DEVELOPMENT OF LINEAR HEAT SOURCE PROBE

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***Abstract**. In this work we use the Levenberg-Marquardt method for the identification of parameters involved in the thermal analysis of a line heat source probe, under development in the Laboratory of Heat Transmission and Technology of PEM/COPPE. The line heat source probe is a variation of the hot wire method and its use is mainly concerned with the identification of the thermal properties of granulated materials and viscous fluids. The probe under development is described below, together with its mathematical formulation. The inverse problem under picture is formulated and the method of solution used to solve the present inverse parameter estimation problem is described. The D-optimum approach is applied to the optimal design of the experiment aiming at the estimation of minimum variance parameters. The use of such an approach reveals that more accurate estimates can be obtained by taking measurements during a period larger than the heating duration. Results based on simulated experimental data are presented.*

***Keywords**. Thermal properties, Line heat source probe, Hot wire, Parameter Estimation*

1. Introduction

The development of techniques for the identification of thermophysical properties of different types of materials has been drawing the attention of several research groups for a long time, because of its capital importance in design and computer simulation procedures. The hot wire method developed by Blackwell (1954) for the identification of thermal conductivity is one of such methods. It consists of a line heat source, usually taken in the form of a heating wire, which is placed inside the material with unknown properties. For large times, the temperature variation of the heat source is shown to be linear with respect to the logarithm of time; the thermal conductivity can be computed from the slope of such linear variation. Despite being susceptible to errors, depending on the thermal contact resistance between the hot wire and the material, as well as on the ratios of thermal conductivity and volumetric heat capacity of the material and of the wire (André et al, 2002; Thomson, 2001), such traditional form of the hot wire method is often used for the identification of thermal conductivity of different types of materials (Tavman, 1996 and 1998, Tavman and Tavman, 1996; Souza et al, 1999).

More recently, inverse analysis techniques of parameter estimation have been applied for the identification of other thermophysical properties, in addition to the thermal conductivity, with the hot wire method (Silva Neto and Carvalho, 1999; André et al, 2002) and with its extensions (Coment et al, 2002; Jarny, 2002). More involved mathematical models, taking into consideration the hot wire (André et al, 2002) or the probe inner region (Coment et al, 2002) have been recently used as well.

This paper describes the continuation of the work advanced by Prof. Roberto de Souza (Souza et al, 1999) on the development of an extension of the hot wire method, similar to the one used by Tavman (1998) and Comment et al (2002). The *Line Heat Source Probe* under development is described below. A mathematical model, considering the probe in the formulation, is presented together with its solution based on the *Classical Integral Transform Technique* (Ozisik, 1993). The inverse problem involving the estimation of the thermal conductivities and volumetric heat capacities of the probe and surrounding material is addressed. Also, the inverse problem of estimating such properties for the heat source (by assuming known the material properties) and the inverse problem of estimating such properties for the material (assuming known the source properties) are examined. The inverse parameter estimation problems considered here are solved with the Levenberg-Marquardt method of minimization of the least-squares norm (Beck and Arnold, 1977; Ozisik and Orlande, 2000) with simulated experimental data. The *D-optimum approach* (Beck and Arnold, 1977; Ozisik and Orlande, 2000) is applied for the design of the experiment.

2. Line Heat Source Probe

The line heat source probe under development, depicted in figure 1 (see also Souza et al, 1999), consists of a stainless steel tube with an outer diameter 2 mm and 200 mm long. An electrical resistance wire (36 AWG Kantal), uniformly wounded on a glass capillary tube, was placed inside the stainless steel tube together with a type-E thermocouple. Before the resistance and the thermocouple were pushed through the stainless steel tube, thermal grease and epoxy were applied in order to fill the voids inside the probe.

The experimental methodology consists in placing the probe inside the material with unknown thermal properties. An electrical current of known and controlled intensity is passed through the electrical resistance and the temperature response of the thermocouple inside the probe is recorded. The temperature measurements are subsequently used to estimate the thermal properties of the material surrounding the probe.

The line heat source probe is portable, reusable and can be applied to the identification of the thermophysical properties of different types of materials. It is particularly suitable to be used with granular materials and viscous liquids, where the temperature measurements are taken during a period when the convective effects are negligible and heat transfer in the material can be modelled as conduction. However, it can also be used with solid materials, where the probe is inserted into a drilled hole. In this case, the thermal contact resistance between the probe and the material has to be considered in the heat transfer analysis. The mathematical formulation of the line heat source probe is presented below.

