INTERPOLATED DATA IN CONDUCTIVITY DOMAIN RECONSTRUCTION: COMPARISON BETWEEN FOURIER AND CUBIC SPLINE TECHNIQUES

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Abstract. Electrical impedance tomography is a non-invasive technique that can be used to monitor patients in intensive care. An image which represents the conductivity distribution of a slice of the thorax, is reconstructed based on boundary information of applied currents and voltages measurements. To guarantee uniqueness a complete Dirichlet-to-Neumann (voltage-to-current) map must be known. This completeness is not possible because of limitations in the total number of electrodes that can be positioned in the torax. Classical literature suggests linear interpolation to improve the boundary information. In this paper, Fourier and Cubic Spline techniques are implemented to improve data voltage. The mean conductivity and standard deviation of the reconsructed images of the homogeneous problem are used to compare the performance of these two interpolation schemes. Cubic Spline interpolation demonstrated to be better than Fourier one. Cubic spline is faster and standard deviation is approximately 4% smaller than Fourier.

***Keywords**. conductivity, backprojection, interpolation, cubic splines, Fourier*

1. Electrical impedance tomography

In the last 20 years different medical devices were developed for diagnosis and monitoring applications. One of these devices which image has a very high quality is the computerized tomography, but besides the expensive price of the equipment it could not be used for monitoring purposes. Radiation and invasive techniques must be avoided in devices developed to continuous observation and portability is an important point allied with confidence and quality of the information.

Blood, tissues, bones and air could be distinguished by their electrical properties. A device based on electrical impedance tomography posses most of these qualities: it is non-invasive, portable, could be used to continuous monitoring and is not expensive (Barber and Brown, 1984). However, hardware and software must be improved to provide useful real time images for medical applications. Hardware limitations include electrode technology, precision of current source and voltage measurement equipments. Besides the small number of independent measurements, run-time and error propagation are the most important software limitations, as the reconstruction algorithms solve an extremely ill-posed inverse problem. Even with such limitations, it has being proved that this technique could be used in several medical applications, for example, to monitor lung perfusion (MacArdle et all, 1988), cardiac function and to detect breast cancer (Cherepenin et all, 2001).

In electrical impedance tomography an image of a slice of the thorax is reconstructed based on the boundary information. The number of electrodes and the pattern of currents define the total number of independent data, which means that voltages between electrodes must be interpolated to complete the information necessary to the reconstruction algorithm. As a very ill-posed inverse problem, small perturbations in boundary data are strongly propagated inside the domain, which means that different interpolation algorithms must produce different reconstructed images.

In this work we compare two different methods of interpolating measured voltages, Fourier and cubic spline. The new set of voltages composed with 32 measured voltages and more 32, 64 and 128 interpolated values are implemented in the backprojection algorithm (Barber, 1990). Instead of the standard linear interpolation (Santosa and Vogeluis, 1990) one of these two algorithms could be used.

To compare the performance of these two schemes of interpolation, mean conductivity and standard deviation of the reconstructed images are calculated. Four different images are used: homogeneous saline water, a glass in saline water and two images of a slice of the thorax taken in two different time of the respiratory cycle. In all cases the standard deviation for cubic spline interpolation is smaller than Fourier interpolation. Specifically, for the images of the thorax, the standard deviation is 5% smaller than Fourier interpolation and for the other two test images it is about 3%. This is an important gain because an error in voltages of the order of 0.1% affects the reconstructed image in approximately 1%. Also, cubic spline interpolation is faster than Fourier one, even with the introduction of pre-calculated values of trigonometric functions.

