ORDER TRACKING METHODS ANALYSIS

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Abstract. *Order tracking analysis is a measurement technique suitable for variable speed machines, in which vibration signals are non-stationary. As a result, in a waterfall plot, the frequency spectrum components can be displayed as stationary lines versus orders (multiples of the shaft rotation) instead of frequency, in Hz. This paper investigates three methods of order tracking: the traditional Computed Order Tracking, Digital Resampling and the TVDFT (Time Variant Discrete Fourier Transform) methods. The performance – computational time and accuracy – of each method is analyzed for different types of simulated operating conditions of rotating systems.*

Keywords: *Order tracking, Vibration, Rotordynamics.*

1. Introduction

Order tracking is a technique capable of identifying the orders amplitudes e phases in variable speed machines. There are different methods of order tracking and, in this paper, three of them are analyzed and compared: the Computed Order Tracking (COT), the Digital Resampling and the TVDFT (Time Variant Discrete Fourier Transform). In digital resampling based order tracking method, the vibration signal is sampled at a constant time increment and then resampled at constant angle increments. In the Computed Order Tracking method both vibration signal and keyphasor pulses are sampled in time domain with a constant sampling time interval. Assuming the shaft is under constant angular acceleration within three consecutive keyphasor pulses, the angular position of the rotating shaft can be determined by a quadratic equation. Using this quadratic equation, one can calculate the time corresponding to a shaft angle where sampling should take place. In the Digital Resampling method, the curve of the instantaneous angular speed is estimated from the tachometer signal and, as a post-processing step, the curve of the angular displacement versus time is obtained by integration. From this curve, the instants of time corresponding to a constant angle interval are derived. In the TVDFT order tracking method it is not necessary to resample the vibration data. This technique based on a discrete Fourier Transform and uses a kernel whose frequency changes with the variation of the machine rotational speed.

This paper present a brief review of the theory of these three methods and investigates numerically their performances for determination of amplitude and phase angle of orders in 3 cases: (i) when a rotating system has only one order, (ii) the system has crossing orders, and (iii) the system has multiple close orders.

2. Order Tracking Techniques

The order tracking analysis is a measurement technique suitable for variable speed machines. Most of the order tracking methods require sampling of the vibration signal at constant angle increments and hence at a sampling rate proportional to the shaft speed of the machine (Fyfe and Munck, 1997; Blough, 1998). As a result, in a waterfall plot, the frequency spectrum components can be displayed as stationary lines versus orders (multiples of the shaft rotation) instead of frequency, in Hz.

The first method of order tracking, the Analog Order Tracking, directly samples the analog vibration signal at constant shaft increment using analog equipments such as a ratio synthesizer and an anti-aliasing tracking filter, as shown in Fig.(1). But the use of these analog instrumentation increases the cost of the measurement process, and also leads to a number of problems in many machines operating with fast run-up rates or where high order numbers are being analyzed (McDonald and Gribler, 1991).

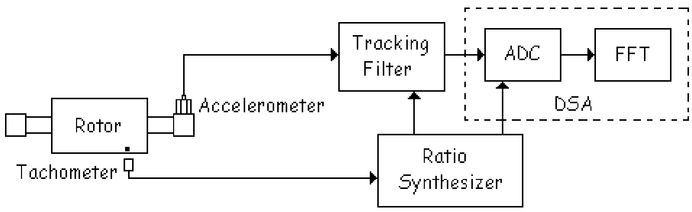


Figure 1. Analog order tracking method.

With the improvement of the digital signal processors (DSP) and microprocessors in high performance digital signal analyzer (SDA), the analog equipments used in the analog order tracking were replaced by digital implementation in software. Hence, the digital version of the method was developed and named Computed Order Tracking (COT). It was developed and patented by Potter (Potter and Gribler, 1989; Potter, 1990) from Helewett-Packard. In this tecnhique both vibration and tachometer signals are sampled in the time domain at a constant time interval and then resampled, through a software, at a desirable constant angular increment Δθ, which corresponds to irregular time increments (resampling times) as shown in Fig. (2). The resampling times are calculated based in the values of the instantaneous rotational speed, which is measured from a tachometer signal (often referred to as keyphasor signal). Generally, once per revolution tachometer signal is used.

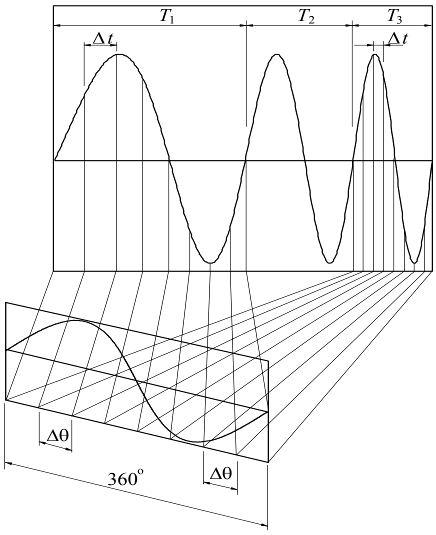


Figura 2. Angular domain representation of a signal.

