

LAMINAR FLOW AROUND A SINUSOIDAL INTERFACE BETWEEN A POROUS MEDIUM AND A CLEAR FLUID

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Abstract. *A number of natural and engineering systems can be characterized by some sort of porous structure through which a working fluid permeates. Boundary layers over tropical forests and spreading of chemical contaminants through underground water reservoirs are examples of important environmental flows. The literature proposes a jump condition in which stresses at both sides of the interface are not of the same value. The objective of this paper is to present a numerical investigation for solving such hybrid medium, considering here a channel partially filled with a porous layer whose interface has a wavy shape, around which fluid flows in laminar regime. One unique set of transport equations is applied to both regions. Results are presented for the mean velocity across both the porous structure and the clear region. The influence of medium properties, such as porosity and permeability, is discussed.*

Keywords: *Porous Media, Numerical Solution, Laminar Flow, Wavy Interface*

1. Introduction

Frequently, homogenization of local properties in porous media is obtained by means of the Volume-Average Theorem (VAT) (Whitaker (1969), Gray and Lee (1977)). When the domain presents a macroscopic interfacial area, the literature proposes the existence of a stress jump interface condition between the clear flow region and the porous medium (Ochoa-Tapia and Whitaker (1995a-b)). Analytical solutions involving such models have been published [Kuznetsov (1996, 1997, 1998a-b, 1999)].

Additionally, purely numerical simulations for two-dimensional hybrid medium (porous region-clear flow) in an isothermal channel have been considered in de Lemos and Pedras (2000) based on the turbulence model proposed in Pedras and de Lemos (2001a-b). That work has been developed under the double-decomposition concept (Pedras and de Lemos (1999, 2000), Rocamora and de Lemos (2000a)). Non-isothermal flow in channels past a porous obstacle [Rocamora and de Lemos (2000b] and through a porous insert have also been presented (Rocamora and de Lemos (2000c)). In all previous work of de Lemos and Pedras (2000) and Rocamora and de Lemos (2000b-d), the interface boundary condition considered a continuous function for the stress field across the interface. Recently, de Lemos and Silva (2002a-b) and Silva and de Lemos (2003a-b) presented numerical solutions for laminar and turbulent flow in a channel partially filled with a flat layer of porous material. There, the authors considered the stress jump condition at the interface. Such works were based on a numerical methodology proposed for hybrid media (de Lemos and Pedras (2000), Rocamora and de Lemos (2000b-c)).

All of the above considered a flat interface dividing the porous substrate and the clear region. However, some natural and engineering flows are better characterized by a porous matrix having an irregular surface rather than a plane geometry. Therefore, the objective of this paper is to extend the previous work on laminar flow in multilayered channels considering now a wavy interface between the porous medium and the clear flow passage.

2. Macroscopic Model

2.1 Geometry

The flow under consideration is schematically shown in Figure 1 where a channel is partially filled with a layer of a porous material. Constant property fluid flows longitudinally from left to right permeating through both the clear region and the porous structure Here, the interface has a sinusoidal shape being characterized by an amplitude a and a wave number, n=L/λ, where λ is the wavelength and L is axial length of the channel. Also, H is the channel height.

2.2 Governing equations

A macroscopic form of the governing equations is obtained by taking the volumetric average of the entire equation set. In this development, the porous medium is considered to be rigid, undeformable and saturated by an incompressible fluid.

The microscopic continuity equation for the fluid phase is given by.

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

Applying the volume-average operator to equation (1), one has (see Pedras and de Lemos (2001a)).

$$\nabla \cdot \mathbf{u}_D = 0 \qquad (2)$$

where the local velocity vector $\mathbf{u}$ is of null value at the local interfacial area A_i^m (not to confuse with the macroscopic interface area A_i) and the Dupuit-Forchheimer relationship, $\mathbf{u}_D = \phi\langle\mathbf{u}\rangle^i$, has been used were the operator "< >" identifies the intrinsic (liquid volume based) average of $\mathbf{u}$ (Gray and Lee (1977)). Equation (2) represents the macroscopic continuity equation for an incompressible fluid in a rigid porous medium.