2. Reconstruction algorithm

Consider a domain Ω with conductivity σ and 32 equally spaced electrodes positioned at the contour $\partial\Omega$. In our case Ω represents a transversal slice of the thorax and is approximated by a circular shape. Currents of 1 miliampere are successively applied in adjacent pairs of electrodes and for each injection pair voltages are measured in all electrodes. For the 2D model two adjacent electrodes simulate a dipole located between the electrodes, as the applied current has the same value with opposite signals. The phenomenon is modeled by simplified Maxwell´ s equations of the Electromagnetism where static conditions are introduced and the influence of internal currents and magnetic field are neglected. For the boundary conditions is assumed that the applied current ψ is normal to the surface. Equations (1) represent the described formulation.

$$\begin{aligned} &\nabla.\sigma\nabla\mathbf{u} = 0, \ \text{ in } \Omega \\ &\sigma.\frac{\partial \mathbf{u}}{\partial \mathbf{n}} = \psi, \ \text{ on } \partial\Omega, \end{aligned} \tag{1}$$

where $\mathbf{u}$ is the voltage potential, $\mathbf{n}$ the normal direction and σ the conductivity.

A simplification of this model is introduced assuming more restrictive hypothesis: the conductivity distribution is considered as a small perturbation $\delta\sigma$ of the homogeneous problem; to a small perturbation in the conductivity corresponds a small perturbation in the voltage potential; the perturbation $\delta\sigma$ is null near the region of the drive pair of electrodes. It means that the new image is a small perturbation of the dipole model. The linearized problem is defined by Eq. (2):

$$\begin{aligned} &\Delta\delta\mathbf{U} = -\nabla(\delta\sigma).\nabla\mathbf{U}, \ \text{ in } \Omega \\ &\frac{\partial \delta\mathbf{U}}{\partial \mathbf{n}} = 0, \ \text{ on } \partial\Omega, \end{aligned} \tag{2}$$

where $\delta\mathbf{U}$ is a small perturbation of the potential of the homogeneous problem $\mathbf{U}$. Therefore, for each drive pair the backprojection algorithm projects the boundary perturbation through the equipotential lines defined by the dipole. As the current is applied in turn in all pairs of electrodes, the final image is a composition of 32 back propagated information each one related to one dipole.

The adjacent configuration gives only a finite number of independent voltages measurements (Tang et all, 2002). This means that the uniqueness hypothesis that depends on the full knowledge of the Dirichlet-to-Neumann (voltage-to-current) map is not verified (Calderón, 1980). The reconstruction algorithm, to be useful for medical applications, must be able to detect a variation of conductivity inside the domain, in an area approximately of 1 cm^2 (Cheney et all, 1999) and this is not possible using only the independent data. Besides, the strong dependence of the uniqueness of the inverse problem solution on the boundary information means that a very precise data must be collected to generate a consistent set of voltages that will be used in interpolation. The analysis of the interpolation schemes is in the next topic.

3. Fourier versus cubic spline

The harmonic nature of the model suggests the introduction of a Fourier interpolation scheme instead of the linear one used in the classic implementation of the backprojection (Santosa and Vogeluis, 1990). For each drive pair a set of 32 voltages is collected and the classic Fourier interpolation algorithm (Frigo and Johnson, 1998) used to generate the values of voltages between the 32 electrodes. To generate the interpolated data all the harmonics are considered and to improve run-time the values of sines and cosines are calculated in advance.

The natural cubic spline interpolation version is implemented (Gerald, 1996). We compare natural with the other classical splines schemes and more consistent results were observed for the natural one. An error about 0.1% is detected in the measured voltages data and this error is certainly propagated to the value of the derivate of the voltage function in the extremes points.

The total number of interpolated data is constrained by run-time and limitations imposed by errors in the measured voltages. For monitoring purposes, 24 images must be generated per second. Therefore, reduction of run-time is an important challenge. The errors in data acquisition restrict also the total number of interpolated data. Therefore, 32 measured more N interpolated voltages are used in the backprojection reconstruction, where N is 32, 64 or 128.

