

MULTIGRID SOLUTION OF LAMINAR NATURAL CONVECTION FLOW IN AN ENCLOSURE FILLED WITH POROUS MATERIAL

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Abstract. *The present work investigates the efficiency of Multigrid Method when applied to solve two-dimensional laminar natural convection flows inside a square domain filled with porous material. Numerical analysis is based on the finite volume discretization scheme applied to structure orthogonal regular meshes. Performance of the correction storage (CS) Multigrid Algorithm is compared for Rayleigh number, $Ra=103$. Up to two grids were used for both V- and W-cycles. Simultaneous and Uncoupled temperature-velocity solutions were also applied. Advantages in using more than one grid are discussed. Results further indicate an increase in computational effort for higher Ra and an optimal number of relaxation sweeps for both V- and W-cycles*

Keywords. Natural convection, Porous Media, Multigrid Method, Heat Transfer

1. Introduction

The analysis of buoyancy-driven flow in an enclosed cavity provides a useful comparison problem for evaluating the robustness and performance of numerical methods dealing with viscous flow calculations. The importance of the enclosure natural-convection phenomena can best be appreciated by noting several application areas. The design of furnaces, in the operation of solar collectors, which contribute to energy losses minimization to increase collector efficiency, nuclear reactor insulation, ventilation rooms and crystal growth in liquids are some examples of applications.

Natural convection occurs in enclosures as a result of gradients in density, which are in turn due to variations in temperature or mass concentration. Natural convection in a infinite horizontal layers of fluid heated from below has received extensive attention since beginning of 20th century, when Bénard, 1901 observed hexagonal roll cells upon the onset of convection in molten spermaceti with a free upper surface. The work of Rayleigh, 1926 was the first to compute a critical value, Ra_c , for the onset of convection. The accepted theoretical value of this dimensionless group is 1708 for rigid upper and lower surfaces.

The study of natural convection in enclosures still attracts the attention of researchers and a significant number of experimental and theoretical works have been carried out mainly from the 80's.

During the conference on Numerical Methods in Thermal Problems, which took place in Swansea, Jones, 1979 proposed that buoyancy-driven flow in a square cavity would be a suitable vehicle for testing and validating computer codes. Following discussions at Swansea, were invited contributions for the solution of the problem. A total of 37 contributions from 30 contributors or groups of contributors in nine countries were received. The summarization and discussion of the main contributions yielded the benchmark of de Vahl Davis, 1983, which is one of the most important reference works in this area.

The thermal convection in porous media has been studied extensively in recent years. Underground spread of pollutants, grain storage, food processing are just some applications of this theme. The monographs of Nield and Bejan, 1992 and Ingham and Pop, 1998 fully document natural convection in porous media.

The case of free convection in a rectangular cavity heated on a side and cooled at the opposing side is an important problem in thermal convection in porous media. Walker and Homsy, 1978, Bejan, 1979, Prasad and Kulacki, 1984, Beckermann et al, 1986, Gross et al, 1986, Manole and Lage, 1992 have contributed with some important results to this problem.

The recent work of Baytas and Pop, 1999, concerned a numerical study of the steady free convection flow in rectangular and oblique cavities filled with homogeneous porous media using a nonlinear axis transformation. The Darcy momentum and energy equations are solved numerically using the (ADI) method.

Further, most iterative numerical solutions, convergence rates of single-grid calculations are greatest in the beginning of the process, slowing down as the iterative process goes on. Effects like those get more pronounced as the grid becomes finer. Large grid sizes, however, are often needed when resolving small recirculating regions or detecting high heat transfer spots. The reason for this hard-to-converge behavior is that iterative methods can efficiently smooth out only those Fourier error components of wavelengths smaller than or comparable to the grid size. In contrast, Multigrid methods aim to cover a broader range of wavelengths through relaxation on more than one grid.

The number of iterations and convergence criterion in each step along consecutive grid levels visited by the algorithm determines the cycling strategy, usually a V- or W-cycle. Within each cycle, the intermediate solution is relaxed before (pre-) and after (post-smoothing) the transportation of values to coarser (restriction) or to finer (prolongation) grids (Brandt (1977), Stüben and Trottenberg (1982), Hackbusch (1985)).