The microscopic Naviek-Stokes equation for an incompressible fluid with constant properties can be written as,

$$\rho\left[\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u})\right] = -\nabla p + \mu\nabla^2\mathbf{u} \qquad (3)$$

Hsu and Cheng (1990) have applied the volume averaging procedure to equation (3) obtaining,

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\mathbf{u}\rangle^i\right) + \nabla \cdot \left(\phi\langle\mathbf{u}\mathbf{u}\rangle^i\right)\right] = -\nabla\left(\phi\langle p\rangle^i\right) + \mu\nabla^2\left(\phi\langle\mathbf{u}\rangle^i\right) + \mathbf{R} \qquad (4)$$

where

$$\mathbf{R} = \frac{\mu}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\mathbf{u})dS - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\,p\,dS \qquad (5)$$

The term $\mathbf{R}$ represents the total drag per unit volume acting on the fluid by the action of the porous structure. A common model for it is known as the Darcy-Forchheimer extended model and is given by

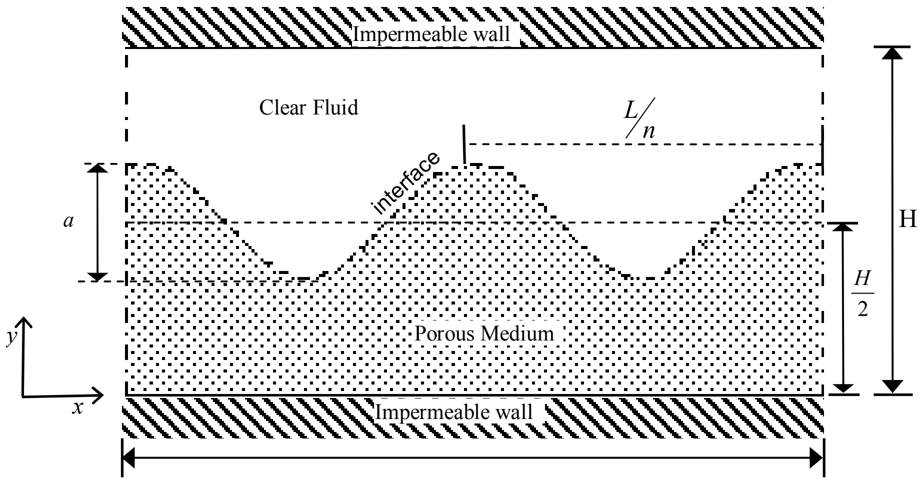


Figure 1: Channel with porous layer of wavy interface.

$$\overline{\mathbf{R}} = -\left[\frac{\mu\phi}{K}\mathbf{u}_D + \frac{c_F\phi\rho|\mathbf{u}_D|\,\mathbf{u}_D}{\sqrt{K}}\right] \qquad (6)$$

where the constant c_F is known in the literature as the non-linear Forchheimer coefficient.

Then, making use again of the expression $\mathbf{u}_D = \phi\langle\mathbf{u}\rangle^i$, equation (6) can be rewritten as,

$$\rho\left[\frac{\partial \mathbf{u}_D}{\partial t} + \nabla \cdot \left(\frac{\mathbf{u}_D\mathbf{u}_D}{\phi}\right)\right] = -\nabla\left(\phi\langle p\rangle^i\right) + \mu\nabla^2\mathbf{u}_D - \left[\frac{\mu\phi}{K}\mathbf{u}_D + \frac{c_F\phi\rho|\mathbf{u}_D|\,\mathbf{u}_D}{\sqrt{K}}\right] \qquad (7)$$

2.2 Interface Condition between the Clear Fluid and the Porous Medium

The equation proposed in Ochoa-Tapia and Whitaker (1995a-b) for describing the stress jump at the interface between the clear flow region and the porous structures is given by,

$$\mu_{eff}\left.\frac{\partial u_{D_\xi}}{\partial \eta}\right|_{\text{Porous Medium}} - \mu\left.\frac{\partial u_{D_\xi}}{\partial \eta}\right|_{\text{Clear Fluid}} = \beta\frac{\mu}{\sqrt{K}}u_{D_\xi}\Big|_{\text{Interface}} \tag{8}$$

where u_{D_ξ} is the Darcy velocity component parallel to the interface aligned with the direction ξ and normal to the direction η, $\mu_{eff}=\mu/\phi$ is the effective viscosity for the porous region, μ is the fluid dynamic viscosity, K is the permeability and β an adjustable coefficient which accounts for the stress jump at the interface. Equation (8) will be later adapted to the geometry and coordinate system here employed.

