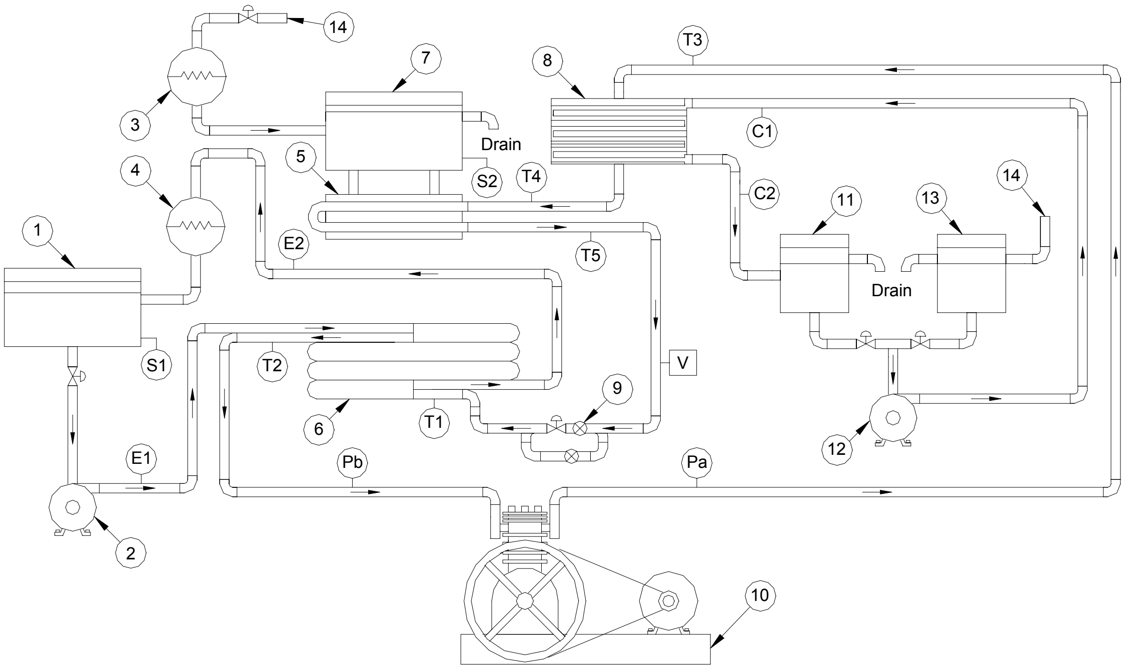

Figure 1 – Schematic configuration of the experimental bench

Legend:

1. Water reservoir (evaporator)
2. Water pump (evaporator)
3. Electric heater (sub-resfriador)
4. Electric heater (evaporator)
5. Cooler
6. Evaporator
7. Water reservoir (sub-resfriador)
8. Condenser
9. Expansion valves
10. Reciprocating compressor
11. Hot water reservoir (condenser)
12. Water pump (condenser)
13. Cold water reservoir (condenser)
14. Water feed

V. Flow meter
Pa. Pressure sensor (condensation)
Pb. Pressure sensor (evaporation)
S1,S2. Temperature sensor (PID)
T1-T5. Temperature sensor of the refrigeration fluid
E1,E2. Temperature sensor of the water at inlet and outlet of the evaporator
C1,C2. Temperature sensor of the water at inlet and outlet of the condenser