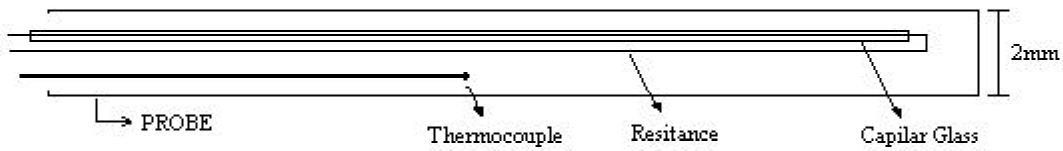


Figure 1 – Sketch of the line heat source probe

3. Physical Problem and Mathematical Formulation

The physical problem consists of a long cylinder of radius *a*, which is inserted into the material with unknown properties. Such material is considered to be a hollow cylinder with internal radius *a* and external radius r_{ex}. For the time scale of interest, the surface at $r = r_{ex}$ is assumed to be thermally insulated, while an imperfect contact between the probe and the material is taken into account through the thermal contact conductance h_c. Physical properties for the cylinder and for the material are supposed constant. The cylinder and the surrounding material are assumed to be initially in thermal equilibrium, at the temperature T_0. For times $t > 0$, the cylinder is uniformly heated with a time dependent heat source of strength $g(t)$. By neglecting end-effects, the physical problem under picture can be formulated as one dimensional in the form:

$$k_s \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_s}{\partial r}\right) + g(t) = \frac{\partial T_s}{\partial t} r_s C_{p_s} \qquad \text{in } 0 \text{ £ } r < a \text{ , and } t > 0 \qquad (1.a)$$

$$k_m \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_m}{\partial r}\right) = \frac{\partial T_m}{\partial t} r_m C_{p_m} \qquad \text{in } a < r < r_{ex} \text{ , and } t > 0 \qquad (1.b)$$

$$-k_s \frac{\partial T_s}{\partial r} = h_c[T_s(r,t) - T_m(r,t)] \qquad \text{at } r = a \text{ , and } t > 0 \qquad (1.c)$$

$$k_s \frac{\partial T_s}{\partial r} = k_m \frac{\partial T_m}{\partial r} \qquad \text{at } r = a \text{ , and } t > 0 \qquad (1.d)$$

$$k_m \frac{\partial T_m}{\partial r} = 0 \qquad \text{at } r = r_{ex} \text{ , and } t > 0 \qquad (1.e)$$

$$T_s(r,t) = T_0 \qquad \text{for } t = 0 \text{ , and in } 0 \text{ £ } r \text{ £ } a \qquad (1.f)$$

$$T_m(r,t) = T_0 \qquad \text{for } t = 0 \text{ , and in } a \text{ £ } r \text{ £ } r_{ex} \qquad (1.g)$$

where r, C_p and k denote density, specific heat and thermal conductivity, respectively, while the subscripts *m* and *s* refer to the surrounding material and the probe, respectively.

The following dimensionless groups are defined here:

$$q(R,t) = \frac{T(r,t) - T_0}{T_0}; \qquad t = \frac{k_{ref}}{r_{ref} C_{p,ref}} \frac{t}{a^2}; \qquad R = \frac{r}{a}; \qquad (2.a\text{-}c)$$

$$C_s = \frac{r_s C_{p,s}}{r_{ref} C_{p,ref}}; \qquad C_m = \frac{r_m C_{p,m}}{r_{ref} C_{p,ref}}; \qquad K_s = \frac{k_s}{k_{ref}}; \qquad K_m = \frac{k_m}{k_{ref}}; \qquad (2.d\text{-}g)$$

$$Bi_c = \frac{h_c a}{k_{ref}}; \qquad G(t) = \frac{a^2}{k_{ref} T_0} g(t) \qquad (2.h,i)$$

where k_{ref} , r_{ref} , $C_{p,\,ref}$ are reference values for thermal conductivity, density and specific heat, respectively.

In this work, following the same idea of Taktak et al (1993), we consider the heat source as a step function in the form:

$$G(t)=\begin{cases} G_0 & , \quad \text{for } 0<t\le t_h \\ 0 & , \quad \text{for } t>t_h \end{cases} \tag{3}$$

where t_h is the dimensionless heating time.

The dimensionless version of problem (1) is solved here with the *Classical Integral Transform Technique* (Ozisik, 1993). For the *heating period* ($0<t\le t_h$) the following expression results for the dimensionless temperature for the probe ($0 < R < 1$) (Thomson, 2001):

$$q_s(R,t)=\frac{G_0}{2\,N(b_0)}+\sum_{n=1}^{\infty}\frac{1}{N(b_n)}y_s(b_n,R)e^{-b_n^2 t}G_0\int_0^t e^{b_n^2 t'}\left(\int_0^1 R' y_s\left(b_n,R'\right)dR'\right)dt' \qquad \text{in } 0<R<1,\ 0<t\le t_h \tag{4}$$

where

$$N(b_0)=\frac{(C_s-C_m)+b'C_m}{2} \tag{5.a}$$

$$N(b_n)=C_s\int_0^1 R'[y_s(b_n,R)]^2\,dR'+C_m\int_1^{b'} R'[y_m(b_n,R)]^2\,dR' \tag{5.b}$$

and $b' = r_{ex}/a$.