Two experimental sets of voltages are collected using 32 electrodes evenly spaced in a tank filled with a homogeneous solution of saline water. One set for the homogeneous case (tank with saline water solution) and other when a glass is introduced in the tank. Simulated voltage data were generated using images from computerized tomography taken in two different time of the respiratory cycle. The time to reconstruct completely an image is shown in Tab. (1) for the homogeneous case and for the different values of N. For these specific numbers of interpolated data, spline routine runs in approximately 60% of the time of the Fourier one.

Table 1. Run-time for Fourier and spline interpolation to reconstruct the homogeneous problem

Number of values of voltages interpolated	Spline (sec)	Fourier (sec)
32	0.41	0.6
64	0.78	1.23
128	1.46	2.46

Mean conductivity and standard deviation of the reconstructed image were calculated for all examples. Significant variation in these numbers could be induced by errors in the interpolated data of the boundary. This could be observed in the homogeneous case shown in Tab. (2). For the homogeneous case a mean conductivity close to zero is expected. The values in all cases have the same magnitude except for spline interpolation of 32 values. Except for the first interpolation of 32 voltages, where standard deviation is greater for spline than for Fourier, standard deviation for Fourier is approximately 12% greater than for spline for the other cases.

Table 2. Mean and standard deviation values for the two interpolation schemes. Homogeneous example.

Number of values of voltages interpolated	Spline		Fourier	
	mean	sd	mean	sd
32	0.00141	0.022208	0.00098	0.015321
64	0.00094	0.011332	0.00099	0.012738
128	0.00092	0.010571	0.00096	0.011739

For each pair of injection electrodes the 32 measured voltages were interpolated with 128 data calculated by Fourier scheme. The new set of voltages is used in the reconstruction of the image relative to this pair of injection. As 32 electrodes are positioned in the boundary, the final image is a mean of 32 images each one related to one specific drive pair. After the reconstruction of the final image using interpolated data by Fourier scheme the process is repeated with cubic spline interpolation.

Figure (1) and Fig. (2) show the reconstructed images of the homogeneous problem for Fourier and cubic spline interpolation, respectively.

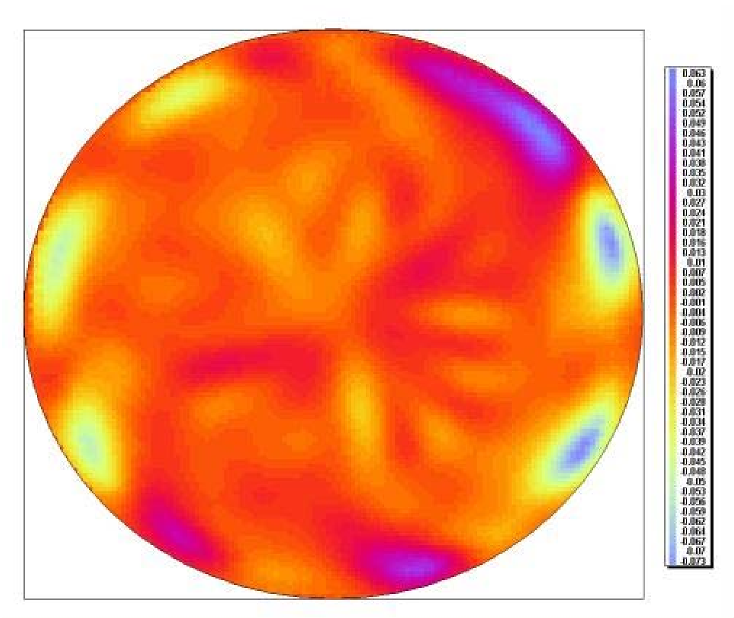

Figure 1. Reconstructed image of the homogeneous problem using Fourier interpolation. The 32 experimental voltages of a tank with saline water were interpolated with 128 new data generated by Fourier algorithm.