The bands of measurements are from 0 to 10 bar on the suction line and from 0 to 34 bar on the discharge line. Two piezoresistive pressure sensors were installed at the inlet and outlet of the expansion valve. At the entrance of the expansion valve, the sensor has a range of measurement from 0 to 24 bar, while at the exit the range is from 1 to 9 bar. Both sensors have an uncertainty of ± 0.2% of the calibrated range. A sensor of differential pressure installed between the inlet and the outlet of the evaporator and the condenser permits the measure of the loss of pressure of the refrigeration fluid flow. Its range of measurement is from 0 to 2.5 bar with uncertainty of ± 0.075% of calibrated range. The refrigerant mass flow was measured with a Coriolis flowmeter. Its range of measurement is from 15 to 200 kg/h. A commercial software computes the maximum uncertainty of this instrument based on the measured mass flow. To determine the mass flow of the secondary fluid in the evaporator, a test tube (with subdivisions of 20 ml) and a digital chronometer (with minimum reading of a hundredth of a second) were used. All the signals generated by the different sensors of the test bench are received and treated by a data acquisition system. A commercial software permits the visualization and registers in a file the temperatures, pressures, and mass flow in real time.

3. Methodology used

In order to characterize the evaporator, it was first necessary to define the variables of inlet and outlet to be used in function of the objective of the work. The variables of inlet correspond to the ones over which it is possible to act directly in order to modify the functioning point, and the variables of outlet characterize this functioning point. In this case, the temperature and the mass flow of the secondary and the refrigeration fluids at the inlet of the evaporator were defined as variables of inlet, while the temperature of the secondary fluid at the outlet of the evaporator and the temperatures of the refrigeration fluids at the inlet and at the outlet of the evaporator were defined as variables of outlet. The other variables are maintained constant. Once these variables were defined, the system was adjusted to operate with constant superheating at the outlet of the evaporator at approximately 7ºC. The adjustment was made by the opening and closing of the manual expansion valve. The boiling temperature was maintained constant by means of temperature control of the secondary fluid of the evaporator. When the variables of outlet presented stable limits, the mass flow was reduced 5% by the closing the expansion valve (step decrease). The time-wise evolutions of these variables were then registered in a data file. When the system reached the new steady state, the test for that operation point was concluded. Tests were made for different operation points corresponding to the boiling temperatures of -10ºC, 0ºC, and 10ºC, and the compressor speeds of 750 rpm, 850 rpm and 950 rpm. In all of the tests made the temperature of condensation was maintained constant at 40ºC, and the mass flow of the secondary fluid of the evaporator maintained at 130 ml/s.

4. Model identification

By definition, identifying a dynamic system consists of determining a model based on the study of the behavior of the inlets and outlets of the system by means of experiments and to determine the numerical values of the parameters used in the model. One of the methods that provides an approximate representation of the system as a transfer function is the Broïda method (Outtagarts, 1994). An identification of the model by this method consists of representing the evaporator by a first order transfer function with a pure delay as shown below:

$$H(p) = \frac{\Delta T(p)}{\Delta \dot{m}(p)} = \frac{K \cdot e^{-\tau \cdot p}}{(1 + \theta \cdot p)} \tag{1}$$

Where τ and θ are, respectively, the delay and the evaporator time constant; p is the Laplace variable. The gain, K, represents the relation between the total variable of degree of superheating, $\Delta(\Delta T_s)$, and the refrigerant mass flow at the inlet of the evaporator, $\Delta \dot{m}_{fe}$. The time constant represents the instant in which the superheating reached 63% of the total value of the variation. The system delay is defined as the period of time existing between the moment of entry of the perturbation and the moment corresponding to the start of the response of the superheating degree.

The evaporator gain can be computed by the expression:

$$K = \Delta(\Delta T_s)/\Delta \dot{m}_{fe} \tag{2}$$

The time constant can by estimated by:

$$\theta = 5.5 \cdot (t_2 - t_1) \tag{3}$$

Where t_2 and t_1 represent respectively the instant of time when 28% and 40% of the final variation of the steady state. Using the inverse Laplace transform in Equation 1, the superheating of the refrigerant is given by:

$$\begin{aligned} &\Delta T_s(t) = \Delta T_s(t_0) + \Delta \dot{m}_f \cdot K \cdot \left(1 - e^{-\frac{t}{\theta}}\right) && \text{for } t > t_0 \\ &\Delta T_s(t) = \Delta T_s(t_0) = \text{constant} && \text{for } t \leq t_0 \end{aligned} \tag{4}$$

Starting from the experimental tests done, the time constant and the gain of the evaporator can be determined for the operational points mentioned.