For hybrid domains, in addition to equation (8), continuity of velocity and pressure fields prevailing at the interface are given by,

$$\mathbf{u}_D\big|_{\text{Porous Medium}} = \mathbf{u}_D\big|_{\text{Clear Fluid}} \tag{9}$$

$$\langle p\rangle^i\Big|_{\text{Porous Medium}} = \langle p\rangle^i\Big|_{\text{Clear Fluid}} \tag{10}$$

Conditions (8), (9) and (10) were proposed in Ochoa-Tapia and Whitaker (1995a-b) using the concept of stress jump at the interface.

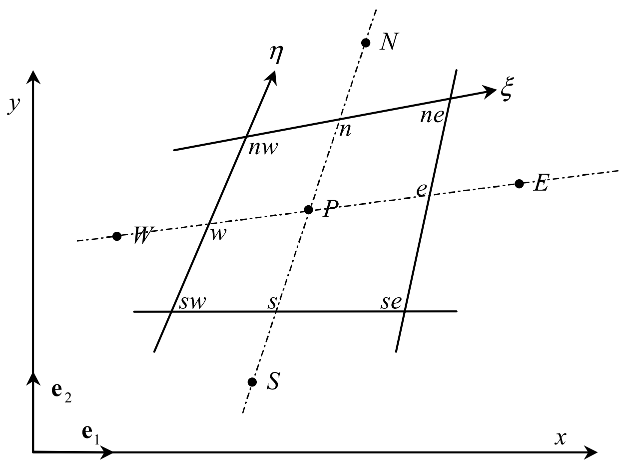


Figure 2: Notation for control volume discretization.

3. Numerical Model

The numerical method used for discretizing the system of equations is the Control volume method of Patankar (1980). In the implementation herein, a system of generalized coordinates was used although all simulation to be shown employed only Cartesian coordinates. Nevertheless, the use of a general system η-ξ for discretizing the equations was found to be adequate for future simulations.

Since the entire derivation herein if set up for solving two-dimensional flows, both cases employ the spatially periodic boundary condition along the x coordinate. This if done in order to simulate fully developed flow for which analytical solutions are available for comparison. The spatially periodic condition is implemented by running the solution repetitively, until outlet profiles in x=L match those at the inlet (x=0).

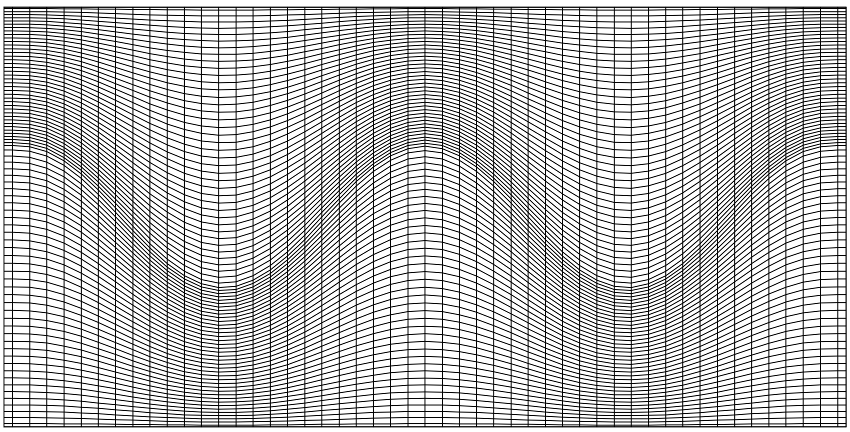


Figure 3: Computational grid.

Figure 2 shows a general control volume in a two-dimensional configuration. The faces of the volume are formed by lines of constant coordinates η-ξ.

