POSITIONAL CONTROL OF A SYSTEM WITH MULTIPLE FLEXIBLE APPENDAGES USING LQR DESIGN

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Abstract. *The objective of this work is to describe the positional control of an unconstrained multi-link flexible structure. The experimental apparatus was designed to be representative of a flexible space structure such as a satellite with multiple flexible appendages. In this work we describe the analytical modeling and the simulation of a position control using a Linear Quadratic Regulator using Kalman State Estimator.*

Keywords *Flexible structures, Modal analysis, Identification, Control of structures*

1. Introduction

This paper presents the analytical modeling of a multibody flexible structure prototype and the simulation of its position control using LQR design using Kalman state estimator. The experimental setup, show in the Fig. (1), was assembled at the Dynamics Laboratory with the aim to investigate the dynamics and the position control of flexible structures representative of aerospace structures such as a satellite with flexible appendages. The experimental setup is composed of two flexible aluminum beams coupled to a central rigid hub. The hub is mounted on a steel disc supported on a gas bearing, in an attempt to minimize the static friction and to simulate the structure's slew motion in space conditions. The steel disc is linked to a brushless DC motor that gives the necessary excitation to the structure. The direct-drive torque actuation avoids the introduction of spurious non-linear effects such as dry friction and backlash in the gear transmission system.

The instrumentation and measurement subsystems consist of collocated and non-collocated sensors and their respective signal conditioning systems. An accelerometer, is used to monitor the vibration displacement of the beam tip. Two full strain-gage bridges are used to measure the elastic deformation at two known positions along the arms. The collocated sensors consist of a tachometer and a potentiometer both fixed to the motor axis.

A schematic view of the experimental set up is shown in Fig (1).

Figure 1- Experimental Setup

2. THE ANALYTICAL MODEL

The generalized Lagrangean approach is used to derive the analytical model of the unconstrained multi-link flexible structure, where the unconstrained characteristic results from the natural motion without external influences, i.e, all the structure is allowed to vibrate and its solution involves both the inertia of the rigid and the flexible parts (Barbieri & Özgüner, 1988). In this study we assume that the elastic deformation of the beams are symmetric with respect to the hub, consequently it is necessary to model only the elastic displacement of one of the arms (Junkins and Kim, 1993). The position of a generic point on the beam is written on a local body fixed coordinate system, as shown in the Fig.(2).

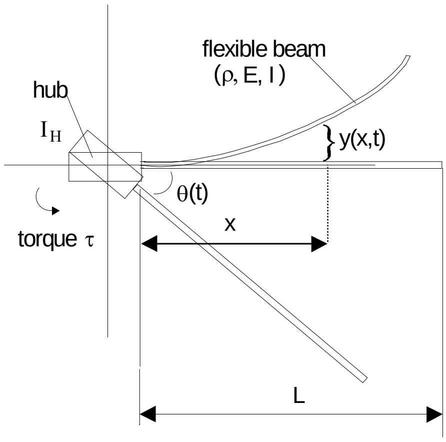

Figure 2. Coordinate system

The kinetic energy of the system is the sum of the kinetic energy of the hub, the arms and the tip mass (boundary elements).

$$T = T_{hub} + T_{beam} + T_{boundary} \tag{1}$$

with

$$T_{hub} = \frac{1}{2} I_{hub} \dot{\theta}^2 \tag{2}$$

$$T_{beam} = \int_o^L \rho \, \underline{\dot{R}}^2 dx \tag{3}$$

$$T_{boundary} = \frac{1}{2} m_t \, \underline{\dot{R}}^2 (L) \tag{4}$$

where I_{hub} is the hub inertia, ρ is the linear mass density of the beam, L is the appendages length and m_t is the mass of the accelerometer located at the tip of the beam.

The potential energy of the distributed parameter system do not take into account the shear deformation and the rotary inertia of the beam and is given by the following expression:

$$V = \int_0^L EI \left[\frac{\partial^2 y(x,t)}{\partial x^2} \right]^2 dx \tag{5}$$

The Lagrangian of the system, is written as the total kinetic energy minus the potencial energy of the structures and the nonconservative work done by the applied torque are respectively:

$$L = T - V \quad ; \quad \delta W_{nc} = \tau \delta\theta \tag{6}$$

From Góes et al. (1998) and Negrão (1998 and 1999) the following matrix equation is obtained for the first three assumed modes:

$$\mathrm{M}\ddot{\mathrm{q}} + \mathrm{K}\mathrm{q} = \mathrm{F} \tag{7}$$

$$M = \begin{bmatrix} I_T & 0 & 0 & 0 \\ I_j & 1 & 0 & 0 \\ I_j & 0 & 1 & 0 \\ I_j & 0 & 0 & 1 \end{bmatrix} \text{ and } K = \begin{bmatrix} 0 & 0 \\ 0 & diag[\omega_1^2 \ldots \omega_3^2] \end{bmatrix} \quad (8)$$