5. Control system of the superheating in the evaporator

In the experimental trials made the delay presented negligible values thus being considered as zero. Therefore, the Eq. (1) can be written as follows:

$$H(p) = \frac{K}{(1 + \theta \cdot p)} \tag{5}$$

For means of computation, it is convenient to represent the control system by a block diagram (Fig. (2)), which also facilitates the visualization of the existing relations between the various signals.

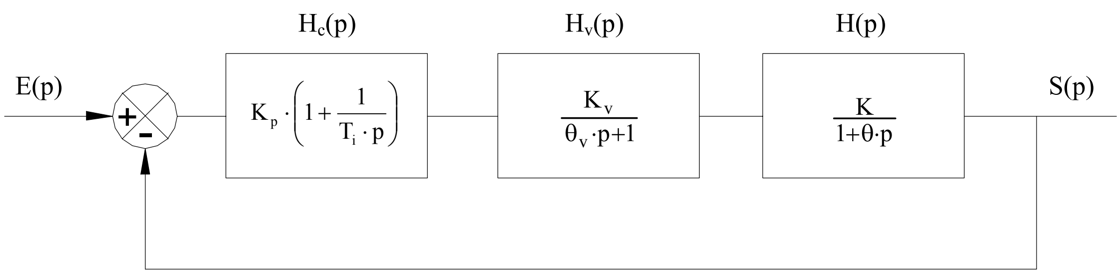

Figure 2 – Representation by a block diagram of the control system of the evaporator

Where $H_c(p)$ and $H_v(p)$ represent respectively the transfers functions of the PI controller of the expansion valve; K_p, T_i, K_v, and θ_v are respectively the proportional gain, the integral time, the gain of the expansion valve and the time constant of the expansion valve.

The global transfer function of the system in a closed loop G(p) represents a relation between the outlet S(p) and the inlet E(p).

$$G(p) = \frac{S(p)}{E(p)} = \frac{K_p \cdot K_v \cdot K \cdot (T_i \cdot p + 1)}{T_i \cdot \theta_v \cdot \theta \cdot p^3 + (T_i \cdot \theta_v + T_i \cdot \theta) \cdot p^2 + (T_i + K_p \cdot K_v \cdot K \cdot T_i) \cdot p + K_p \cdot K_v \cdot K} \tag{6}$$

In equation (6), the polynomial of the denominator is known as the characteristic equation of the system. The roots of the characteristic equation are the poles of the closed loop.

6. Controller tuning

Tuning a controller means defining values for the constants T_i and K_p that lead the system to a stable behavior and that, at the same time, offers an action of rapid control with the smallest over signal possible. As the system in study works in various different points of operation, it is desirable that T_1 and K_p are defined in a way to be readjusted to each alteration in the functioning point of the evaporator. This is known as adaptive control or adaptive tuning. The criterion of Naslin (Outtagarts, 1994) is a robust method of easy application used in order to obtain the adaptive tuning and, for that reason, it was utilized in this work. The application of the criterion of Naslin in the present study consists of using the coefficients of the characteristic equation to obtain expressions for T_i and K_p in function of the gain and of the time constant of the evaporator, among other variables.

