MEASUREMET AND PREDICTION OF MOBILITY TRANSFER FUNCTION OF A PLATE COVERED WITH A FOAM LAYER.

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Abstract. *In this work the mobility of a rectangular plate covered with a layer of porous material was analyzed, structure commonly found in the aeronautic and naval industry, as well as in offshore platforms. In this kind of stucture, the coating has the aim of reducing the strucutural sound radiation as well as the acoustical energy of the medium. Since in this work the foam is assumed attached to the plate, it was modeled based on the Biot's theory that considers the porous material as a flexible structure, which allows the propagation of three different wave types in the material and the coupling between the both foam phases. The analyzed system consists of a simply supported thin plate covered with a foam layer driven by a point harmonic force, assuming only flexural vibration. Commonly the foam mechanical properties are assumed constant in frequency, it will be seen that this assumption is not totally correct depending on the foam properties and the frequency range of interest.*

Keywords*. Porous materials, mobility, covered plate.*

1. Introduction

The measurement and prediction of mobility transfer function of a rectangular plate covered with a foam layer is presented. This kind of structures is usually found in industrial applications made of metal plates having isolation coverings as automotive, naval and aeronautics structures. Generally, the theoretical models of porous materials are divided in two categories: with rigid frame, that supports only longitudinal waves, and with flexible frame which account for the propagation of three different waves, two longitudinal and one rotational wave. Since the foam layer used in this study is attached to the plate, the contact between the foam and the vibrating plate induces vibrations not only in the fluid but also in the solid phase providing wave propagation in both phases simultaneously. To account for this effect a model that considers flexible frame is needed and thus one should use the Biot theory to simulate the porous material behavior (Biot 1956). In this model, besides the three wave types, the solid and fluid phase movements are coupled by the porous tortuosity and the viscosity of the saturating fluid.

Several pieces of works were publicized concerning to the properties and behavior of the porous material. Allard and colleagues (Allard 1993, Brouard 1997) presented various studies about the foam properties and the wave propagation in porous materials developing a method based on matrix representation to predict the behavior of multi layer structures, generally applied to determine the transmission loss of such structures. Lauriks and colleagues (Kelders 2001, Kelders 1994, Leclaire 1996) worked on the development of the analytical and experimental characterization of the porous materials, this is, on methods to determine the characteristic properties of these materials. In 1987, Bruer presented a study to determine the mechanical properties and loss factor of a foam layer using the standard procedure based on the transfer function of a sandwich beam (ASTM E 754-83). Some simplified models and the guidelines for a general one were presented. Numerical methods were also used to predict not only the porous material, but also the sandwich structures behavior (Dauchez 1999, Vigram 1997, Kang 1996).

In this piece of work, the structure will be modeled analytically using the Biot's theory to model the porous layer. To achieve reasonable functions to represent the structure behavior, some simplifying hypotheses to the problem will be presented in the first part of this study. After that, the equations governing the propagation in the foam as well as the displacement components will be presented. In the third part, the experimental apparatus is introduced followed by the obtained results and some discussions about them. In the last part the conclusions are presented.

1. Problems simplifications and material properties

To simplify the model the porous material is considered as being isotropic; the size of the unit elements is larger than that of the pores. Due to the complexity of the material structure it is difficult to model the material in the pore level. On the other hand, at a macroscopic level one can characterize the micro geometry in a statistical context and then assume the material as homogeneous. With this, the displacement vector is defined as the displacements of the material considered as uniform, averaged over the element; the element is assumed to be small relative to the wavelength of the elastic waves, which allows the use of the properties of continuum mechanics; one assumes small displacements the stress-strain relations being linear and harmonic motion with sign convention $e^{j\omega t}$ is assumed.

Assuming the base plate as simply supported and a foam attached to it as depicted in figure 1.

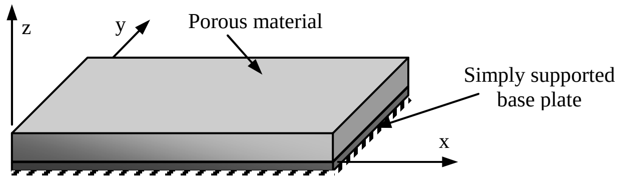

Figure 1. Simply supported covered plate.