where:

$$I_T = I_{Hub} + I_{beam} + m_t\, l^2 \quad ; \quad I_j = \frac{\left(-\left(I_{Hub} + I_{beam} + m_t\, l^2\right)\theta_j\right)}{\left(\int_0^l \rho\phi_j^2\, dx + m_t\phi_j^2(l) + I_{Hub}\theta_j^2\right)} \quad ; \quad j = 1,2,3 \quad (9)$$

$$q = \left[\Theta \quad \eta_1 \quad \eta_2 \quad \eta_3\right]^T \; ; \quad F = \left[\tau_m \quad \phi_1'(0)\tau_m \quad \phi_2'(0)\tau_m \quad \phi_3'(0)\tau_m\right]^T \quad (10)$$

$$I_{beam} \equiv \int_0^L \rho\, x^2\, dx \quad ; \quad \theta_i = -\frac{\rho\int_0^L (x)\phi_i(x)dx + m_t(L)\phi_i(L)}{I_{hub}} \quad ; \quad i,j=1,2,\ldots \quad (11)$$

$\phi_j(x)$ are the eigenfunctions of the hub-beam system.

Now it is simple to get the state-space representation of the system in the form:

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{B}\,\underline{u} \quad (12)$$

where the $\underline{\underline{A}}$ e $\underline{B}$ matrix are:

$$\underline{\underline{A}} = \begin{bmatrix} \underline{0} & \underline{\underline{I}} \\ \underline{\underline{M}}^{-1}\underline{\underline{K}} & \underline{0} \end{bmatrix} \quad ; \quad \underline{B} = \begin{bmatrix} \underline{0} \\ \underline{\underline{M}}^{-1}\underline{F} \end{bmatrix} \quad (13)$$

We define the observation matrix, $\underline{\underline{C}}$, that describe the measured signals in terms of the state variables. This matrix is obtained from the model of the available sensors. The accelerometer is located at the free tip of the beam and, its signal is conditioned by a pre-amplifier and a double integrator filter with a global coefficient of sensitivity given by G_a, in V/cm units. Thus, we can write:

$$e_{ac} = Ga(L\theta + y(L,t)) \quad (14)$$

Rewriting the integrated accelerometer equation, as in (Negrão, 1998):

$$e_{ac} = Ga[L \quad \phi_1(L) \quad \phi_2(L) \quad \phi_3(L) \quad 0 \quad 0 \quad 0 \quad 0][\theta(t) \quad \eta_1(t) \quad \eta_2(t) \quad \eta_3(t) \quad \dot{\theta}(t) \quad \dot{\eta}_1(t) \quad \dot{\eta}_2(t) \quad \dot{\eta}_3(t)]^T \quad (15)$$

The potentiometer provides a voltage proportional to the angular position of the hub, $e_p = G_p\,\theta(t)$. The full strain-gage bridge gives a signal proportional to the axial strain of the beam (ε_s), which can be related with the elastic deformation y(x, t), at the point were it is located by the Eq. (16),

$$\varepsilon_s\big|_x = \left[\frac{e}{2}\right]\left(\frac{\partial^2 y}{dx^2}\right)\Bigg|_x \quad (16)$$

where *e* is the thickness of the beam. The strain-gage sensor is rewritten as:

$$\varepsilon_s = \left[\frac{e}{2}\right]\left[0 \quad \frac{d^2\phi_1(x_1)}{dx^2} \quad \frac{d^2\phi_2(x_1)}{dx^2} \quad \frac{d^2\phi_3(x_1)}{dx^2} \quad 0 \quad 0 \quad 0 \quad 0\right][\theta(t) \quad \eta_1(t) \quad \eta_2(t) \quad \eta_3(t) \quad \dot{\theta}(t) \quad \dot{\eta}_1(t) \quad \dot{\eta}_2(t) \quad \dot{\eta}_3(t)]^T \quad (17)$$

where x_1 is the position where the sensor is located on the beam. The tachometer gives a signal proportional to the angular velocity of the hub, $e_t = \dot{\theta}(t)$, which combined with the other sensor equations, gives the observation vector $\underline{y} = \underline{\underline{C}}.\underline{x}$, where:

$$\underline{y} = [e_{ac} \quad e_p \quad e_s \quad e_t]^T \tag{18}$$

and,

$$\underline{\underline{C}} = \begin{bmatrix} L & \phi_1(L) & \phi_2(L) & \phi_3(L) & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{e}{2}\left[\frac{d^2\phi_1(x_1)}{dx^2}\right] & \frac{e}{2}\left[\frac{d^2\phi_2(x_1)}{dx^2}\right] & \frac{e}{2}\left[\frac{d^2\phi_3(x_1)}{dx^2}\right] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