Accordingly, Multigrid methods can be roughly classified into two major categories. In the CS formulation algebraic equations are solved for the corrections of the variables whereas, in the full approximation storage (FAS)

scheme, the variables themselves are handled in all grid levels. It has been pointed out in the literature that the application of the CS formulation is recommended for the solution of linear problems being the FAS formulation more suitable to non-linear cases (Brandt (1977), Stüben and Trottenberg (1982), Hackbusch (1985)). An exception to this rule seems to be the work of Jiang, et al (1991), who reported predictions for the Navier-Stokes equations successfully applying the Multigrid CS formulation. In the literature, however, not too many attempts in solving non-linear problems with Multigrid linear operators are found.

Acknowledging the advantages of using multiple grids, Rabi and de Lemos (1998a) presented numerical computations applying this technique to recirculating flows in several geometries of engineering interest. There, the correction storage (CS) formulation was applied to non-linear problems. Later, Rabi and de Lemos (1998b) analyzed the effect of Peclet number and the use of different solution cycles when solving the temperature field within flows with a given velocity distribution. Optimal multigrid studies have also been conducted (Rabi and de Lemos, 2001, 2003). In all those cases, the advantages in using more than one grid in iterative solutions were confirmed. Furthermore, de Lemos and Mesquita (1999), introduced the solution of the energy equation in their Multigrid algorithm. Temperature distribution was calculated solving the whole equation set together with the flow field as well as uncoupling the momentum and energy equations. A study on optimal relaxation parameters was there reported.

More recently Mesquita and de Lemos (2000a-b) analyzed the influence of the increase of points of the mesh and optimal values of the parameters of the Multigrid cycle for different geometries.

The present contribution extends the early work on CS Multigrid methods to the solution of temperature field in porous media. More specifically, steady-state laminar flows in a square cavity totally filled with a porous material are calculated with up to 4 grids. A schematic of such configurations is show in Figure 1, refers to the two-dimensional flow of a Boussinesq fluid of Prandtl number 1 in a square cavity of $H=1$ m completely filled with porous medium. The cavity, assumed to be of infinite depth along the z-axis, is isothermally heated from the left and cooled from the opposing side. The other two walls are insulated. The no-slip condition is applied on the velocity at all walls and the resulting flow is treated as steady. The controlling parameter is the Rayleigh number, $Ra = g\beta H\Delta T K / \nu\alpha_{eff}$

where $\alpha_{eff} = k_{eff} / (\rho c_p)_f$. The particle diameter is given by $D_p = \sqrt{\frac{144K(1-\phi^2)}{\phi^3}}$.

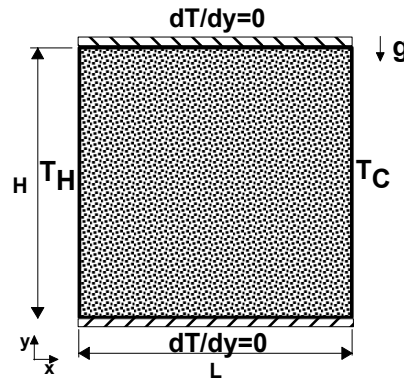


Figure 1 – The cavity under consideration

2. Mathematical Formulation and Numerics

The equations used are demonstrated in the work of Pedras and de Lemos, 2001, Pedras and de Lemos (2000) and de Lemos and Braga (2003). This work extends the development therein in order to include the buoyancy term in the governing equations.

Accordingly, the Boussinesq hypothesis can be written as,

$$\rho = \rho_{ref} [1 - \beta(T - T_{ref})].$$

Substituting this term in the momentum equation, the buoyancy term reduces to,

$$\rho_{ref} g \beta (T - T_{ref}), \quad (1)$$

$$\text{where, } \nabla p^* = \nabla p - \rho_{ref} g$$

Applying the volumetric average one has,

$$\langle \rho_{ref} \mathbf{g} \beta (T - T_{ref}) \rangle^v = \frac{\Delta V_f}{\Delta V} \frac{1}{\Delta V_f} \int \rho_{ref} \mathbf{g} \beta (T - T_{ref}) dV \quad (2)$$

Therefore, the buoyancy term becomes,

$$\rho_{ref} \beta_\phi \mathbf{g} \phi (\langle T \rangle^i - T_{ref}) \quad (3)$$