In this method, the shaft is considered under a constant angular acceleration within three consecutive tacho pulses (if there is only one pulse per revolution). Then, the angular position of the rotating shaft can be calculated by the following quadratic equation:

$$\theta(t) = b_o + b_1 t + b_2 t^2 \tag{1}$$

where θ is the shaft angle and b_0, b_1, and b_2 are unknown coefficients. Thus, three time points are needed to solve for these unknown coefficients, i.e., these coefficients are found by fitting three sucessive keyphasor arrival times (t_1, t_2, t_3), which occur at known shaft angle intervals, ΔΦ. In the case of once per revolution event, this angle interval is 2π radians. Thus:

$$\begin{aligned} \Phi(t_1) &= 0 \\ \Phi(t_2) &= \Delta\Phi \\ \Phi(t_3) &= 2\Delta\Phi \end{aligned} \qquad \therefore \qquad \begin{pmatrix} 0 \\ \Delta\Phi \\ 2\Delta\Phi \end{pmatrix} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{bmatrix} \begin{Bmatrix} b_o \\ b_1 \\ b_2 \end{Bmatrix}. \tag{2}$$

Once the vector b is known, Eq.(1) is rearranged to solve for the resampling time:

$$t = \frac{1}{2b_2}\left[\sqrt{4b_2(\theta - b_o) + b_1^2} - b_1\right] \tag{3}$$

There is another process to estimate resampling times in this method: the amplitude interpolation process, i.e., the estimation of the amplitudes for the required new times or shaft positions. Cubic splines, piecewise cubic interpolation, and even simple linear interpolation are used to determine the best amplitude fitting. In Fig. (3) the steps of the COT method are ilustred.

In the COT method, the data in angle domain are obtained after each three tachometer pulses have been measured. That is why this method is referred to as digital resampling in on-line way. Fyfe and Munck (1997) analyze this technique in details. Other papers that also explain this technique are those from McDonald and Gribler (1991), Bossley and McKendrick (1999) and Mesquita et al (2002).

A variation of the above method is obtained when the curve of the instantaneous angular speed is estimated from tachometer signal and, as a post processing step, the curve of the angular displacement is obtained by integration. There is no assumption of constant angular acceleration. With the information of the angular displacement versus time, the resampling times can be calculated, corresponding to a desired constant angle interval.

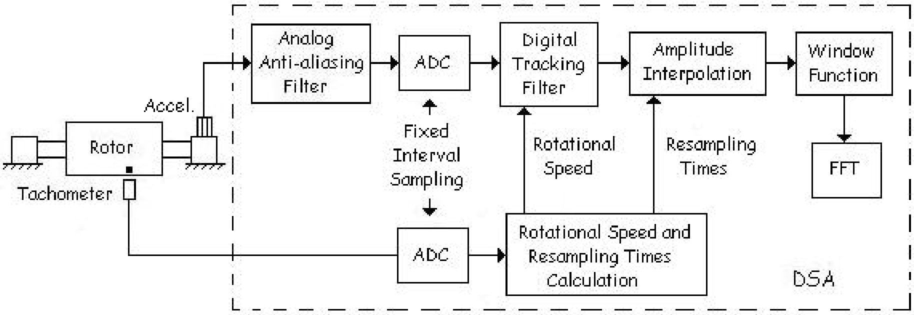


Figure 3. Steps of the COT method.

In this new method, referred to only as Digital Resampling, the vibration signal is filtered by a fixed frequency low-pass anti-aliasing filter, and it is sampled at constant increments of time, Δt. Once the signal has been sampled, it is resampled computationally using the tachometer signal to extract the signal amplitude at a constant angle increment, $\Delta\theta$.

The basic steps of this method are ilustred in the block diagram in Fig. (4). The tacho signal is measured and the instantaneous frequency versus time curve is obtained (step 1a). Then, this curve is integrated to give the angular displacement versus time (step 2a). From this curve, the desired (constant) resolution in the angle domain defines the time intervals at which data samples of the vibration measures should be resampled, i.e., the resample times (step 3a).

On the other hand, the digital vibration data is upsampled (Crochiere and Rabiner, 1983) in order to improve the process of amplitude interpolation as well as the signal to noise ratio (Blough, 1998) (step 1b). Then, the data are low-pass filtered to eliminate the spectral images obtained in the up-sampling process (step 2b). The next step (3b) is the amplitude interpolation process, i.e., the estimation of the amplitudes of vibration corresponding to the resample times. After the amplitudes are determined, a type of window function is applied to the data in order to avoid leakage (when the number of revolution is not an integer in the time record (block size) (step 4). Finally, the resampled data are transformed from the angle domain to the order domain by means of the FFT (steps 5 and 6). It should be noted that a decimation [11] (anti-aliasing filtering and downsampling) operation could be performed to obtain a desired sample rate in the angle domain. The procedure of decimation is done between the steps 3b and 4.

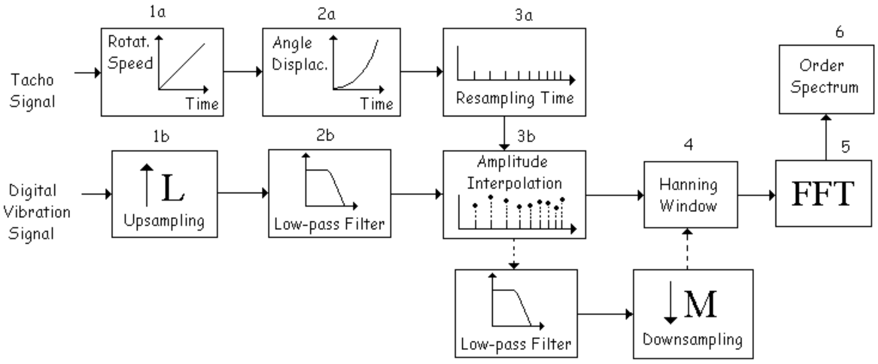


Figure 4. Steps in the digital resampling method.

The last two methods (COT and Digital Resampling) described in this section can be classified as digital resampling based order tracking, and they are the most common order tracking methods in use in commercial softwares and dynamic signal analyzers (Blough, 1998).