The eingenfunctions are given by:

$$y_s(b_n,R)=J_0(g_n R) \tag{6.a}$$

$$y_m(b_n,R)=A_{2n}J_0(h_n R)+B_{2n}Y_0(h_n R) \tag{6.b}$$

where

$$A_{2n}=\frac{\{-[K_s\,g_n\,J_1(g_n)-Bi_c\,J_0(g_n)]g_n\,Y_1(b'h_n)\}}{Bi_c\,g_n\,[\,J_0(h_n)Y_1(b'h_n)-Y_0(h_n)J_1(b'h_n)\,]} \tag{7.a}$$

$$B_{2n}=\frac{\{\,[K_s\,g_n\,J_1(g_n)-Bi_c J_0(g_n)]g_n\,J_1(b'h_n)\}}{Bi_c g_n\,[\,J_0(h_n)Y_1(b'h_n)-Y_0(h_n)J_1(b'h_n)\,]} \tag{7.b}$$

$$g_n=\frac{b_n}{\sqrt{\dfrac{K_s}{C_s}}} \qquad\qquad h_n=\frac{b_n}{\sqrt{\dfrac{K_m}{C_m}}} \tag{7.c, d}$$

The eigenvalues are obtained from the solution of the following equation:

$$\begin{vmatrix} g_n K_s J_1(g_n)-Bi_c J_0(g_n) & Bi_c\ J_0(h_n) & Bi_c Y_0(h_n) \\ \dfrac{K_s}{K_m}\sqrt{\dfrac{K_s C_m}{K_m C_s}}\ J_1(g_n) & -J_1(h_n) & -Y_1(h_n) \\ 0 & g_n\ J_1(b'h_n) & g_n\ Y_1(b'h_n) \end{vmatrix}=0 \tag{8}$$

where | . | denotes the determinant.

Since the radial position of the thermocouple inside the probe cannot be accurately determined and the radius of the probe is small we assume that the thermocouple readings correspond to the average temperature inside the probe. The average temperature inside the probe, defined as:

$$q_{av}(t) = \frac{1}{p}\int_0^1 2p\; R q_s(R,t)\, dR \tag{9}$$

is given for the heating period ($0 < t \le t_h$) as:

$$q_{av}(t) = \frac{G_0}{2\,N(b_0)} + \sum_{n=1}^{\infty} \frac{2G_0}{N(b_n)} y_s(b_n, R)\left[\frac{e^{b_n^2 t} - 1}{b_n^2}\right]\left(\int_0^1 R' y_s(b_n, R')\, dR'\right)^2 \qquad \text{for } 0 < t \le t_h \tag{10}$$

Similarly, the average temperature inside the probe for the non-heating period ($t > t_h$), can be obtained as (Thomson, 2001):

$$q_{av}(t) = \frac{G_0}{2\,N(b_0)} + \sum_{n=1}^{\infty} \frac{2G_0}{N(b_n)} e^{-b_n^2 t}\left[\frac{1 - e^{-b_n^2 t_h}}{b_n^2}\right]\left(\int_0^1 R' y_s(b_n, R')\, dR'\right)^2 \qquad \text{for } t > t_h \tag{11}$$

4. Direct Problem and Inverse Problem

The *direct problem*, associated with the formulation given above for the physical problem, consists in determining the average temperature inside the probe from the knowledge of the heat source term, G_0, the heating time, t_h, the geometry of the probe and of the material, and the thermal properties of the probe (K_s and C_s) and of the material (K_m and C_m).

Three different types of *inverse parameter estimation problems* are examined in this paper, referred hereafter, for the sake of brevity, as types (i), (ii) and (iii). They are respectively defined as:

(i) The estimation of the probe thermal properties (K_s and C_s) by assuming the material thermal properties as exactly known (K_m and C_m). Such type of inverse problem is of interest during the initial use of the manufactured probe, when materials with known properties are used to calibrate the probe (i.e., estimate its thermal properties).

(ii) The estimation of the material thermal properties (K_m and C_m) by assuming the probe thermal properties (K_s and C_s) as exactly known. Such type of inverse problem is of interest for the estimation of unknown material's properties, after the probe has been initially calibrated by following the procedure of the inverse parameter estimation problem of type (i).