Therefore, for steady laminar flow and making $\rho_{ref} = \rho$, the macroscopic equations for continuity, momentum and temperature take the form:

$$\nabla \mathbf{u}_D = 0 \quad (4)$$

$$\rho \left[\nabla \left(\frac{\mathbf{u}_D \mathbf{u}_D}{\phi} \right) \right] = -\nabla (\phi \langle p^* \rangle^i) + \mu \nabla^2 \mathbf{u}_D - \left[\frac{\mu \phi}{K} \mathbf{u}_D + \frac{c_F \phi \rho |\mathbf{u}_D| \mathbf{u}_D}{\sqrt{K}} \right] + \rho \beta_\phi \mathbf{g} \phi (\langle T \rangle^i - T_{ref}) \quad (5)$$

$$(\rho c_p)_f \nabla (\mathbf{u}_D \langle T \rangle^i) = \nabla \cdot [k_f \phi + k_s (1 - \phi)] \nabla \langle T \rangle^i \quad (6)$$

where \mathbf{u}_D is the Darcy velocity, ρ is the density of the fluid, p is the total pressure and μ is the dynamic viscosity. The gravity acceleration vector is defined by \mathbf{g} and β_ϕ is the macroscopic thermal expansion coefficient. $\langle T \rangle^i$ and T_{ref} are the macroscopic and the reference temperatures respectively. The thermal conductivity for the fluid and solid are labeled k_f and k_s respectively. Finally, c_p is the specific heat and ϕ is the porosity, K is the permeability and c_F is the Forchheimer coefficient.

2.1. Numerical Model

The solution domain is divide into a number of rectangular control volumes (CV), resulting in a structure orthogonal non-uniform mesh. Grid points are located according to a *cell-centered* scheme and velocities are store in a *collocated* arrangement (Patankar, 1980). A typical CV with its main dimensions and internodal distances is sketched in Figure 2 Writing equations (2)-(4) in terms of a general form ϕ

$$\frac{\partial}{\partial x} \left(\rho U \phi - \Gamma_\phi \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\rho V \phi - \Gamma_\phi \frac{\partial \phi}{\partial y} \right) = S_\phi \quad (7)$$

where ϕ stands for U, V, and P. Integrating the equation 7 over the control volume of Figure2,

$$\int_{\delta v} \left[\frac{\partial}{\partial x} (\rho U \phi) + \frac{\partial}{\partial y} (\rho V \phi) \right] dv = \int_{\delta v} \left[\frac{\partial}{\partial x} \left(\Gamma_\phi \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\Gamma_\phi \frac{\partial \phi}{\partial y} \right) \right] dv + \int_{\delta v} S_\phi dv \quad (8)$$

Integration of the three terms in 8, namely: convection, diffusion and source, lead to a set of algebraic equations. These practices are described elsewhere (e.g. Patankar, 1980) and for this reason they not repeated here. In summary, convective terms are discretized using the upwind differencing scheme (UDS), diffusive fluxes make use of the central differencing scheme.

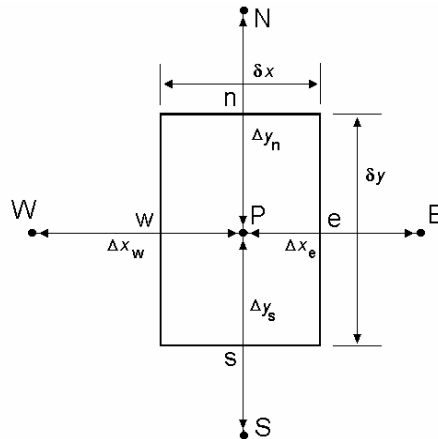


Figure 2 - Control Volume for discretization

Substitution of all approximate expressions for interface values and gradients into the integrated transport equation 8, gives the final discretization equation for grid node P

$$a_P \phi_P = a_E \phi_E + a_W \phi_W + a_N \phi_N + a_S \phi_S + b \quad (9)$$

with the east face coefficient, for example, being define as

$$a_E = \max[-C_e, 0] + D_e \quad (10)$$

In (10) $D_e = \mu_e \delta_y / \Delta x_e$ and $C_e = (\rho U)_e \delta_y$ are the diffusive and convective fluxes at the CV east face, respectively, and