At the lateral sides of the coating the displacement components must satisfy,

$$\begin{cases} u_z = 0 \quad \text{at } x = 0,\ x = L_x,\ y = 0,\ y = L_y, \\ u_x = 0 \quad \text{at } y = 0,\ y = L_y, \\ u_y = 0 \quad \text{at } x = 0,\ x = L_x, \end{cases} \tag{1}$$

It means that the normal component is null at the borders. The x-component is null at the x-direction and free at the y-direction and the y-component is null at the y-direction and free at the x-direction.

All the parameters for the porous material, these are, porosity ϕ, resistivity σ, tortuosity α_∞, Young's modulus E, shear modulus G, characteristic viscous Λ and thermal Λ' lengths were determined experimentally. The foam mechanical properties, E and G, are assumed constant in frequency. The characteristics of the porous materials parameters and the methods to obtain them are given in references (Leclaire 1996, Litwinczik 2003, Vigram 1997). The air characteristics and the data for the aluminum plates are the commonly tabulated ones. The plate is assumed thin, homogeneous and isotropic following the classical Euler-Bernoulli theory (Junger 1993). Table 1 and Tab. 2 show the foam and the aluminum plate properties, respectively.

Table 1 – Material parameters of the porous layer.

ρ [kg/m³]	ϕ	α_∞	σ [Ns/m^4]	Λ [m]	Λ' [m]	h [m]	E [N/m²]	G [N/m²]
30	0.97	1.2	2530.0	0.00015	0.0005	0.0254	$1.15\text{x}10^5$+j$1.0\text{x}10^4$	$0.6\text{x}10^5$+j$0.6\text{x}10^4$

Table 2 – Dimensions and material properties of the aluminum plate.

L_x [m]	L_y [m]	h [m]	E [N/m²]	ρ [kg/m³]	ν	η
0.6	0.5	0.003	$69\text{x}10^9$	2690	0.33	0.03

2. Wave propagation into the foam

With the considerations made above, the elastic displacement in the solid phase, $\mathbf{u}(u_x, u_y, u_z)$, can be written as a sum of the volumetric and rotational strains in the material,

$$\mathbf{u} = \nabla\varphi_{1,2} + \nabla\times\boldsymbol{\psi}. \tag{2}$$

The fluid phase displacement, $\mathbf{U}(U_x, U_y, U_z)$, is analogous and given by $\mathbf{U} = \mu_i\mathbf{u}$, i = 1, 2, t, where the index 1 and 2 relate the longitudinal waves and t the transversal one. The first term of the right hand side of the Eq. (2) corresponds to the volumetric strain (longitudinal waves), which can be expressed as function of a scalar potential φ. The second term, $\nabla\times\boldsymbol{\psi}$, corresponds to the rotational strain (transverse wave) and can be expressed by a vector potential $\boldsymbol{\psi} = (\psi_x, \psi_y, \psi_z)$. The scalar potentials are governed by the wave equations

$$\nabla^2\varphi_{1,2} + k_{1,2}^2\varphi = 0 \tag{3}$$

and the vector one by

$$\nabla^2\boldsymbol{\psi} + k_t^2\boldsymbol{\psi} = 0 \tag{4}$$

Since the rotational components of the displacement does not contribute to the volumetric strain one can write,

$$\nabla\cdot\boldsymbol{\Psi} = \frac{\partial\psi_x}{\partial x} + \frac{\partial\psi_y}{\partial y} + \frac{\partial\psi_z}{\partial z} = 0 \tag{5}$$

Through Eqs. (3) and (4) the scalar and vector potentials can be written as

$$\varphi_1(z) = A_1 \exp(-j\, k_{1z}\, z) + A_2 \exp(j\, k_{1z}\, z) \tag{6}$$

$$\varphi_2(z) = A_3 \exp(-j\, k_{2z}\, z) + A_4 \exp(j\, k_{2z}\, z) \tag{7}$$

$$\psi_x(z) = A_5 \exp(-j\, k_{tz}\, z) + A_6 \exp(j\, k_{tz}\, z) \tag{8}$$

$$\psi_y(z) = A_7 \exp(-j\, k_{tz}\, z) + A_8 \exp(j\, k_{tz}\, z) \tag{9}$$

where the wavenumbers are related by $k_b^2 = k_x^2 + k_y^2$, $k_i^2 = k_b^2 + k_{iz}^2$ and the dependence at the in plane direction x and y will be given according to the boundary conditions (1). For a simply supported plate *in vacuum*, the displacement is given by the superposition of its normal modes