(iii) The simultaneous estimation of the material thermal properties (K_m and C_m) and of the probe thermal properties (K_s and C_s), which correspond to a more general case involving the estimation of all thermal properties appearing in the formulation of the dimensionless form of problem (1).

For the solution of the inverse problems referred to above, we suppose as exactly known the other quantities appearing in the formulation. In addition, temperature measurements of the average probe temperature are assumed available. Such measurements may contain random errors, which are supposed to be additive, uncorrelated, normally distributed with zero mean and with known and constant standard deviation.

For the solution of the inverse problems described above, we consider the minimization of the ordinary least squares norm:

$$S(\mathbf{P}) = [\mathbf{Y} - \Theta(\mathbf{P})]^T\,[\mathbf{Y} - \Theta(\mathbf{P})] \tag{12}$$

where the superscript T denotes the transpose, $\mathbf{P}^T \equiv [P_1, P_2, ..., P_N]$ is the vector of unknown parameters and $[\mathbf{Y}\text{-}\Theta(\mathbf{P})]^T$ is the vector containing the differences between measured and estimated average probe temperatures, that is,

$$[\mathbf{Y} - \Theta(\mathbf{P})]^T = [Y_1 - q_{av,1}, Y_2 - q_{av,2}, \cdots, Y_I - q_{av,I}] \tag{13}$$

In equation (13), I denotes the number of transient measurements, while $Y_i \equiv Y(t_i)$ and $q_{av,i} \equiv q_{av}(t_i)$ are the measured and estimated average probe temperatures at time t_i, respectively.

We use in this paper the *Levenberg-Marquardt Method* (Beck and Arnold, 1977, Ozisik and Orlande, 2000) for the minimization of the objective function given by Eq. (12). The iterative procedure of such method is given by:

$$\mathbf{P}^{k+1} = \mathbf{P}^k + (\mathbf{J}^T\mathbf{J} + m^k\Omega^k)^{-1}\mathbf{J}^T[\mathbf{Y} - \Theta(\mathbf{P}^k)] \tag{14}$$

where m^k is the damping parameter and $\mathbf{\Omega}^k$ is a diagonal matrix, which can be taken as the identity matrix or as the diagonal of $\mathbf{J}^T\mathbf{J}$. The *sensitivity matrix* $\mathbf{J}$ is defined as:

$$\mathbf{J}(\mathbf{P}) = \left[\frac{\partial \Theta^T(\mathbf{P})}{\partial \mathbf{P}}\right]^T = \begin{bmatrix} \frac{\partial q_1}{\partial P_1} & \frac{\partial q_1}{\partial P_2} & \frac{\partial q_1}{\partial P_3} & \cdots & \frac{\partial q_1}{\partial P_N} \\ \frac{\partial q_2}{\partial P_1} & \frac{\partial q_2}{\partial P_2} & \frac{\partial q_2}{\partial P_3} & \cdots & \frac{\partial q_2}{\partial P_N} \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial q_I}{\partial P_1} & \frac{\partial q_I}{\partial P_2} & \frac{\partial q_I}{\partial P_3} & \cdots & \frac{\partial q_I}{\partial P_N} \end{bmatrix} \quad (15)$$

The purpose of the matrix term $\mu^k \, \mathbf{\Omega}^k$ in equation (14) is to damp oscillations and instabilities due to the ill-conditioned character of the problem, by making its components large as compared to those of $\mathbf{J}^T\mathbf{J}$, if necessary. The damping parameter is made large in the beginning of the iterations. With such an approach, the matrix $\mathbf{J}^T\mathbf{J}$ is not required to be non-singular in the beginning of iterations and the Levenberg-Marquardt Method tends to the *Steepest Descent Method*, that is, a very small step is taken in the negative gradient direction. The parameter μ^k is then gradually reduced as the iteration procedure advances to the solution of the parameter estimation problem and then the Levenberg-Marquardt Method tends to the *Gauss Method* (Beck and Arnold, 1977). However, if the errors inherent to the measured data are amplified generating instabilities on the solution, as a result of the ill-conditioned character of the problem, the damping parameter is automatically increased. Such an automatic control of the damping parameter makes the Levenberg-Marquardt method a quite robust and stable estimation procedure, so that it does not require the use of the *Discrepancy Principle* in the stopping criterion to become stable, like the conjugate gradient method (Ozisik and Orlande, 2000).