2.2. Multigrid Technique

Assembling equation 9 for each control volume of Figure in the domain of Figure 2 defines a linear algebraic equation system of the form,

$$A_k T_k = b_k \quad (11)$$

where A_k is the *matrix of coefficients*, T_k is the *vector of unknowns* and b_k is the vector accommodating source and extra terms. Subscript “k” refers to the grid level, with k=1 corresponding to the coarsest grid and k=M to the finest mesh. defined as

As mentioned, Multigrid is here implemented in a *correction storage* formulation (CS) in which one seeks coarse grid approximations for the *correction* defined as $\delta_k = T_k - T_k^*$ where T_k^* is an *intermediate value* resulting from a small number of iterations applied to (11). For a linear problem, one shows that δ_k is the solution of (Brandt (1977), Stüben and Trottenberg (1982), Hackbusch (1985)),

$$A_k \delta_k = r_k \quad (12)$$

where the *residue* is defined as

$$r_k = b_k - A_k T_k^* \quad (13)$$

Eq. (10) can be approximated by means of a coarse-grid equation,

$$A_{k-1} \cdot \delta_{k-1} = r_{k-1} \quad (14)$$

with the *restriction operator* I_k^{k-1} used to obtain

$$r_{k-1} = I_k^{k-1} r_k \quad (15)$$

The residue restriction is accomplished by summing up the residues corresponding to the four fine grid control volumes that compose the coarse grid cell. Thus, equation 15 can be rewritten as,

$$r_{k-1}^{IJ} = r_k^{IJ} + r_k^{ij+1} + r_k^{i+1j} + r_k^{i+1j+1} \quad (16)$$

Diffusive and convection coefficients in matrix A_k need also to be evaluated when changing grid level. Diffusive terms are recalculated since they depend upon neighbor grid node distances whereas coarse grid mass fluxes (*convective terms*) are simply added up at control volume faces. This operation is commonly found in the literature (Peric, et al (1989), Hortmann et al (1990)).

Once the coarse grid approximation for the correction δ_{k-1} has been calculated, the *prolongation operator* I_{k-1}^k takes it back to the fine grid as

$$\delta_k = I_{k-1}^k \delta_{k-1} \quad (17)$$

In order to update the intermediate value

$$T_k = T_k^* + \delta_k \quad (18)$$

Figure 3 illustrates a 4-grid iteration scheme for both the V - and W -cycles where the different operations are: s =smoothing, r =restriction, cg =coarsest grid iteration and p =prolongation. Also, the number of domain sweeps before and after grid change is denoted by v^{pre} and v^{post} , respectively. In addition, at the coarsest k level ($k=1$), the grid is swept v^{cg} times by the error smoothing operator

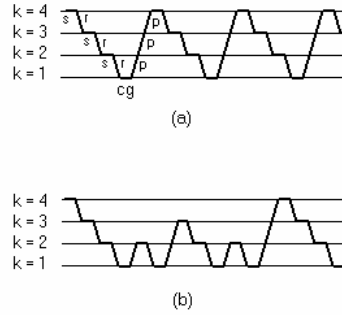


Figure 3 - Sequence of Operation in a 4-grid iteration (a)V-cycle, (b)W-cycle

3. Results and Discussion

The computer code developed was run on a IBM PC machine with a double Pentium 1GHz processor. Grid independence studies were conducted such that the solutions presented herein are essentially grid independent. For both V - and W -Cycles, pre- and post-smoothing iterations were accomplished via the Gauss-Seidel algorithm while, at the coarsest-grid, the TDMA method has been applied (Patankar, 1980). Also, cases in Figure 1 were run with the finest grid having size 80×80 .

Figure 4 shows isotherms for a flow in a square cavity filled with porous material for Rayleigh numbers ranging from 10^3 to 10^4 . The cavity is heated from the left side and cooled from the opposing side. The results are compared with the numerical work of Baytas and Pop, 1999 and show reasonable agreement.

At $Ra=10^3$, Fig. (4), the streamlines are a single flattened vortex, with its center in the center of the cavity. In contrast with the clear cavity case the porous matrix makes the flow be more intense near the heated and cooled walls and damped in the center. Corresponding isotherms, Fig. (5), indicate that the most of the heat transfer is due to convection mechanism.

The vortex is generated due to the horizontal temperature gradient across the section. This gradient, $\delta T / \delta y$, is negative everywhere, giving a clockwise vertical rotation.

For higher values of Ra , not shown here, the flow tends to be stratified as in the clear medium, but in a lesser value of Ra .