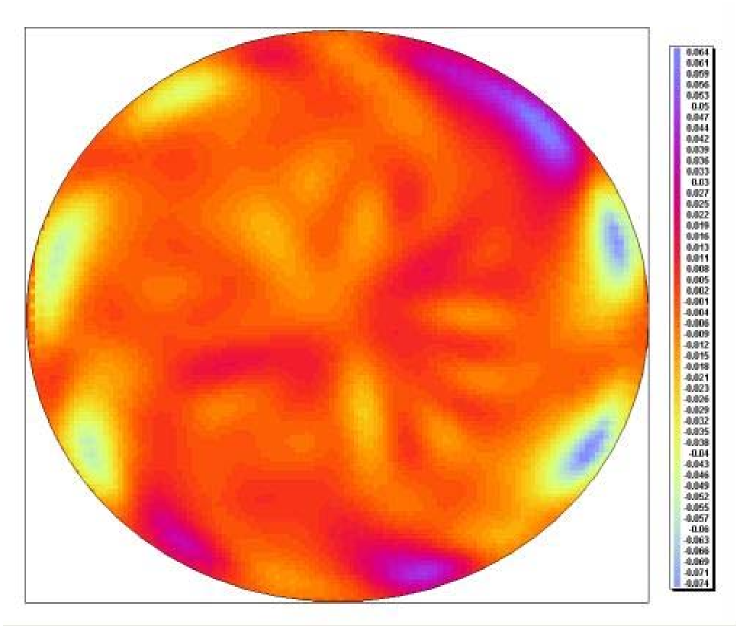

Figure 2. Reconstructed image of the homogeneous problem using cubic spline interpolation. The 32 experimental voltages of a tank with saline water were interpolated with 128 new data generated by cubic spline algorithm.

Comparing the two reconstructed images of Fig. (1) and Fig. (2) is not clear which interpolation scheme is the best one. But from the results showed in Tab. (2) the advantage of spline interpolation could be observed.
The same interpolations were compared in the reconstruction of the image of another experimental data: a glass is introduced inside the tank with saline water.

Figure (3) and Fig. (4) show the reconstructed images of the glass in the tank homogeneous problem, using Fourier and cubic spline interpolation, respectively. The position of the glass corresponds to the circular blue region near the center of the image.

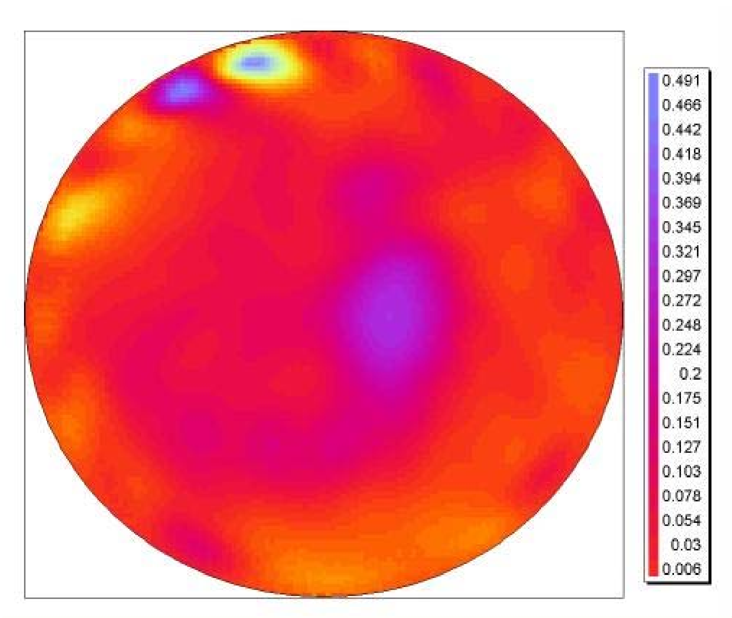

Figure 3. Reconstructed image of the glass inside the tank with saline water problem using Fourier interpolation. The 32 experimental voltages of a glass inside the tank with saline water were interpolated with 128 new data generated by Fourier algorithm.