Recently, some new methods of order tracking have been developed such as the Kalman filter based methods (Vold and Leuridan, 1993; Vold 1997) and the TVDFT method (Blough et al., 1996; Blough, 1998). The former will not be treated in this paper. The Time Variant Discrete Fourier Transform (TVDFT) is an order tracking method but does not resample the vibration data. It is performed directly on data that is sampled with a constant time interval, reducing considerably the computational effort.

The TVDFT is a special case of the chirp-z transform. The chirp-z transform is defined as a type of Fourier transform with a kernel whose frequency and damping vary as a function of time. The TVDFT is defined as a discrete Fourier transform whose kernel frequency varies as a function of time defined by the rpm of the machine, but the damping does not vary as a function of time. This kernel is a cosine or sine function of unitary amplitude and an instantaneous frequency equal to that of the tracked order at each instant of time. Its expressions are presented in Eqs. (4a) and (4b):

$$a_m = \frac{1}{N}\sum_{n=1}^{N} x(n.\Delta t)\cos\left(2\pi\int_0^{n.\Delta t}(O_m.\Delta t.\frac{rpm}{60})dt\right), \tag{4a}$$

$$b_m = \frac{1}{N}\sum_{n=1}^{N} x(n.\Delta t)\,\mathrm{sen}\left(2\pi\int_0^{n.\Delta t}(O_m.\Delta t.\frac{rpm}{60})dt\right), \tag{4b}$$

where O_m is the order which is being analyzed, x is the operating data, N is the block size of the transform, Δt is the sampling interval, a_m is the Fourier coefficient of the cosine term for O_m, b_m is the Fourier coefficient of the sine term for O_m, and rpm is the instantaneous rpm of the machine.

In order to obtain better results when orders are either very close together or crossing one another, an orthogonality compensation matrix (OCM) may be applied. The application of the OCM also allows faster sweep rates to be analyzed more accurately. The OCM may be applied in a post-processing step to the order estimates from a TVDFT analysis. Very close orders are normally difficult to be separated using resampling techniques, as well as in the TVDFT. Nevertheless, if the OCM is used, these close orders can be separeted effectively (Blough, 1998).

The formulation of OCM is a set of linear equations formulations that must be solved for each rpm value:

$$\begin{bmatrix} e_{11} & e_{12} & e_{13} & \cdots & e_{1m} \\ e_{21} & e_{22} & e_{23} & & e_{2m} \\ e_{31} & e_{32} & e_{33} & & e_{3m} \\ \vdots & & & & \vdots \\ e_{m1} & & \cdots & & e_{mm} \end{bmatrix} \begin{Bmatrix} o_1 \\ o_2 \\ o_3 \\ \vdots \\ o_m \end{Bmatrix} = \begin{Bmatrix} \tilde{o}_1 \\ \tilde{o}_2 \\ \tilde{o}_3 \\ \vdots \\ \tilde{o}_m \end{Bmatrix}. \tag{5}$$

where e_{ij} is the cross orthogonality contribution of order i in the estimate of order j, O_i is the compensated value of order i, and $\tilde{O}_i$ is the estimated value of order i obtained using the TVDFT.

The cross orthogonality terms, e_{ij}, are calculated by applying the kernel of order i to the kernel of order j, as shown in Eq. (6)

$$e_{ij} = \frac{1}{N}\sum_{n=1}^{N}\left\{\exp\left(2\pi\int_0^{n.\Delta t}(o_i.\Delta t.\frac{Rpm}{60})dt\right).(window).\exp\left(2\pi\int_0^{n.\Delta t}(o_j.\Delta t.\frac{Rpm}{60})dt\right)\right\}. \tag{6}$$

Equation (5) is solved by multiplying both sides by the inverse of OCM. The resulting order estimates are linearly independent to one another:

$$\begin{Bmatrix} o_1 \\ o_2 \\ o_3 \\ \vdots \\ o_m \end{Bmatrix} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & \cdots & e_{1m} \\ e_{21} & e_{22} & e_{23} & & e_{2m} \\ e_{31} & e_{32} & e_{33} & & e_{3m} \\ \vdots & & & & \vdots \\ e_{m1} & & \cdots & & e_{mm} \end{bmatrix}^{-1} \begin{Bmatrix} \tilde{o}_1 \\ \tilde{o}_2 \\ \tilde{o}_3 \\ \vdots \\ \tilde{o}_m \end{Bmatrix}. \tag{7}$$

As any order tracking method, the TVDFT is also very sensitive to the quality of the instantaneous frequency measurement, i.e., its accuracy depends on the quality of the tachometer signal processing.

3. NUMERICAL RESULTS

In this section it is presented the numerical results of the identification of orders amplitudes and phase angles in three different conditions: the first case is the unbalance response of a De Laval rotor during run-up (in this case one has only one order to be estimated), the second one simulates a rotating system with two crossing orders and the third case is represents a rotating system with multiple close orders. For each case, the COT, Digital Resampling, TVDFT and TVDFT with OCM are applied and the results are discussed.

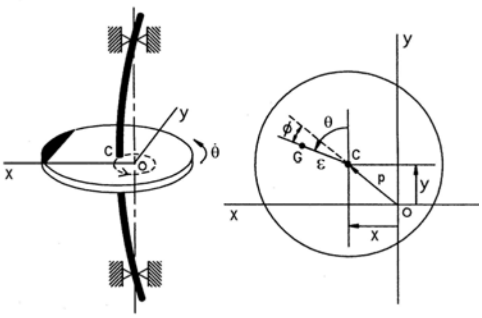
In Tab.(1) the required time for the processing of each order tracking method is presented for each simulated cases. Note that the COT and Digital Resampling methods, that are resampling based techniques, present similar computational time, while TVDFT method is almost five times faster than the previous techniques. The main difference between the resampling techniques and the direct method (TVDFT) is the interpolation step, which increases considerably the processing time. Oftentimes this step limits the application of the resampling techniques to slow sweep rate cases. For faster sweep rates, the number of required interpolations grows significantly once the interpolation is performed for each revolution of the rotating system. The biggest advantage of the TVDFT method is that it estimates the order with a reduced amount of computational complexity and, as a consequence, faster than the resampling techniques, as discussed by Blough, 1996.