5. Statistical Analysis and Design of Optimum Experiments

By performing a statistical analysis it is possible to assess the accuracy of $\hat{P}_j$, j = 1, …, N, which are the values estimated for the unknown parameters P_j, j = 1, …, N . By assuming the errors to be additive, uncorrelated and normally distributed, with a zero mean and a known constant standard-deviation s , as well as no previous information regarding the parameter values, the *covariance matrix*, **V**, of the estimated parameters $\hat{P}_j$, j = 1, …, N, is given by (Beck and Arnold, 1977):

$$\mathbf{V} \equiv (\mathbf{J}^T\mathbf{J})^{-1} s^2 \quad (16)$$

where **J** is the sensitivity matrix and σ is the standard deviation of the measurement errors, which is assumed to be constant. We note that equation (16) is exact for linear estimation problems and is approximately used for nonlinear parameter estimation problems.

The *standard deviations* for the estimated parameters can thus be obtained from the diagonal elements of **V** as

$$s_{\hat{P}_j} \equiv \sqrt{\mathrm{cov}(\hat{P}_j, \hat{P}_j)} = \sqrt{V_{jj}} \qquad \text{for } j = 1, ..., N \quad (17)$$

where V_{jj} is the j^{th} element in the diagonal of **V**.

Confidence intervals at the 99% confidence level for the estimated parameters can be obtained as

$$\hat{P}_j - 2.576\, s_{\hat{P}_j} \le P_j \le \hat{P}_j + 2.576\, s_{\hat{P}_j} \qquad \text{for } j = 1,...,N \quad (18)$$

The *joint confidence region* for the estimated parameters can be given by (Beck and Arnold, 1977):

$$(\hat{\mathbf{P}} - \mathbf{P})^T \mathbf{V}^{-1} (\hat{\mathbf{P}} - \mathbf{P}) \le c_N^2 \quad (19)$$

where c_N^2 is the value of the chi-square distribution with N degrees of freedom for a given probability.

Optimum experiments can be designed by minimizing the hypervolume of the confidence region of the estimated parameters, in order to ensure minimum variance for the estimates. The minimization of the confidence region given by equation (19) can be obtained by maximizing the determinant of $\mathbf{V}^{-1}$, in the so-called *D-optimum design* (Beck and Arnold, 1977). Since the covariance matrix **V** is given by equation (16), we can then *design optimum experiments by maximizing the determinant of the matrix* $\mathbf{J}^T\mathbf{J}$. Therefore, experimental variables such as the duration of the experiment and the duration of the heating period are chosen based on the criterion $\max\left|\mathbf{J}^T\mathbf{J}\right|$.

For cases involving a single sensor, each element $\mathbf{F}_{m,n}$, m,n = 1, …, N, of the matrix $\mathbf{F} \equiv \mathbf{J}^T\mathbf{J}$ is given by:

$$\mathbf{F}_{m,n} \equiv \left[\mathbf{J}^T \mathbf{J}\right]_{m,n} = \sum_{i=1}^{I} \left(\frac{\partial T_i}{\partial P_m} \right) \left(\frac{\partial T_i}{\partial P_n} \right) \qquad \text{for } m,n = 1, \ldots, N \tag{20}$$

where I is the number of measurements and N is the number of unknown parameters. The matrix with elements given by equation (20) is the so-called *Fisher Information Matrix*. If we take into account restrictions, such as a large but fixed number of transient measurements and also the maximum average temperature for the probe, $q_{av,max}$, we can choose to maximize the determinant of a dimensionless form of **F**, here denoted as $\mathbf{F}_I$ (Beck and Arnold, 1977), the elements of which are given by

$$[\mathbf{F}_I]_{m,n} = \frac{1}{t_f} \int_{t=0}^{t_f} \left(P_m \frac{\partial q_{av}}{\partial P_m} \right) \left(P_n \frac{\partial q_{av}}{\partial P_n} \right) \left(\frac{1}{q_{av,\max}} \right)^2 dt \qquad \text{for } m,n = 1, \ldots, N \tag{21}$$

6. Results and Discussions

For the results presented below, we have used simulated experimental data generated from the solution of the direct problem with *a priori* defined values for the unknown parameters. To generate the simulated data, and consequently for the analysis of the sensitivity coefficients and of the determinant of the information matrix, the values of the properties were taken as $K_s = C_s = K_m = C_m = 1$,while the value of the dimensionless contact conductance was taken $Bi_c = 10^8$ and the dimensionless heat source term during the heating period as $G_0 = 1.25$. If we consider the following physical variables: $k_{ref} = 0.2$ W/m^{o}C and $(\rho_{ref}\, C_{p,ref}) = 1255200$ J/m^3 ^{o}C, such a value of the heat source term corresponds to 9.2×10^6 W/m^3 and, for the time scale, the dimensionless time of t=10 corresponds to t = 62.8 s.