Table (1) shows, for some Rayleigh numbers the average Nusselt number \overline{Nu} on the vertical boundary of the cavity at $x=0$. It is seen from Table (1) that the present computations fall within the range of values presented in the literature. The heat transfer coefficient is seen to increase with Rayleigh number, as convection becomes dominant.

It is important to emphasize, that the Darcy number was considered as a constant and several runs were performed for different permeabilities. Although not shown here, one observes that, the lower the permeability, the higher the average Nusselt number. In comparison with results of Tab (1), more accurate simulations were obtained for low permeability media. The present results were performed with $D_p=1$ mm and $\phi=0.8$. The Prandtl number and the conductivity ratio between the solid and fluid phases are assumed to be a unit.

Table 1 – Average Nusselt number for laminar flow for a square cavity filled with porous material for Ra ranging from 10 to 10^4 , $\phi=0.8$ and $D_p=1$ mm.

	Ra			
	10	10^2	10^3	10^4
Walker and Homsy, 1978	-	3.097	12.96	51.0
Bejan, 1979	-	4.2	15.8	50.8
Beckerman, 1986	-	3.113	-	48.9
Gross et al, 1986	-	3.141	13.448	42.583
Manole and Lage, 1992	-	3.118	13.637	48.117
Moya et al, 1987	1.065	2.801	-	-
Baytas and Pop, 1999	1.079	3.160	14.06	48.330
Presents results	1.090	3.086	12.931	38.971

The residue is normalized and calculated according to

$$R_T = \sqrt{\sum_{ij} (R_{ij}^2)} \quad (19)$$

$$\text{with } R_{ij} = A_p T_p - \left(\sum_{nb} A_{nb} - T_{nb} \right)$$

Where ij identifies a given control volume on the finest grid and nb refers to its neighboring control volumes. Figure 6 shows residue history for the square cavity filled with porous material case following the two cycles pictured on Figure 3, namely the V - and W -cycles. The solution follows a simultaneous approach in the sense that the temperature is always relaxed after the flow field, within the multigrid cycle. It is interesting to note that for the V -cycle (Figure 6a), the computational effort related to values among too many grids became relevant.

Using a W -cycle (Figure 6b) for this Rayleigh number seems to bring more savings to iterative simultaneous solution procedure. When recalling the nature of the W -cycle in comparison with the V strategy (Figure 3), one can see that the number of grid transfers per cycle is less in the former algorithm. In addition, the more work done in the lower frequency range of the error spectrum with the W -cycle contributes for a faster overall solution. Also, undetectable advantages of the W -cycle for 2 grids could be associated with the idea that the loss due to the number of additional data transfer per cycle, required in the W strategy, outweighs the gain in relaxing the variables in the low-frequency range. Figure 6 also indicates the advantages in using 2 grids, in both cycles.

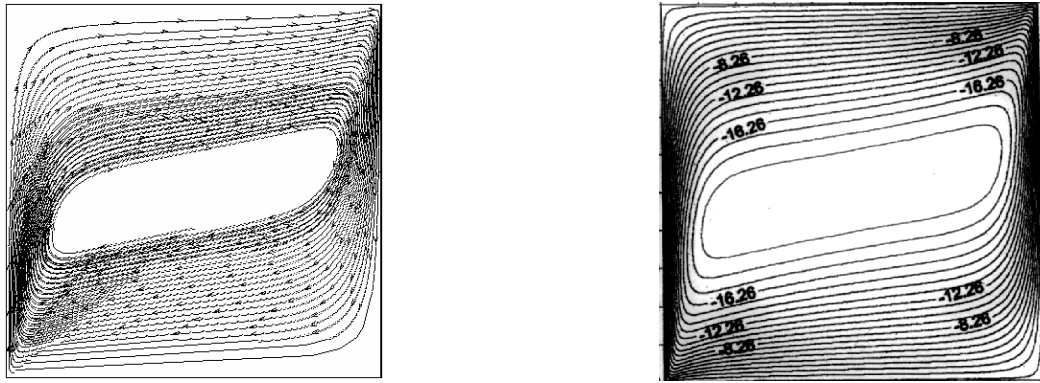


Figure 4 – Streamlines for $Ra=10^3$ with $Dp=1\text{mm}$ and $\phi=0.8$, Present Results and Baytas and Pop, 1999, respectively.