Table 1: Required time for each method processing.

Order tracking methods	Anisotropic (Direction x)	Crossing orders (Direction x)	Multiple orders (Direction x)	Number of Points/ Revolution per block
COT	5 min 01 sec	6 min 36 sec	5 min 04 sec	1500 points
Digital Resampling	5 min 03 sec	6 min 38 sec	5 min 06 sec	1500 points
TVDFT	1 min 05 sec	1 min 06 sec	1 min 08 sec	61 revolutions

Case 1) Simulation of an anisotropic De Laval rotor

A simulation of a variable speed rotor permits to compare the order tracking methods. The non-stationary signals are the unbalance response, obtained by integration of the De Laval rotor equations of motion, shown in Fig. (5).



$$\begin{cases} m\ddot{x} + c_{xx}.\dot{x} + k_{xx}.x = m_G.\varepsilon\left(\dot{\theta}^2.\cos(\theta) + \ddot{\theta}.\text{sen}(\theta)\right) \\ m\ddot{y} + c_{yy}.\dot{y} + k_{yy}.y = m_G.\varepsilon\left(\dot{\theta}^2.\text{sen}(\theta) - \ddot{\theta}.\cos(\theta)\right) \end{cases} \quad (8)$$

Figure 5. DeLaval rotor system and equations of motion.

For the anisotropic De Laval rotor a set of parameters k_{xx} and k_{yy} are chosen in order to adjust the natural frequencies to 25 and 50 Hz, for the direction x and direction y, respectively. The rotor acceleration is 20 rad/s^2. The order map parameters are: maximum order tracked equals to 10; order sampled for TVDFT method equals to 0.01 and 40 time lines. The unbalance response, in the *x* and *y* directions, and the waterfall plot of the response in the *x* direction are presented in Fig.(6) and Fig.(7), respectively. The results in the *y* direction are not presented because they are to those of the *x* direction.

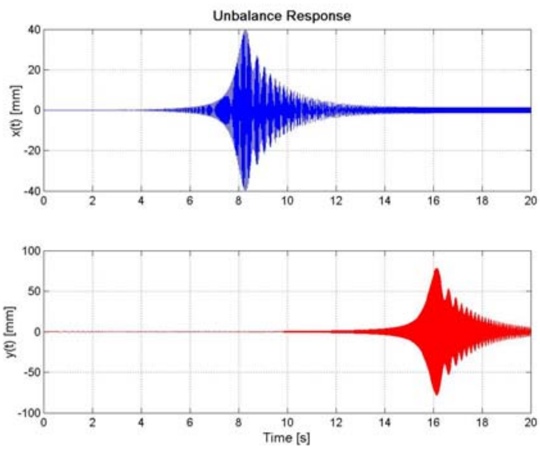


Figure 6. Unbalance response in *x* and *y* directions.

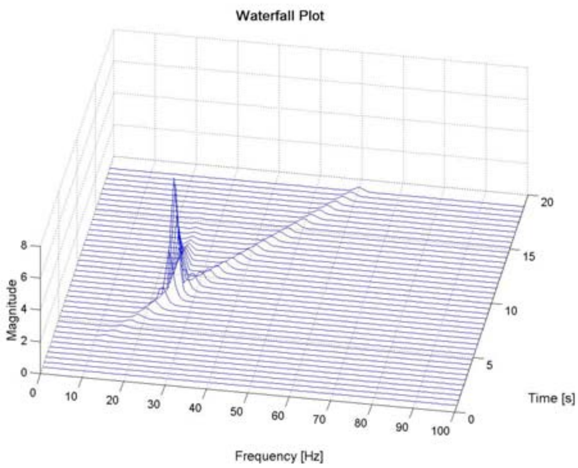


Figure 7. Waterfall Plot in the *x* direction.

The order map, amplitude and phase angle of the order in the *x* direction, estimated for each order tracking method, are presented in Figs.(8) to (13).

It can be clearly observed in the order maps the first order, which represents the rotor unbalance, and another curve which evolves gradually out of the order line and that is related to the natural frequency of the system. This situation can also be observed in the waterfall plot but the order tracking procedure makes the distinction between the order and the natural frequency contributions easier.

The estimated amplitude values are the same for the COT and digital resampling but are different from those obtained through the TVDFT technique due to the influence of the natural frequency with the order. As it was presented before, the OCM shall improve the quality of the estimates for closely spaced and crossing orders as will be seen in the next example.

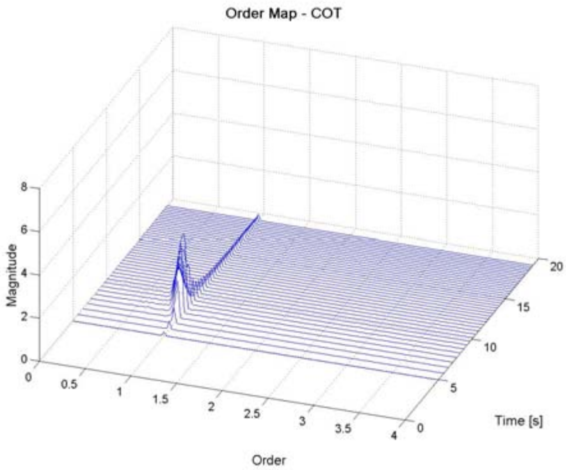


Figure 8. Order map. COT.