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin(k_x x)\sin(k_y y), \tag{10}$$

where $k_x = m\pi/L_x$ and $k_y = n\pi/L_y$. Similarly, the potentials $\varphi_{1,2}$ and ψ can be written as

$$\varphi_{1,2}(x,y,z) = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \varphi_{1,2\ pq}(z)\sin(k_x x)\sin(k_y y), \tag{11}$$

$$\psi_x(x,y,z) = \sum_{p=1}^{\infty}\sum_{q=0}^{\infty} \psi_{x,pq}(z)\sin(k_x x)\cos(k_y y), \tag{12}$$

$$\psi_y(x,y,z) = \sum_{p=0}^{\infty}\sum_{q=1}^{\infty} \psi_{y,pq}(z)\cos(k_x x)\sin(k_y y). \tag{13}$$

The value of ψ_z is expressed in terms of ψ_x and ψ_y according Eq. (5) and $k_p = p\pi/L_x$ and $k_q = q\pi/L_y$. The displacement components in the foam are obtained applying Eqs. (11) (12) and (13) into Eq. (2). The amplitudes of the fluid and solid phases displacement, can be determined through the stress–train relations (Allard 1993)

$$\sigma^s_z = (P - 2N)\,\theta^s + Q\theta^f + 2N\partial u_z/\partial z \tag{14}$$

$$\sigma^f_z = Q\theta^s + R\theta^f \tag{15}$$

$$\tau_{xz} = N(\partial u_x/\partial z + \partial u_z/\partial x) \tag{16}$$

$$\tau_{yz} = N(\partial u_y/\partial z + \partial u_z/\partial y) \tag{17}$$

and the boundary conditions between the plate and porous layer are presented in Fig. 2 (Litwinczik 2003)

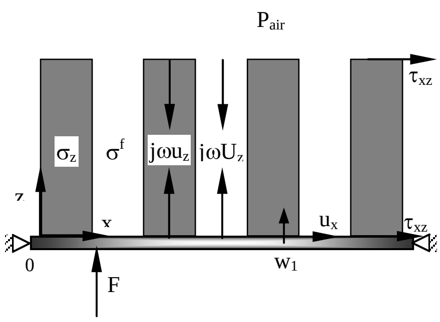

Figure 2. Exaggerated view of the problem's cross-section.

$$z = 0 \rightarrow \begin{cases} \text{i) } \left(D_{t1}k_b^{\,4} - \omega^2 m_{s1}\right) w_1 = -F + \sigma^s{}_z + \sigma^f - \dfrac{h_{p1}}{2}\left(\dfrac{\partial\tau_{xz}}{\partial x} + \dfrac{\partial\tau_{yz}}{\partial y}\right); \\ \text{ii) } w_1 = u_z \qquad \text{iii) } w_1 = U_z; \\ \text{iv) } \dfrac{h_{p1}}{2}\dfrac{\partial w_1}{\partial x} = u_x; \quad \text{v) } \dfrac{h_{p1}}{2}\dfrac{\partial w_1}{\partial y} = u_y. \end{cases} \qquad z = L \rightarrow \begin{cases} \text{vi) } v_z = j\omega\left[(1-\phi)u_z + \phi U_z\right]; \\ \text{vii) } \sigma^s{}_z = (1-\phi)P_{air}; \\ \text{viii) } \sigma^f = \phi P_{air}; \\ \text{ix) } \tau_{xz} = 0; \quad \text{x) } \tau_{yz} = 0. \end{cases} \tag{18}$$

where P, N, Q and R are elastic constants (Allard 1993), and θ^s and θ^f are the solid and fluid volumetric strains, respectively.

Once amplitudes $A_1 - A_8$ were determined it is possible to predict the acoustic field anywhere in the porous material.