Therefore, the generic expression of the Eq. (6) is written as shown bellow:

$$G(p) = \frac{\sum_{j=0}^{m} b_j p^j}{\sum_{i=0}^{n} a_i p^i} = \frac{b_0 + b_1 \cdot p + \cdots + b_{m-1} \cdot p^{m-1} + b_m \cdot p^m}{a_0 + a_1 \cdot p + \cdots + a_{n-1} \cdot p^{n-1} + a_n \cdot p^n} \tag{7}$$

The constants T_i and K_p are determined by existing relations between the coefficients a_i of the denominator of the transfer function in closed loop of the system and the constant α; function of the maximum over signal of the response to a step increase.

$$a_i^2 \geq \alpha_i \cdot (a_{i-1} \cdot a_{i+1}) \qquad 1 \leq i \leq n-1 \tag{8}$$

Where n is the degree of the characteristic polynomial. For n = 3 we can write starting from Eq. (8):

$$\frac{(T_i + K_p \cdot K_v \cdot K \cdot T_i)^2}{(K_p \cdot K_v \cdot K) \cdot (T_i \cdot \theta_v + T_i \cdot \theta)} \geq \alpha_1 \qquad \text{for } i = 1 \tag{9}$$

$$\frac{(T_i \cdot \theta_v + T_i \cdot \theta)^2}{(T_i + K_p \cdot K_v \cdot K \cdot T_i) \cdot (T_i \cdot \theta_v \cdot \theta)} \geq \alpha_2 \qquad \text{for } i = 2 \tag{10}$$

Making $\alpha_1 = \alpha_2 = \alpha$ the Eq. (9) and Eq. (10) can be written as:

$$K_p = \frac{(\theta_v + \theta)^2 - \alpha \cdot \theta_v \cdot \theta}{\alpha \cdot \theta_v \cdot \theta \cdot K_v \cdot K} \tag{11}$$

$$T_i = \frac{\alpha \cdot (K_p \cdot K_v \cdot K) \cdot (\theta_v + \theta)}{(1 + K_p \cdot K_v \cdot K)^2} \tag{12}$$

Equations (11) and (12) depend on the coefficient α, which is an empirical function of maximum over signal (M_p) of the response to a step.

$$\alpha = 2.4 - \frac{\log M_p}{2} \tag{13}$$

This expression is only valid for characteristic equations of the orders 2 to 8, and the value of M_p must be given in %. It is common in control engineering to utilize a damping coefficient of 0.7. For a system of second order this is equivalent to a maximum over signal of 6%.

Note that the characteristic equation obtained in this work is of the third order but, once the effect of the additional pole is to reduce the maximum over signal and to increase the settling time**,** we can assume the maximum over signal of a third order system the same as the one of a second order system. Therefore, for M_p = 6% the value of the constant α reaches 2.

7. Results analysis

Figure (3) presents the response of the degree of superheating to a step increase perturbation on the mass flow of the refrigeration fluid. The functioning point presented corresponds to a medium evaporation temperature of 0.18ºC and a medium rotation of the compressor of 950 rpm.

The mass flow of the refrigeration fluid in the evaporator stabilizes 14 s after the application of the perturbation over the inlet mass flow, while the degree of superheating changes gradually stabilizing 161 s after the start of the perturbation.

The mass flow at the outlet of the evaporator ($\dot{m}_{fs}$) was mathematically obtained based on experimental data; it is presented in Fig. (2). It was observed that at the instant of the step increase, the mass flow at the outlet of the evaporator was superior to the mass flow at the inlet of the evaporator. The increase of the degree of the superheating is justified by the decrease of mass of refrigerant in the evaporator in consequence of the differences of the mass flow at the inlet and at the outlet.

Starting from the tests made, the time constant and the gain of the evaporator were determined for the points of operation mentioned. The static gain of the evaporator diminished with the increase of the temperature of evaporation as much as with the increase of the rotation of the compressor (Fig. (5)). This fact is mainly due to the increase of mass flow with the increase of these two scalars (Fig. (4)). As the mass flow rate is the denominator of the function that defines the gain, it is natural that the permanent gain of the evaporator decreases as the mass flow rate increases (Maia, 2000). The variation of the superheating was not sufficient to interfere in this tendency.