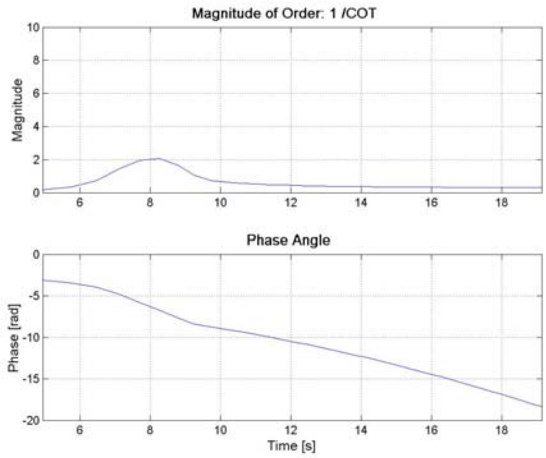


Figure 9. Magnitude and phase of the order. COT.

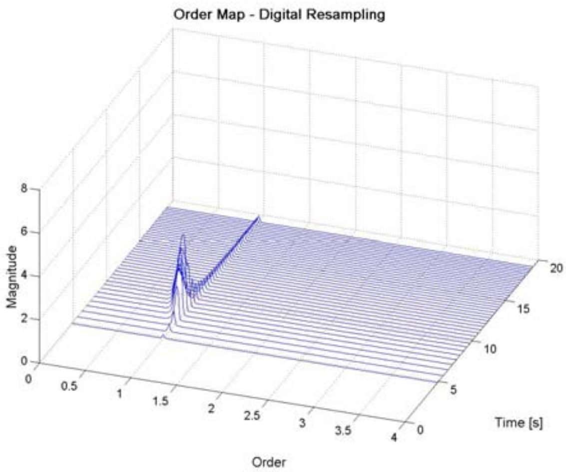


Figure 10. Order map. Digital resampling.

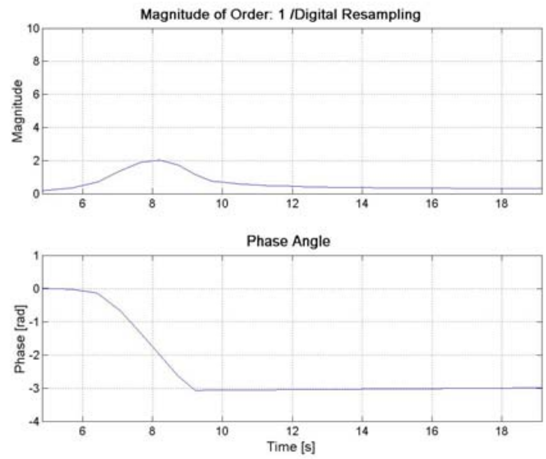


Figure 11. Magnitude and phase. Digital resampling.

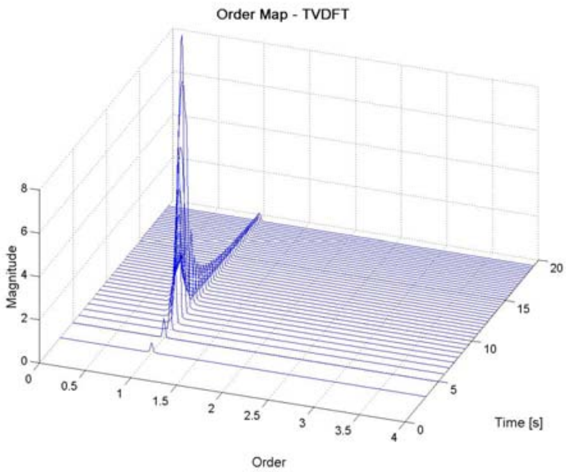


Figure 12. Order map. TVDFT.

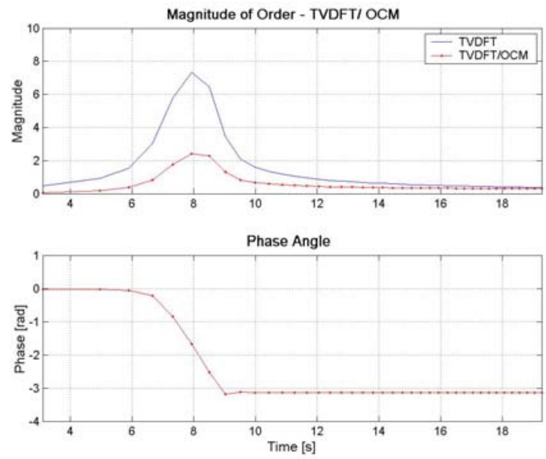


Figure 13. Magnitude and phase. TVDFT.

As can be seen in Figs.(8) to (13), the phase angles were also estimated. In this case, the COT did not produced good results. On the other hand, the digital resampling and the TVDFT technique did estimated the phase angle precisely. It is worth noting that technical papers rarely discuss the phase angle curves and their physical meaning.

Case 2) Simulation of crossing orders

In this case, two unbalance responses of different acceleration are added (Fig.(14)) for simulation of crossing orders, as occurs, for example, when two independent unbalanced rotors are influencing the same structure. The first acceleration is 10 rad/s^2 with initial velocity of 200 rad/s, and the second acceleration is 22 rad/s^2 with zero initial condition. The rotor is isotropic and the natural frequency is 50 Hz. The order map parameters are the same of the first simulation. The waterfall plot of the system is shown in Fig. (15).