3. Experimental validation

To simulate a simply supported boundary condition, the aluminum plate was mounted on a vertical wall composed of two layers of gypsum board, 1.5 cm thick each, separated by a metal framework, working as a rigid baffle, between two rooms. The plate was mounted in direct contact with the metal strips that act as simple supports and was mechanically driven by a suspended shaker mounted in the source room side, as depicted in Fig. 3. A very thin glue line was applied at the interface to prevent any transversal displacement at the borders.

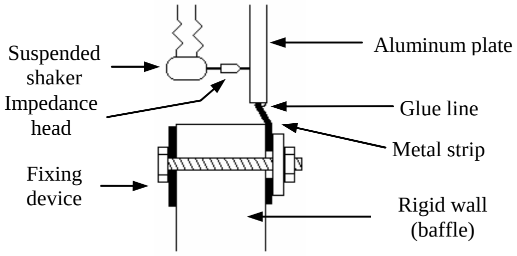

Figure 3. Set-up scheme for the simply supported boundary condition.

The plate was driven using a white noise signal controlled via software (Cool Edit Pro) and the responses were saved and processed in a two-channel FFT analyzer (B&K 2144). The plate responses were measured using a LASER Doppler transducer. A scheme of the arrangement of the equipments is showed in Fig. 4.

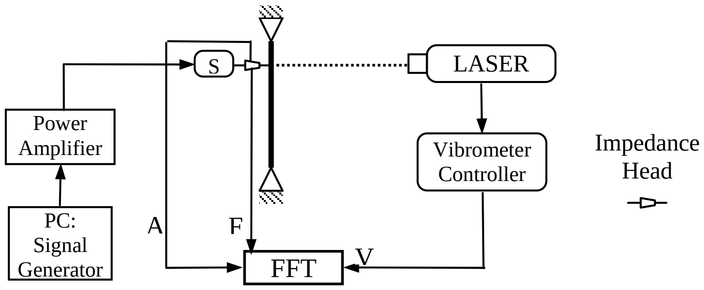

Figure 4. Arrangement of equipment for measuring and recording plate mobility.

Figure 5 shows a comparison between the simulated and measured point mobility for the plate presented in Tab. (2).

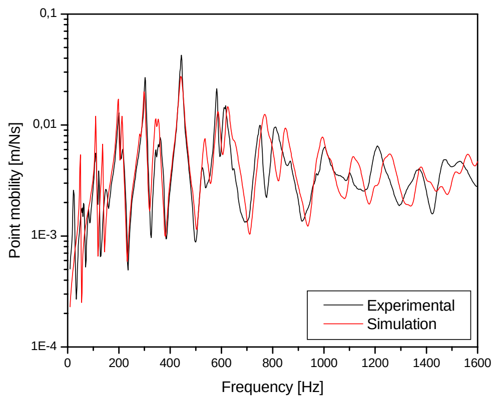

Figure 5. Comparison between measured and simulated point mobility.

For the point mobility the coordinates of the excitation and measuring points were (0.535m, 0.105m), assuming the left upper corner as the origin of the coordinates. Considering the difficulty in reproducing experimentally the simply supported boundary condition, the results presented above show a reasonable agreement between numerical simulations and experimental data.

3. Results and discussions

Using the same set-up presented above, a foam layer was attached to the plate and the measurements repeated. To use the LASER Doppler transducer one need to attach small pieces of a reflective tape to the analyzed structure to improve the LASER sensitivity. For the case of not sealed surface, as a foam surface, these tapes are too small to be influenced by the fluid foam phase. Assuming this, the responses obtained by such method only represents the response of the foam solid phase and thus the simulation should consider only the solid displacement component of the foam in the response. A good agreement can be noted until the region near the first resonance frequency of the foam solid phase (Allard 1993, Dauchez 1999):

$$f_1 = 1/(4L)\,(K_s/\rho_1)^{1/2} = 653 \text{ Hz}. \quad (19)$$

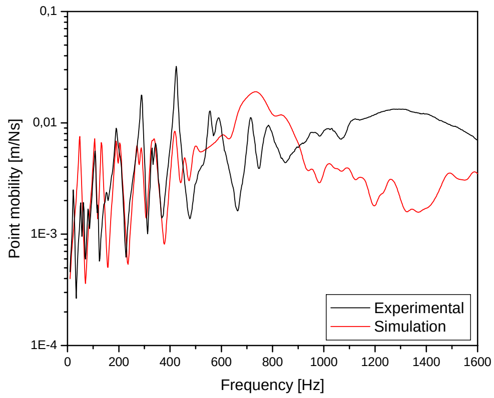

Figure 6. Point mobility of a simply supported covered plate.