3. THE ANALYTICAL TRANSFER FUNCTIONS

To obtain the analytical transfer functions, for the unconstrained multi-link flexible system, we used the physical parameters listed in table 1,

Table 1. Model parameter of the unconstrained flexible beams

Aluminum density	ρ	$2.7950\ 10^{3}$	Kg/m³
Aluminum Young's modulus	E	$6.8900\ 10^{10}$	N/m²
Beams width	Eb	$4.1200\ 10^{-3}$	M
Beams height	Hb	$8.0780\ 10^{-2}$	M
Beams length	L	$9.7150\ 10^{-1}$	M
Beams cross-section area	A	$3.3281\ 10^{-4}$	m^2
Beams moment of inertia	I	$4.7070\ 10^{-10}$	m^4
Beams mass moment of inertia	I_b	$2.8430\ 10^{-1}$	Kg m^2
Hub mass moment of inertia	I_{hub}	$7.6749\ 10^{-1}$	Kg m^2
Hub radius	r	$9.0000\ 10^{-2}$	M

Applying the Laplace transform into Eq. (12) with zero initial conditions and using the model parameters listed in Tab. (1), we can obtain the analytical transfer functions for each sensor.

$$\underline{Y}(s) = \underline{\underline{C}}.(s\underline{\underline{I}} - \underline{\underline{A}})^{-1}.\underline{B}.U(s) \tag{20}$$

4. **Position Control**

Position control of mechanical systems with structural flexibility has been an important research topic in recent years. We show a simulation results of a position control using LQR design. Consider the system:

$$\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}\underline{x} + \underline{B}\underline{u} \\ y &= \underline{\underline{C}}\underline{x} \end{aligned} \tag{21}$$

and

$$J = \frac{1}{2}\int_0^T (\underline{x}'\underline{\underline{Q}}\,\underline{x} + \underline{u}'\underline{\underline{R}}\underline{u})dt$$

The solution of the LQR problem is to minimize J with respect to the control input $u(t)$ and J represents the weighted sum of energy of the state and control and Q and R represent respective weights on the different states and control channels. The problem is a solution of the algebraic Riccati equation :

$$A'P + PA + Q - PBR^{-1}B'P = 0 \tag{22}$$

and the optimal control law is:

$$u = -k\underline{x}, \tag{23}$$

where $k = R^{-1}B'P$ (24)

The implementation of the state feedback law requires the state vector $\underline{x}$ available for measurement and feedback. Unfortunately, that is not the case. The Optimal Observer Design Kalman-Bucy Filter was utilized for this purpose.

Consider the system in Eq.(21) with the additional random terms ω and ν:

$$\dot{\underline{x}} = \underline{\underline{A}}\underline{x} + \underline{B}\underline{u} + \Gamma\omega$$
$$y = \underline{\underline{C}}\underline{x} + \nu \quad (25)$$

Where ω represents random noise disturbance input and ν represents random measurement noise.
Assuming that ω and ν are both white Gaussian zero-mean stationary processes with know covariances, we have:

$$E\{\omega(t)\} = 0, E\{\nu(t)\} = 0 \quad (26)$$

$$E\{\omega(t) * \omega(t+\tau)'\} = Q_0\delta(t-\tau) \quad (27)$$

$$E\{\nu(t) * \nu(t+\tau)'\} = R_0\delta(t-\tau) \quad (28)$$

$$E\{\omega(t) * \nu(t+\tau)'\} = 0 \quad (29)$$

The goal is to estimate *x*(t), based on noise-corrupted measurement. On this paper, it was used the Kalman-Bucy Filter approach. That is means to minimize the following cost function:

$$J_0 = E\{\tilde{x}(t)' * \tilde{x}(t)\} = error\ variance \quad (30)$$

That is, the goal is to minimize the variance of the estimated state less the real plant state. Under these assumptions, the optimal estimator (Kalman-Bucy filter) is given by:

$$\hat{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$
$$L = \Sigma C' R^{-1} \quad (31)$$

Where Σ is found from:

$$A\Sigma + \Sigma A' + \Gamma Q_0 \Gamma' - \Sigma C' R_0^{-1} C \Sigma = 0 \quad (32)$$

This is shown in the schematic below:

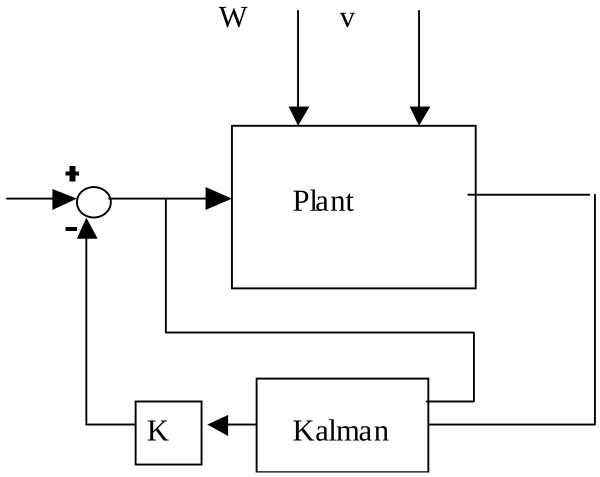

Figure 3. LQR Control Scheme with optimum state observer

Using a step reference of 1 [Volts], the results of the position control using LQR design with Kalman State Estimator are illustrated in Fig. (4)-(7):