The order map, amplitude and phase angle of the order in the *x* direction, estimated for each order tracking method, are presented in Figs.(16) to (19).

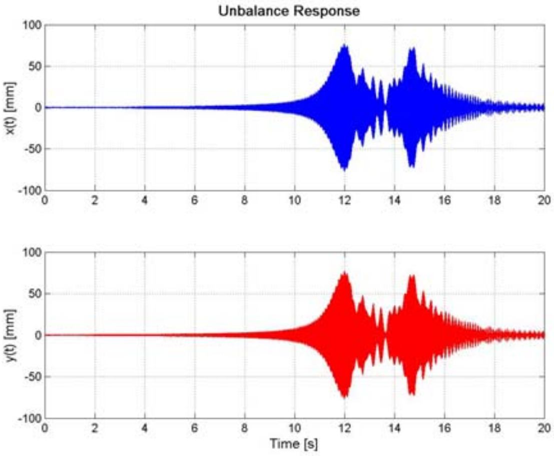


Figure 14. Unbalance response of isotropic rotor with two crossing orders.

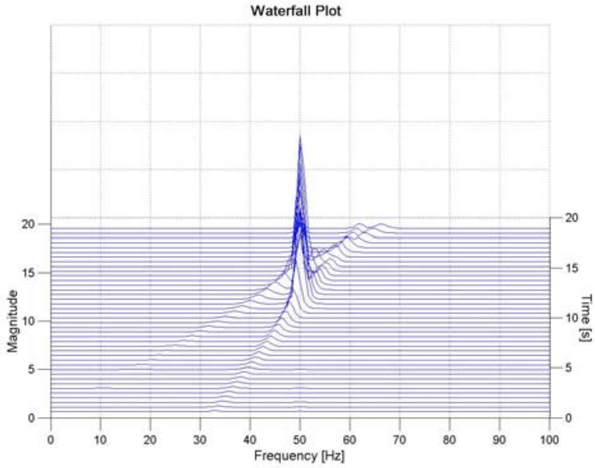


Figure 15. Waterfall Plot of isotropic rotor with two crossing orders.

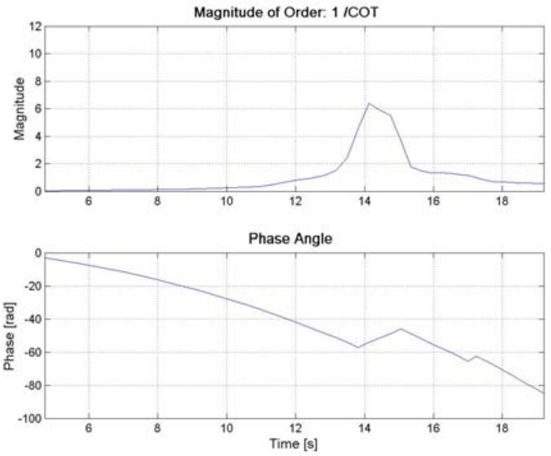


Figure 16. First COT order.

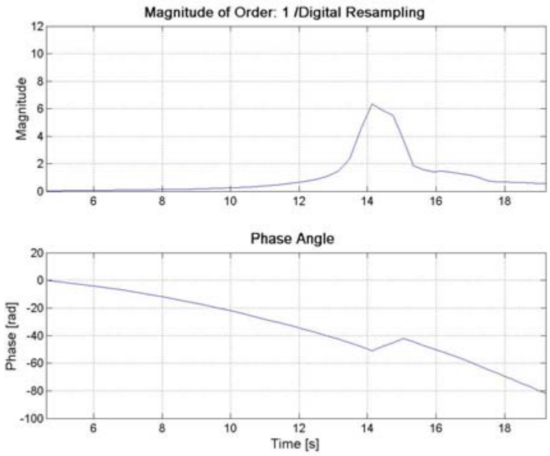


Figure 17. First Digital resampling order.

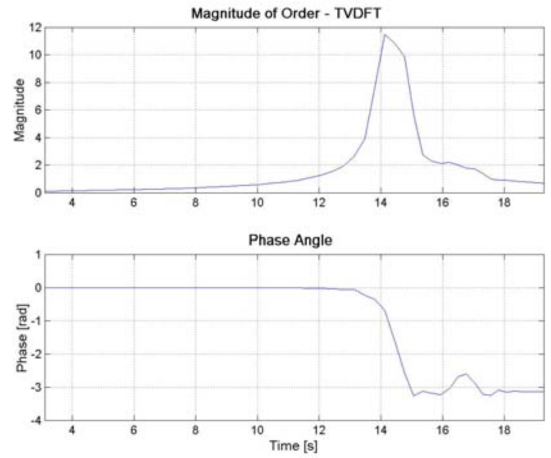


Figure 18. First TVDFT order.

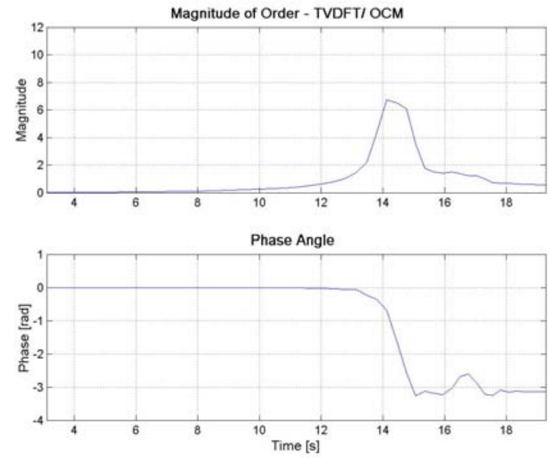


Figure 19. First TVDFT order with OCM.