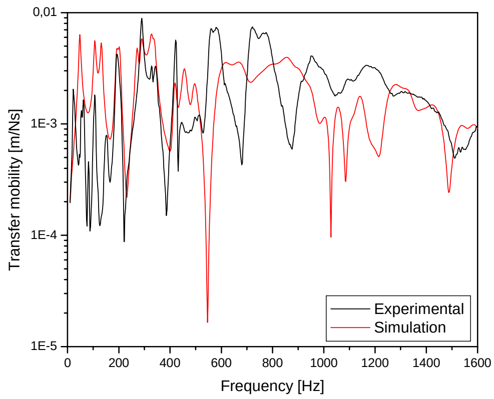

Figure 7. Transfer mobility of a simply supported covered plate.

Based on Fig. 6 and Fig. 7 can be said that the foam resonance in the experimental curve occurs at a frequency higher than that in the simulated one, around two times f_1, characterized by the reduction of the peaks. In Eq. (19) there are four possibilities to double the frequency. The thickness and the density are fixed parameters and so they cannot change. The Poisson ratio does not vary too much for common porous material, being commonly set as 0.3. The Young's modulus is therefore the only parameter possible to be changed. To double the frequency the Young's modulus must be multiplied by 4 making $E_1 = 4.6\text{x}10^5 + j4.0\ \text{x}10^4$ [N/m^2], that will be called E_4. This change in the Young's modulus results in a resonance frequency f_1 of 1306.5 Hz.

Now plotting the mobility with E_4 (Fig. 7) a better agreement between both curves up to higher frequencies can be seen. It can also be noted that, at the mid frequency range, this increase in the stiffness causes a small shift in the peaks to higher frequencies.

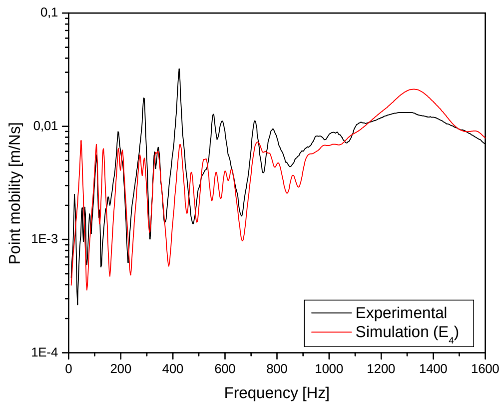

Figure 8. Point mobility for simply supported covered plate with corrected Young's modulus.

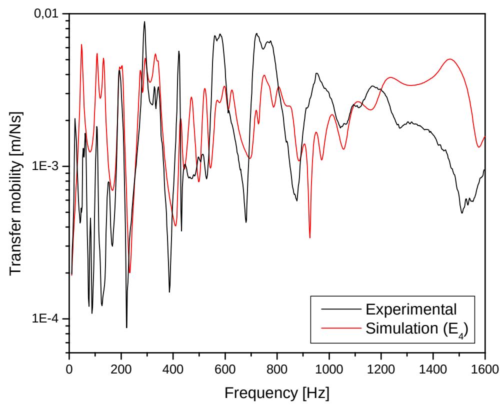

Figure 9. Transfer mobility for simply supported covered plate with corrected Young's modulus.

The results showed in Fig. 6 – Fig. 9 indicate that the foam stiffness is not constant in frequency, as assumed in this work.

4. Conclusions

This study investigated the behavior of a simply supported covered plate modeling the core based on the Biot theory. An analytical model of such structure, assuming the core with finite dimensions, was successfully tested comparing the simulated and experimental point mobility of the structure. It could be seen that the assumed frequency independence of the foam mechanical properties are not valid in the whole spectrum. For the foam tested, this behavior is valid only at low frequency range. From mid-to-high frequency range these properties are frequency dependent. The assumption that the LASER Doppler transducer is not sensitive to the fluid phase shown to be correct since the comparisons with the analytical model were performed considering only the solid phase of the porous material.

5. Acknowledgement

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6. References

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