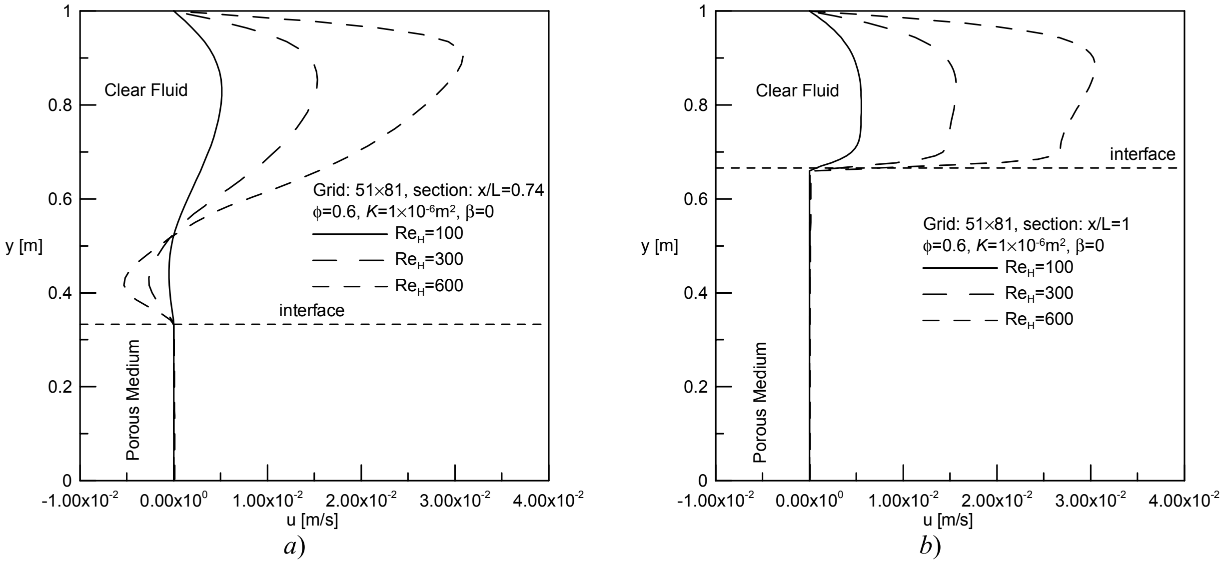


Figure 4: Effect of Re_H on velocity profiles for n=2, a=1/3: a) x/L=0.74 (valley), b) x/L=1 (peak).

For steady-state, a general form of the discrete equations for a general variable $\boldsymbol{\varphi}$ becomes,

$$I_e + I_w + I_n + I_s = S_{\boldsymbol{\varphi}} \tag{11}$$

where I_e, I_w, I_n e I_s are the fluxes of $\boldsymbol{\varphi}$ at faces *east*, *west*, *north* and *south* of the control volume of Figure 2, respectively, and S_φ is a source term. Details on the numerical methodology employed in obtaining (11) and for discretization for (8) can be found respectively in (Pedras and de Lemos (2001b-c-d) and Silva and de Lemos (2003)). Here, all computations were carried out until the normalized residue of the algebraic equations was brought down to 10^{-7}, where the residue was defined as the difference between the right and left sides of the discretized equations.

Figure 3 shows the used computational, having 51 grid nodes in the longitudinal x-direction and 81 nodes along the cross-stream y coordinate. As mentioned, the spatially periodic boundary condition was applied along the main flow direction in order to simulate fully developed flow.

4. Results and discussion

Figure 4 shows the effect of the Reynolds number, Re_H, on profiles of the axial velocity for ϕ=0.6, K=1×10^{-6}m^2, β=0, n=2 and a=1/3 for the axial location, namely x/L=0.74 (valley) and x/L=1 (peak). Is can be noted the increased mass flow rate at the unobstructed region with little change of the mass flow rate inside the porous substrate. It is also interesting to note that recirculation region at the valley for the given material properties.