Figure 2 presents the transient variation of the dimensionless normalized sensitivity coefficients with respect to the various parameters. The normalized dimensionless sensitivity coefficients were obtained by multiplying the dimensionless sensitivity coefficients by the parameters that they are referred to. The variation of the average dimensionless temperature in the probe and the sensitivity coefficient with respect to Bi_c were also included in figure 2. The results shown in figure 2 were obtained for a dimensionless heating time of t_h=8. We can note that the sensitivity coefficients with respect to the thermal properties of the probe and of the material are of the same order of magnitude of the average temperature of the probe, except for the sensitivity coefficient with respect to Bi_c, which is quite small. This was an expected result because such a high value for Bi_c corresponds to perfect contact between the probe and the material, which is a reasonable assumption for granular materials and for viscous liquids. Figure 2 shows that the sensitivity coefficients with respect to C_s and K_s are linearly dependent during the heating period ($0 < t \le t_h$). On the other hand, these sensitivity coefficients tend to zero at different rates after the heating is stopped. Therefore, the analysis of the sensitivity coefficients reveals that the simultaneous estimation of the four unknown parameters may be possible if the heating time is considered to be smaller than the duration of the experiment, that is, measurements are taken during a period larger than the heating time. Also, from the analysis of the sensitivity coefficients we note that the estimation of the pairs (K_s, C_s) or (K_m, C_m) is also possible if measurements are taken during a period larger than the heating time. We note that similar conclusions can be obtained for $K_s = C_s = 1$ and $K_m = C_m = 10$.

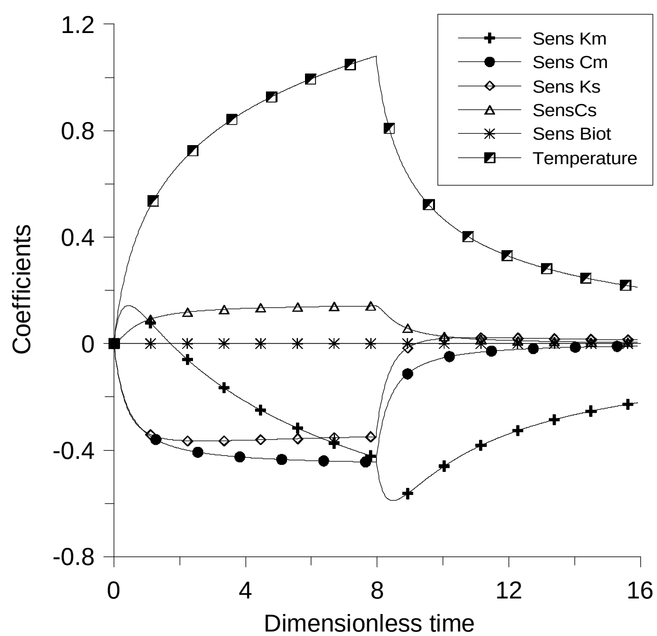
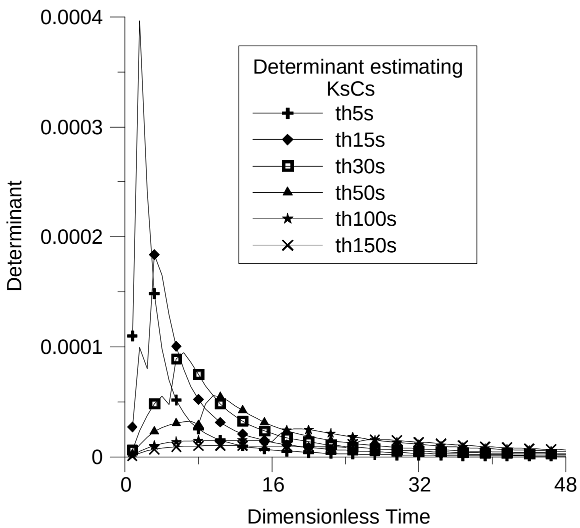


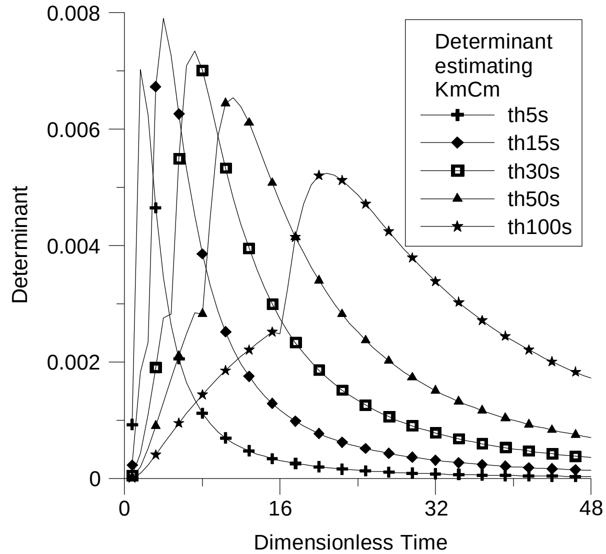
Figure 2. Sensitivity Coefficients and average probe temperature for t_h=8.