As can be seen, different amplitude values were estimated by the TVDFT technique and by the COT and digital resampling methods. This difference is caused by the influence of the natural frequency on the first order and also by crossing order. The OCM procedure can correct the order magnitude estimated by the TVDFT, which is overestimated. In this simulation, the tracked phase is not well estimated by the resampling based methods. On the other hand, the TVDFT method still shows good results; the plot presents a constant zero phase before the critical velocity and a phase inversion for $-\pi$ radian after that. In the crossing of the orders, the phase presents a small peak, returning to $-\pi$ radian after the crossing.

Case 3) Simulation of multiples orders

A multiple order condition is obtained by a sum of unbalance responses (Fig.(20)) with different accelerations and same initial conditions. This kind of signal is found in power transmission machines. The simulated signal has three components, a fundamental order (1.0), and orders 1.2 and 2.0. The configuration parameters of the order maps are equal to the first case.

The order map, amplitude and phase angle of the three orders for each order tracking method are presented in Figs. (22) to (28). The waterfall plot showing the three components is presented in Fig. (21).

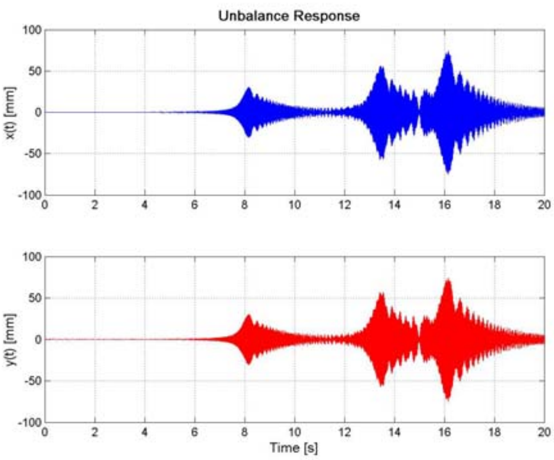


Figure 20. Unbalance response of isotropic rotor.

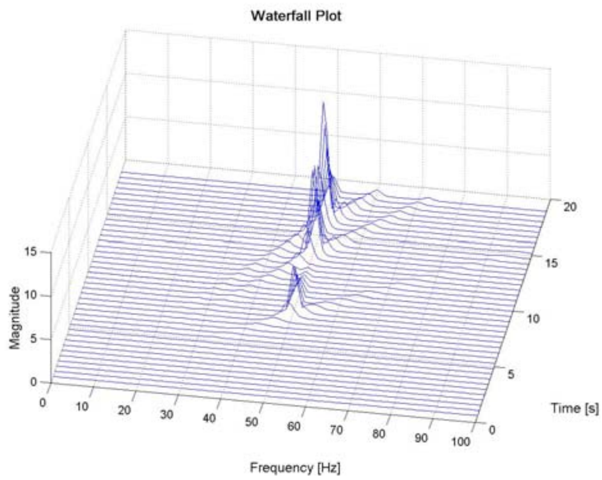


Figure 21. Waterfall Plot with multiple orders.

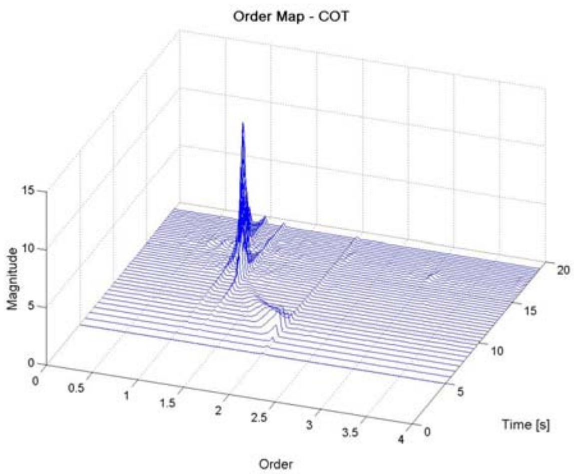


Figure 22. COT map of multiple orders.

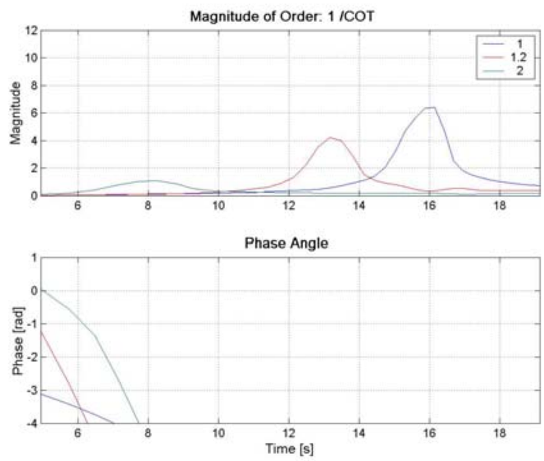


Figure23. COT orders.

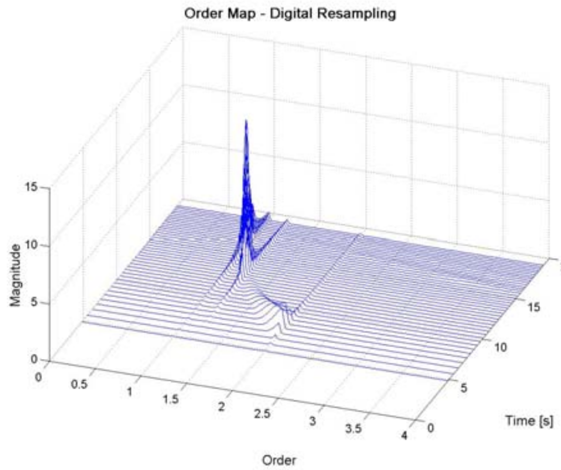


Figure 24. Digital resampling map of multiple orders.