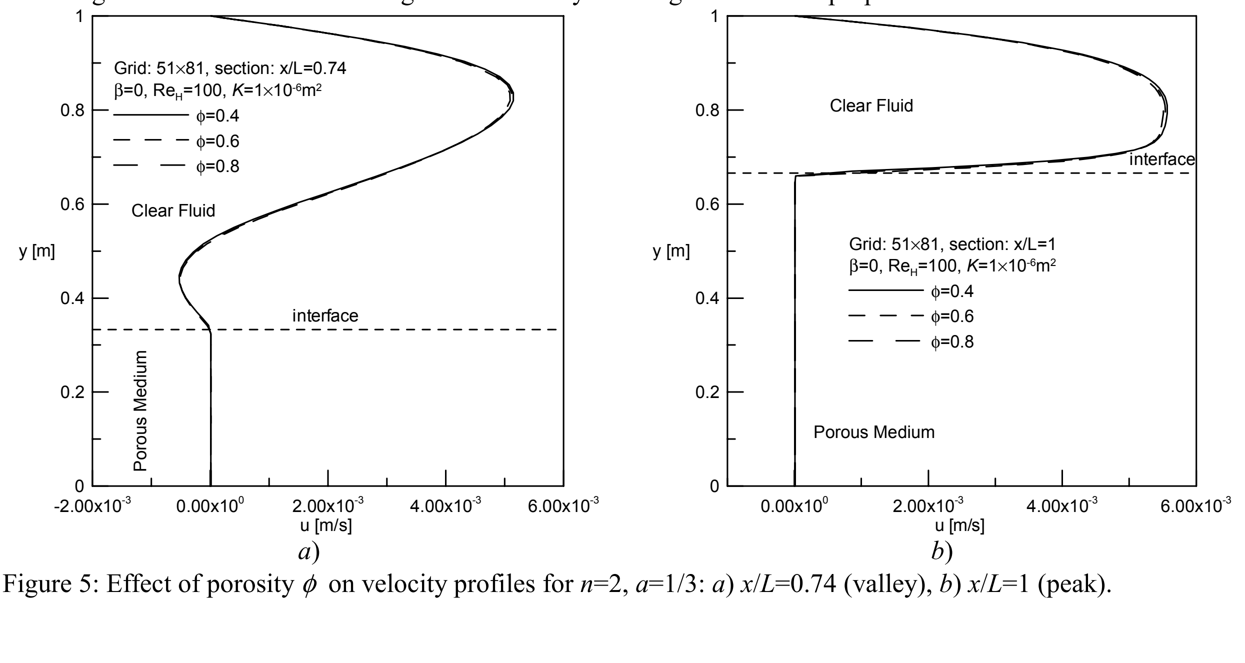
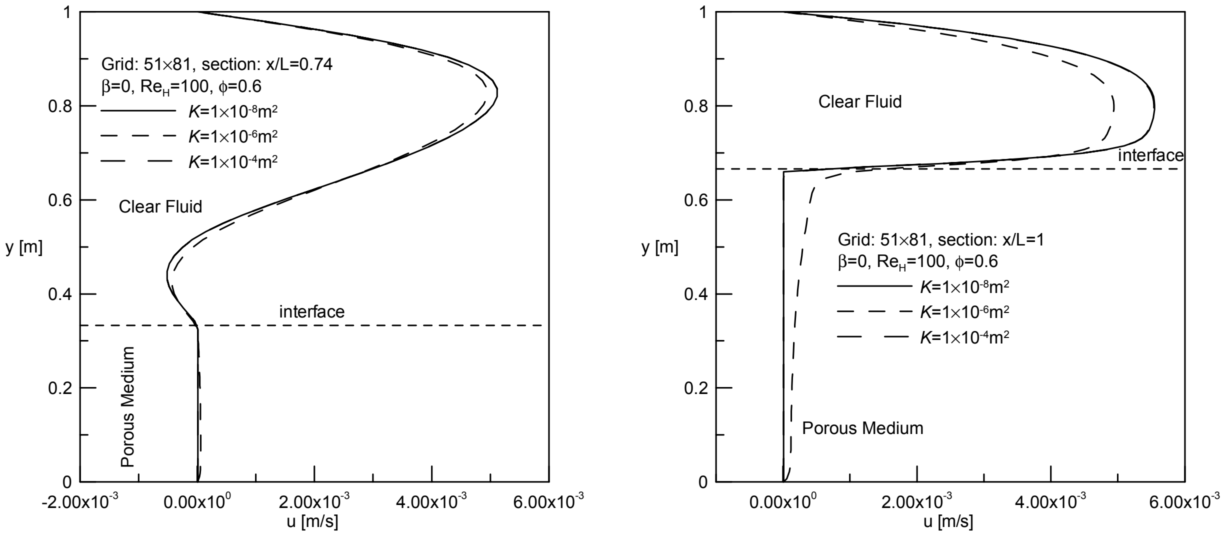


Figure 5: Effect of porosity ϕ on velocity profiles for n=2, a=1/3: a) x/L=0.74 (valley), b) x/L=1 (peak).