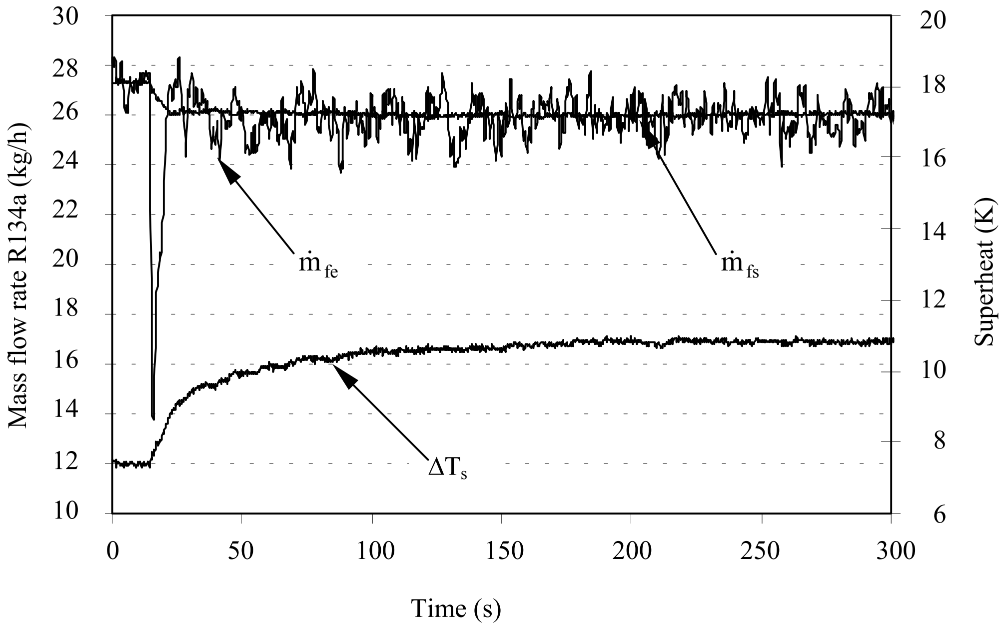

Figure 3 - Response of the degree of superheating to a perturbation of the step type on the mass flow of the refrigeration fluid

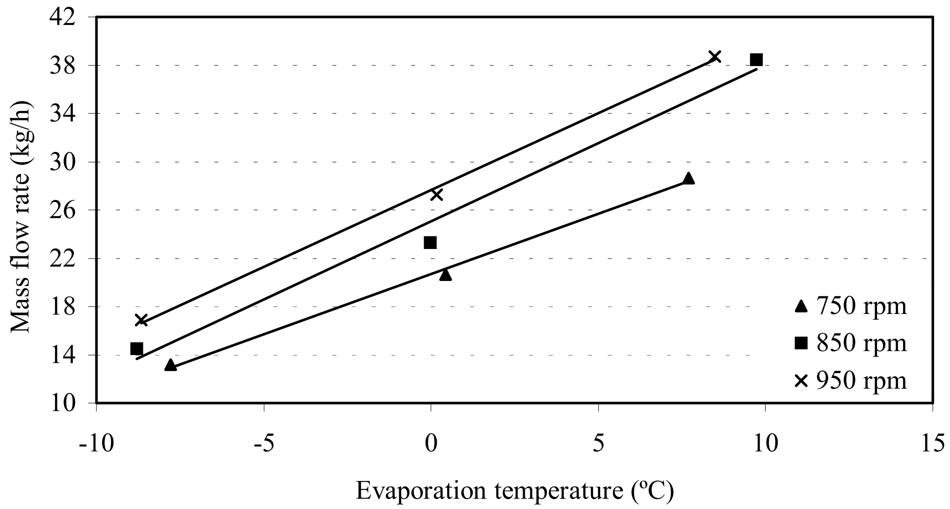

Figure 4 – Evolution of the mass flow rate in function of the evaporation temperature and the compressor speed.