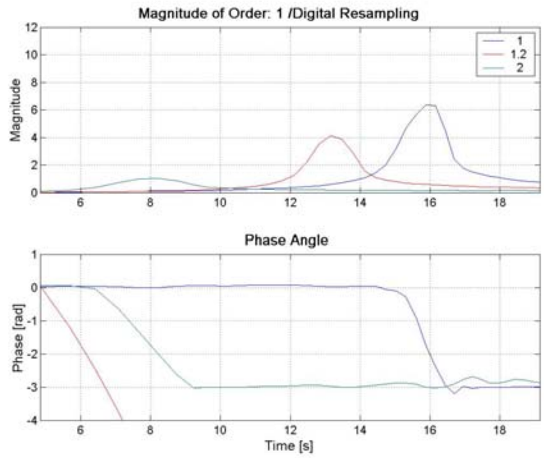


Figure 25. Digital resampling orders.

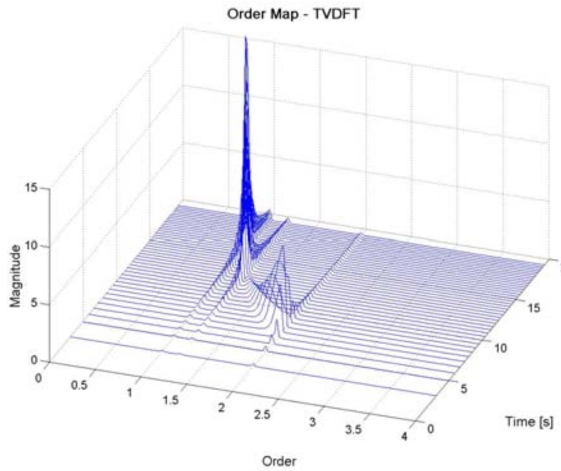


Figure 26. TVDFT map of multiple orders.

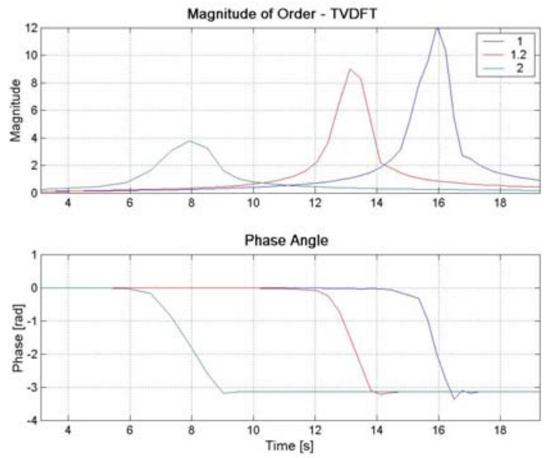


Figure 27. TVDFT orders.

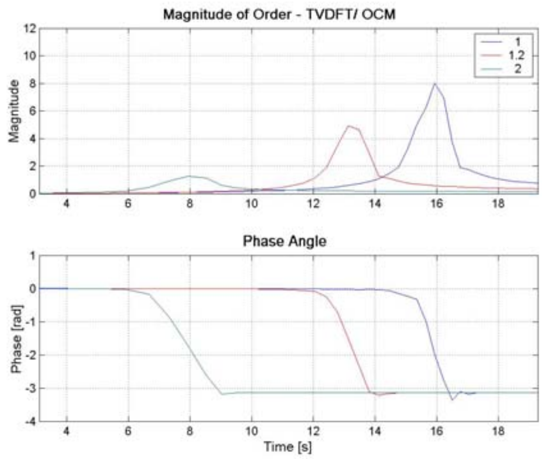


Figure 28. TVDFT orders with OCM processing.

The order maps estimates for the three components are similar for COT and Digital Resampling methods, but there is a visible difference when compared with TVDFT method. The application of the OCM treatments can correct the amplitude value, although the final result still do not exhibit the same magnitude than those found in the resampling techniques.

As usual, the phase angle estimated by the COT did not converge. This is also the case for the 1.2 order estimated by the digital resamplig. For all other cases (1^{st} and 2^{nd} orders for digital resampling and all orders for TVDFT) the orders phase angles were correctly estimated. It is also worth mentioning that the TVDFT orders phase estimation is not changed by the OCM procedure.

4. CONCLUSIONS

The order tracking techniques (COT – Computed Order Tracking, Digital Resampling and TVDFT – Time Variant Discrete Fourier Transform) were applied to a De Laval rotor operating in three different operational conditions. In all cases, the COT and the digital resampling produced the same estimated orders amplitude values and required almost the same computational time to extract the orders. The TVDFT technique presented different values, compared with the digital resampling methods, in the case of crossing and closely spaced orders. The OCM proved to be effective in correcting the orders amplitudes in such cases. The computational time required to extract the orders with the TVDFT technique is much lower than the digital resampling methods once in the former it is not needed the interpolation step. This characteristic makes the TVDFT suitable for studying systems with fast sweep rate. The digital resampling methods presented very poor phase angle estimation. On the contrary, the order phase angles were precisely estimated by the TVDFT technique. Despite the fact that it is necessary to post-process (using OCM) the TVDFT results in order to get good order estimates, this order tracking method proved to be the most efficient among the three techniques analyzed.

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