As in the homogeneous case, the visual effect is almost imperceptible if a comparison between the reconstructed image using Fourier or cubic spline is made. However, a gain is observed in standard deviation of the spline column, as is exposed in Tab. (3).

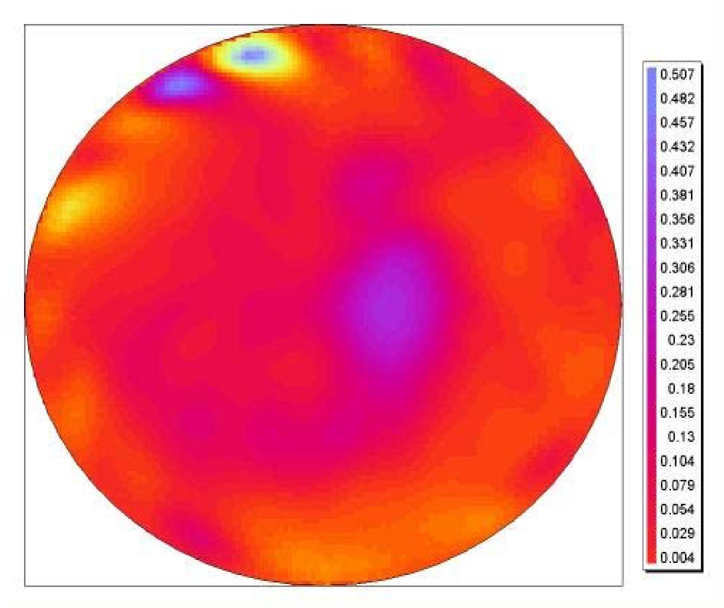

Figure 4. Reconstructed image of the glass inside the tank with saline water problem using cubic spline interpolation. The 32 experimental voltages of a glass inside the tank with saline water were interpolated with 128 new data generated by cubic spline algorithm.

Table 3. Mean and standard deviation values for the two interpolation schemes. Glass inside the tank example.

Number of values of voltages interpolated	Spline		Fourier	
	mean	sd	mean	sd
32	0.040095	0.116839	0.039705	0.12178
64	0.039909	0.105484	0.039618	0.11055
128	0.039815	0.103152	0.039579	0.106919

A more significant visual effect occurs in the simulated data obtained from computerized tomography. In this case a tomography image is used as a base of distribution of conductivity in the direct problem and a set of 32 voltages is generated, simulating the experimental event of 32 electrodes positioned in the thorax. The same interpolations were compared for the simulated data. Fig. (5) and Fig (6) show the reconstructed images for the tomography data, using Fourier and cubic spline interpolation, respectively.

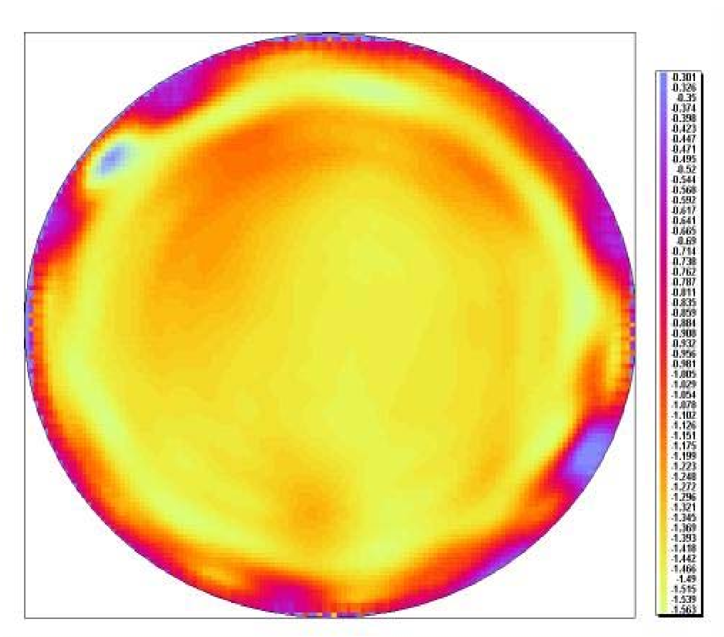

Figure 5. Reconstructed image of thorax using Fourier interpolation of 128 new values of voltages.