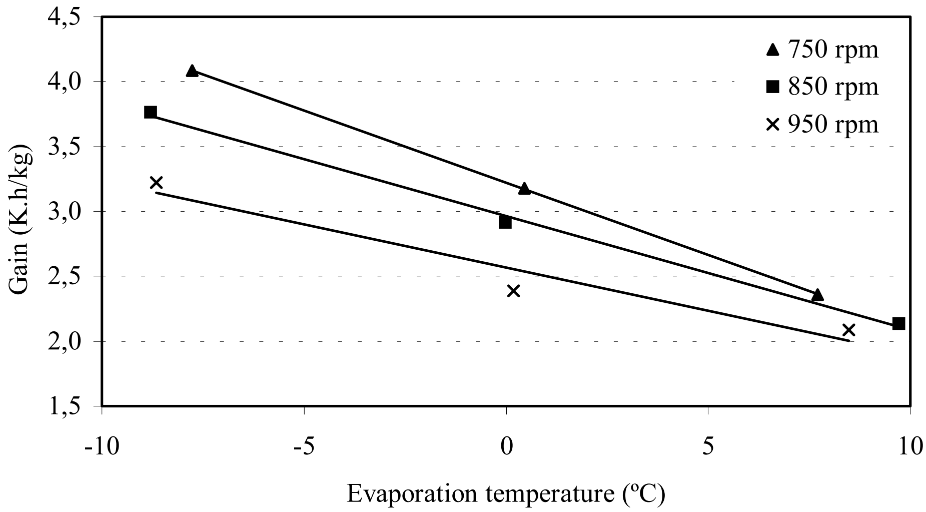

Figure 5 – Evolution of the gain of the evaporator in function of the evaporation temperature.

The time constant presented a reduction with the increase of the temperature of evaporation (Fig. (6)) because, for a fixed rotation of the compressor, the specific mass of the refrigeration fluid at the inlet of the compressor grows with the evaporation temperature, causing an increase on the mass flow rate of the fluid. The increase of the mass flow rate, in return, makes the thermal changes occur with greater efficiency, causing the system to reach the steady state more rapidly (Maia, 2000).

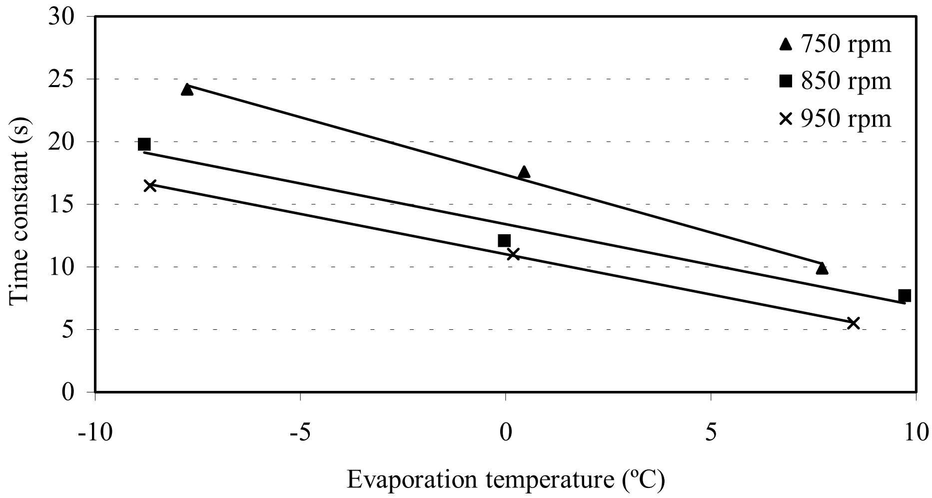

Figure 6 – Evolution of the time constant of the evaporator in function of the evaporation temperature.

The average uncertainty in the computation of the gain is 40%. As it is a matter of a direct measure, which supposedly has a low uncertainty of measurement, the uncertainty of the computation of the time constant was not considered.