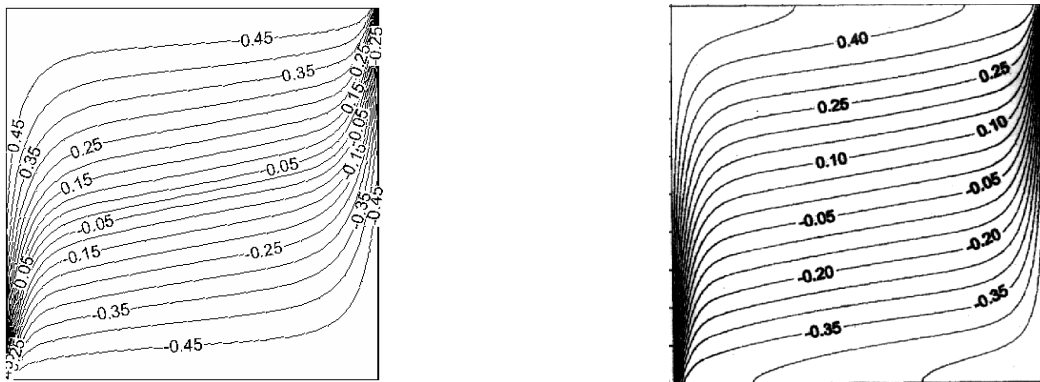


Figure 5 - Isotherms for $Ra=10^3$ with $Dp=1\text{mm}$ and $\phi=0.8$, Present Results and Baytas and Pop, 1999, respectively.

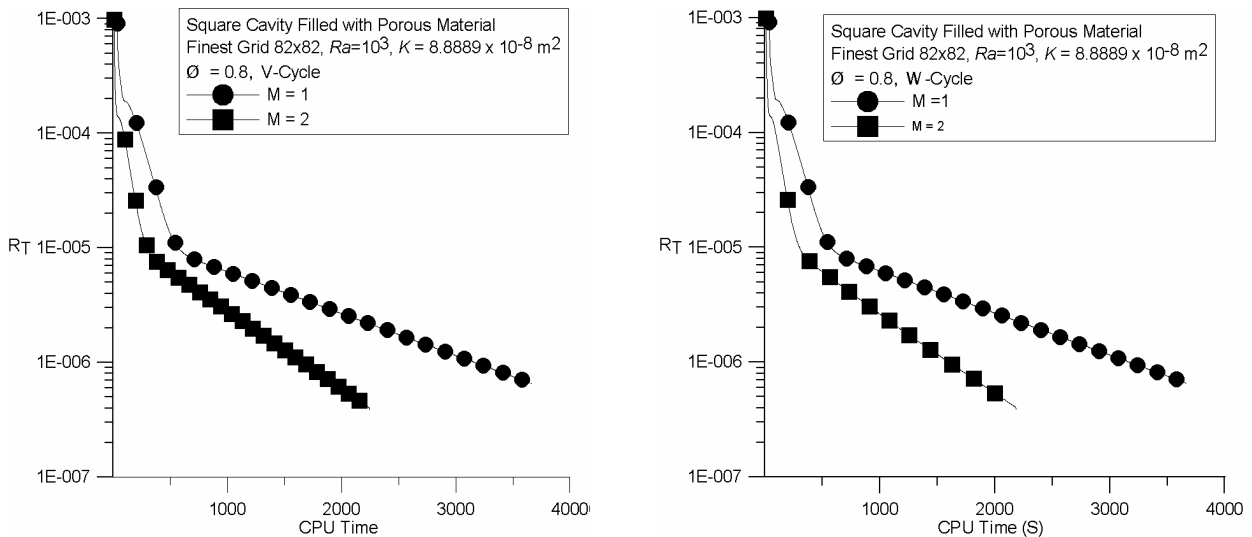


Figure 6 – Residue history of temperature for square cavity filled with porous material $Ra = 10^3$ – V -cycle and W – Cycle, respectively.

4. Conclusion

The efficiency of Multigrid Method when applied to solve two-dimensional laminar natural convection flows inside a square domain filled with porous material was investigated. In a general manner the code yields satisfactory agreement with others works. Results proved the superiority of the multigrid method against single grid calculations. Ultimately, it is expected that additional research on this new subject be stimulated by the work here presented.

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