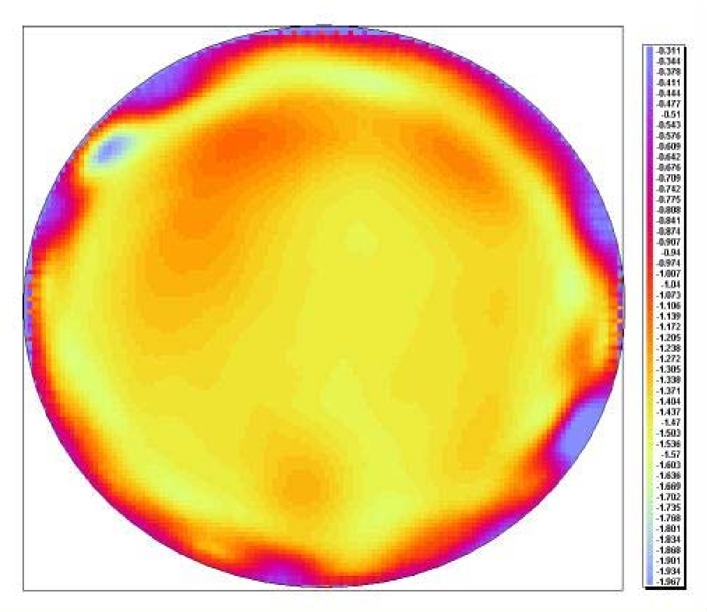

Figure 6. Reconstructed image of thorax using cubic spline interpolation of 128 new values of voltages.

In this case a new effect is observed for the reconstruction using Fourier interpolation. Values of higher conductivity are scattered in arcs around the center of the image. These arcs are not present in the reconstruction made with spline interpolation as can be seen in Fig. (6). This effect in not observed in the physiological image.

Mean conductivity and standard deviation is also calculated for these images. Table (4) shows the values of mean and standard deviation for the reconstructed image of the thorax. Again, standard deviation is 5% smaller for natural spline interpolation. The same results are observed for the other image of the thorax that represents a different time in the respiratory cycle.

Table 4. Mean and standard deviation values for the two interpolation schemes. Simulated data from computerized tomography.

Number of values of voltages interpolated	Spline		Fourier	
	mean	sd	mean	sd
32	1.34644	0.319597	1.34339	0.339823
64	1.34673	0.276606	1.3434	0.294227
128	1.34661	0.266635	1.34272	0.282

5. Concluding remarks

The mathematical model assumes that a small perturbation of the voltages in the boundary induces a small perturbation of the conductivity distribution inside the domain. The reference problem is the homogeneous distribution of conductivity with Neumann conditions in the boundary and the potential is defined by the dipole solution. Therefore, is expected that the interpolation by harmonic functions will generate more consistent results.

Fourier and natural cubic spline interpolation are used to introduce new values of voltages before the reconstruction of the image by backprojection algorithm. The mean conductivity and standard deviation are used to compare the reconstructed images for four different problems. For the homogeneous problem mean conductivity is smaller when interpolated data uses cubic spline interpolation scheme. In all cases the standard deviation for cubic spline interpolation is smaller than Fourier interpolation, about 5% smaller for the images of the thorax and 3% for the other cases. These results, are not in perfect agreement with the nature of the mathematical model of the electrical dipole with suggests that the voltages in the boundary are represented by harmonic functions.

Also, cubic spline interpolation is faster than Fourier one, even with the introduction of pre-calculated values of trigonometric functions.

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