The laws of variation of these variables were identified, starting from the results, by an expression of the following form:

$$y = a_0 + a_1 N + a_2 T_{eb} + a_3 T_{eb}^2 + a_4 T_{eb} N + a_5 T_{eb}^2 N + a_6 T_{eb} N^2 + a_7 T_{eb}^2 N^2 \quad (14)$$

In this equation, T_{eb} is expressed in K and N in rps. The coefficients of the equation (14) are given in Tab. (1). The coefficients for the time constants and the coefficients for the gains of the evaporator are represented respectively on the first and second lines.

Table 1 – Coefficients for the constants of time and for the gains of the evaporator.

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
-1624.85	158.2617	23.7414	-0.06375	-2.62545	0.007346	0.048247	-0.00017
-995.644	79.00461	7.542197	-0.01424	-0.58294	0.001081	-0.00039	1.13×10^{-6}

8. Evaluation of the controller performance

Starting from the transfer function in a closed loop, Eq. (6), a diagram of simulation was elaborated in Matlab-Simulink environment to evaluate the controller performance. The system was submitted to a step increase disturbance and the step-wise evolution at the outlet was obtained for the nine points of operation proposed in this work.

Figure (7) presents the response of the control system of the degree of superheating in the evaporator in a closed loop due to a unitary step increase at the inlet. It was observed that the system converges about 20 seconds after the perturbation. The maximum over signal obtained was of the order of 30% and occurred 6 seconds after the perturbation.

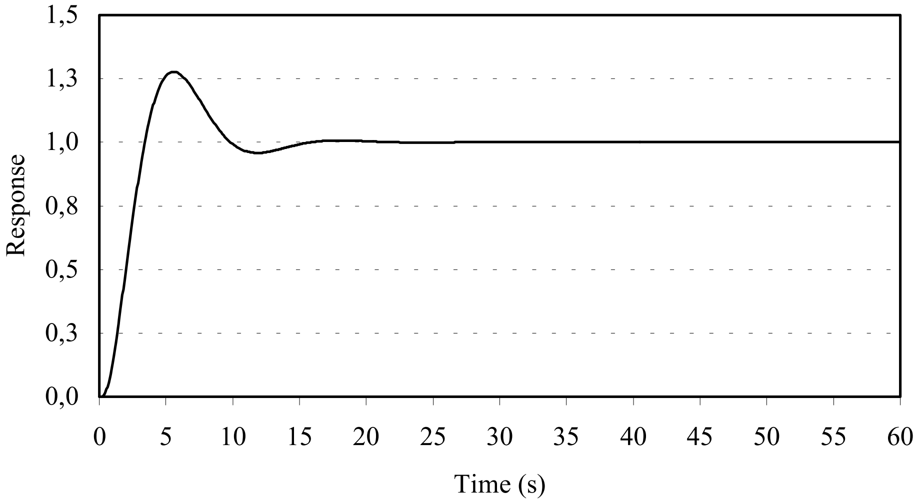

Figure 7 – Response of the control system of the step increase of the superheating in the evaporator in a closed loop to a unitary step at the inlet for T_{eb} = 10ºC and N = 950 rpm.

The controller was also evaluated for the other points of operation. In all the points tested the system reached a stable behavior. The greatest over signal obtained was of the order of 30%. The time of convergence was almost the same in all the simulations made.

9. Conclusion

A study to determine the response of the degree of superheating to a step increase perturbation in the mass flow rate of the refrigeration fluid at the inlet of the evaporator was performed. This study permitted the determination of laws of variations of the characteristic variables of the evaporator (gain and time constant) as functions of the evaporation temperature and of the compressor speed. The equations that describe these relations were used in the modeling of a PI controller with adaptive gains, to act in the regulation of the degree of superheating at the outlet of the evaporator.

The system of control in the resulting closed loop was submitted to simulations in Matlab-Simulink environment for various operation points. The control model proved to be effective to lead the system to a stable behavior after a step increase perturbation. The maximum over signal detected in the simulations was of 37%. A process of optimization of controller gains can reduce the value of the maximum over signal.

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