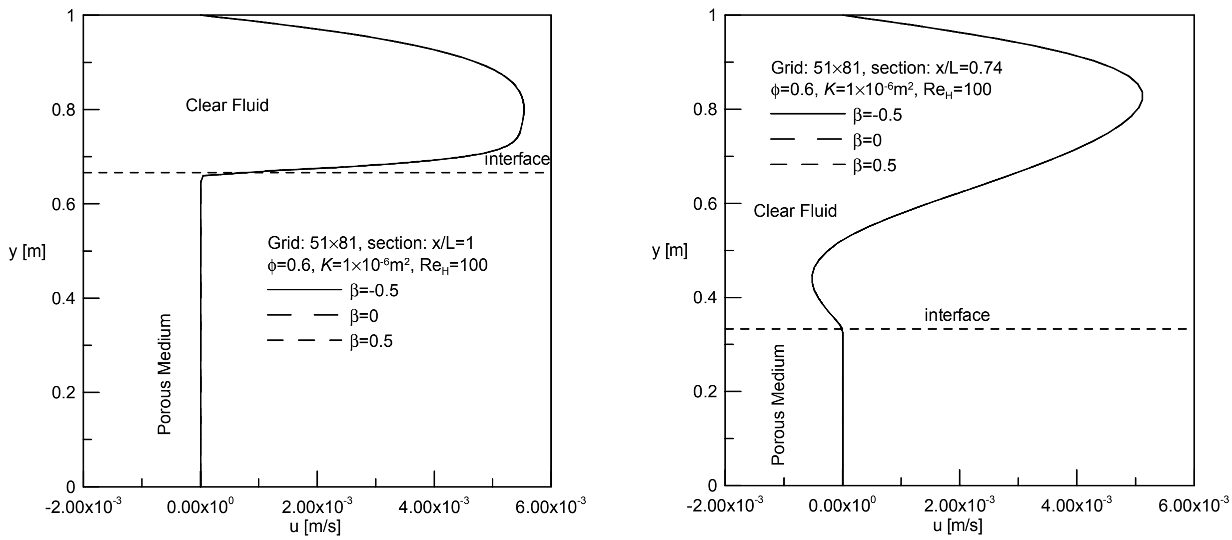
The effect of porosity ϕ of the porous bed on the on the flow is presented in Figure 5, K=1×10^{-6}m^2, Re_H=100, wave number n=2 and amplitude a=1/3 (see Figure 1 for dimensions), for positions at x/L=0.74 (valley) and at x/L=1 (peak). Here, it is interesting to observe that nearly no change in the mass flow rate across the channel is noted. Figure 6 shows a little more sensitivity of velocity values when the permeability K is varied, with higher mass flow rates within the porous substrate as the permeability increases.



a) *b*)

Figure 6: Effect of permeability K on velocity profiles for n=2, a=1/3: *a*) x/L=0.74 (valley), *b*) x/L=1 (peak).

Velocity profiles at the same axial locations account for he influence of the jump coefficient β in Figure 7. As observed in the cases of flat surfaces (Silva and de Lemos (2003)), there little influence of β on the flow pattern.



a) *b*)

Figure 7: Effect of parameter β on velocity profiles for n=2, a=1/3: *a*) x/L=0.74 (valley), *b*) x/L=1 (peak).

Pressure losses along the channel are calculated according to,

$$\overline{\Delta p} = \frac{1}{\phi A_t} \int_{A_t} \phi (p_{\text{inlet}} - p_{\text{outlet}}) dA \tag{12}$$

where A_t is the channel transversal area, p_{inlet} is the local inlet pressure and p_{outlet} is the pressure at the exit for each cell height. The Table shows that stronger pressure gradients occurs when values of β, ϕ are higher, noting that for all these cases the overall flow rate in the channel was forced to be the same (same Re_H). For higher K, however, the pressure drop is reduced. Also noted is that, as expected, for a higher Reynolds number the pressure loss become of a higher value.

Table 1: Pressure drop for periodic flow

Re_H=100, K=1×10^{-6}m^2, ϕ=0.6, n=2, a=1/3	$\overline{\Delta p}\,[N/m^2]\times 10^{-5}$
β=-0.5	2.869
β=0	2.875
β=0.5	2.880
β=0, Re_H=100, K=1×10^{-6}m^2, n=2, a=1/3	
ϕ=0.4	2.691
ϕ=0.6	2.875
ϕ=0.8	2.995
β=0, Re_H=100, ϕ=0.6, n=2, a=1/3	
K=1×10^{-8}m^2	2.880
K=1×10^{-6}m^2	2.875
K=1×10^{-4}m^2	2.306
β=0, K=1×10^{-6}m^2, ϕ=0.6, n=2, a=1/3	
Re_H=100	2.875
Re_H=200	17.187
Re_H=300	3115.542

5. Concluding Remarks

Numerical solutions for laminar flow in a composite channel were obtained for different values of Re_H and ϕ-K properties. The interface between the porous medium and the clear flow was assumed to be of a sinusoidal form. Governing equations were discretized and solved for both domains making use of one unique numerical methodology. Results herein my contribute to the analysis of important environmental and engineering flows where an irregular interface surrounding a porous body may be identified.

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