Figures 3.a-c present the determinant of the information matrix for inverse problems types (i), (ii) and (iii), respectively, by considering the constraints of a large and fixed number of measurements and the maximum average temperature of the probe. These figures show the determinant for different heating times. We can clearly notice in figures 3.a-c an increase in the determinant at the moment that the heating is stopped, as a result of the sudden change in the shape of the sensitivity coefficients, as observed in figure 2. For the inverse problem type (i), the optimum dimensionless heating and final times are 0.8 and 1.6. However, such heating time corresponds to 5 seconds, which is quite short and can result in small increases of the probe temperature, and, hence, large temperature measurement uncertainties. Therefore, for practical reasons, the dimensionless heating and final times of 2.4 and 3.2 were chosen for inverse problem type (i), which correspond to 15 s and 20 s, respectively. Basically for the same reason, the heating and final times for inverse problem type (iii) were taken as 2.4 and 4.0, which corresponds to 15 and 25 seconds, respectively. For inverse problem type (ii), the optimum dimensionless heating and final times are 2.4 and 7.2, which correspond 15 s and 45 s, respectively.

A comparison of figures 3.a and 3.b reveals that the maximum values of the determinant of the information matrix are larger for inverse problem type (ii) than for the estimation of inverse problem type (i). Hence, more accurate estimates are expected for (K_m, C_m) than for (K_s, C_s). Also, figure 3.c shows that the maximum values of the determinant of the information matrix for the estimation of the 4 parameters is quite small, although a comparison of figure 3.c with figures 3.a,b is avoided here, since the number of unknown parameters is different, which influence the analysis because of the constraints of a large and fixed number of measurements and of the maximum average temperature of the probe. On the other hand, such a small value of the determinant reveals that parameters with low accuracy are expected for inverse problem type (iii).



(a)



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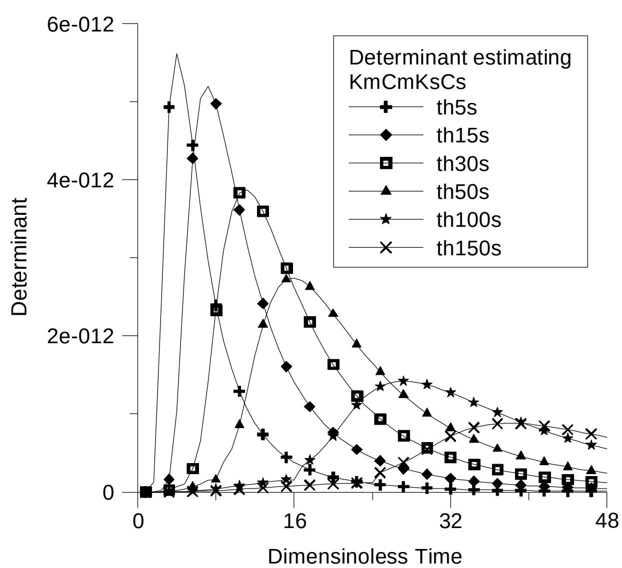


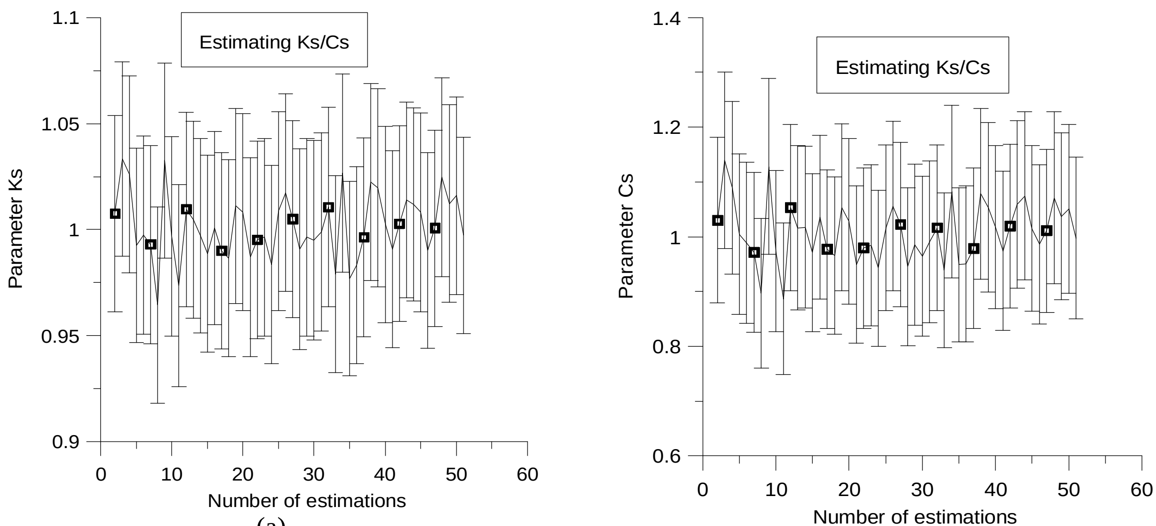
(c)