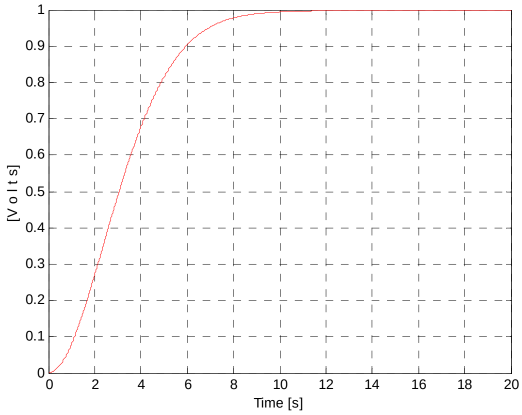

Figure 4. Angular position for a step reference

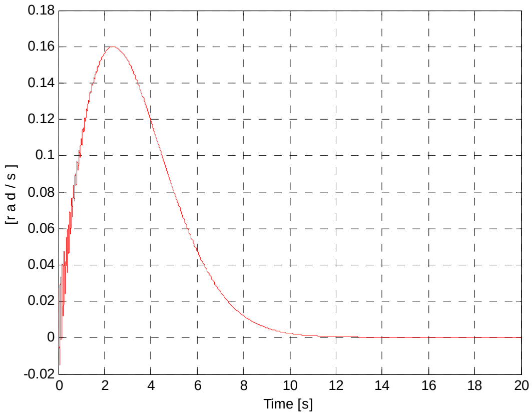

Figure 5. Angular velocity for a step reference

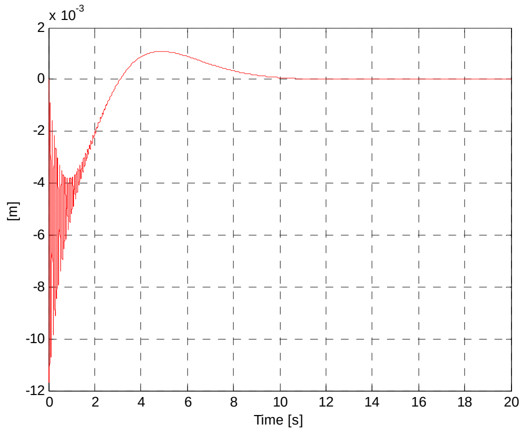

Figure 6. Transversal deformation for a step reference

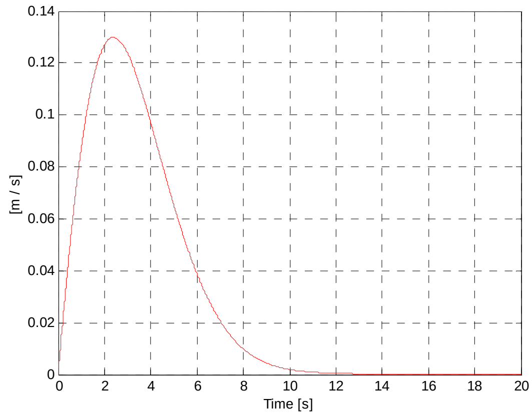

Figure 7. Tip acceleration for a step reference

As one can see in the Figures (4) and (7), that the positional control is efficient. The final position was reached in 10 seconds. This was the best performance that could be achieved without excitation of the higher vibrations modes of the beam considering noise-corrupted measurement. This work is still in progress, and we are implementing an experimental real-time control using the platform program MATLAB/ SIMULINK. We also intend to implement others control strategies including the LQG/LTR, and $H_{infinity}$ which due to the system inaccuracies, could be proven to be more robust to the unmodelled dynamics.

5. CONCLUSIONS

This paper reports preliminaries results obtained with an experimental apparatus with multiple flexible bodies. The model was derived using the Lagrangean approach and its discretization was done with the Assumed Modes Method as defined in Negrão(1999). The results in control position using LQR design using Kalman State Estimator shown that the controller reach the reference position in 10 seconds. This work is still in progress and using MATLAB to implement experimentally this control as well as we intend to implement other control strategy, such as robust control. Due to the system inaccuracies a robust control synthesis like LQG/LTR and $H_{infinity}$ should be more suitable for this system (Soares, Goes and Souza, 1996).

6. References

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