Figures 3. (a) Determinant of the information matrix for inverse problem type (i)
(b) Determinant of the information matrix for inverse problem type (ii)
(c) Determinant of the information matrix for inverse problem type (iii)

Figures 4.a,b present the results obtained for inverse problem type (i), for 50 different runs with simulated measurements with different random measurement errors of standard deviation of 0.5 ^{o}C. Figures 4.a,b show that the probe thermal properties can be accurately estimated with the present approach, if the materials properties are known. For errorless measurements, the iterative procedure of the Levenberg-Marquardt method converged to the exact parameters with initial guesses ranging from 0.1 to 10 in this case.

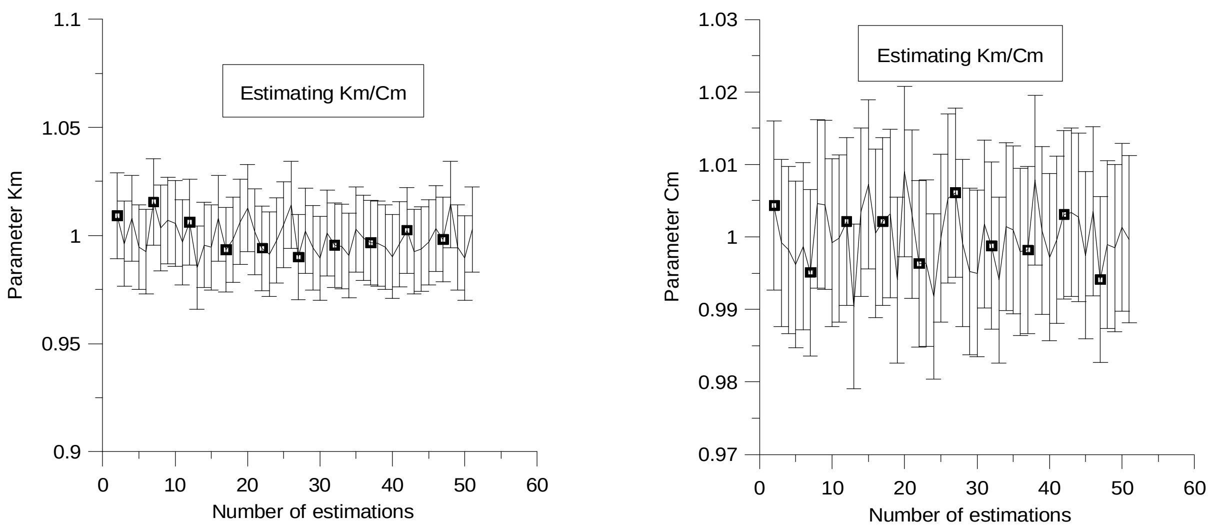
Similarly, figures 5.a,b present the results obtained for inverse problem type (ii), for 50 different runs with simulated measurements with different random measurement errors of standard deviation of 0.5 ^{o}C. As for inverse problem type (i), quite accurate results were obtained for the material properties when the probe properties are known. For errorless measurements, the iterative procedure of the Levenberg-Marquardt method converged to the exact parameters with initial guesses ranging from 0.1 to 3 in this case.

The joint rectangular confidence region, obtained from the multiplication of the 99% confidence intervals for the two parameters, for the results shown in figure 4 for inverse problem type (i) is 0.007. The joint rectangular confidence region for the results shown in figure 5 for inverse problem type (ii) is 0.0002. As expected from the analysis of the determinant of the information matrix, more accurate parameters were estimated for inverse problem type (ii) than for inverse problem type (i). We also examined the case where the inverse problem type (ii) was solved with non-optimal heating and final times. In this case, we have used the t_h=t_f=16 and the joint rectangular confidence region for the parameters was 0.0003, because of the lower value of the determinant of the information matrix.



(a) (b)

Figures 4. Results for inverse problem type (i)



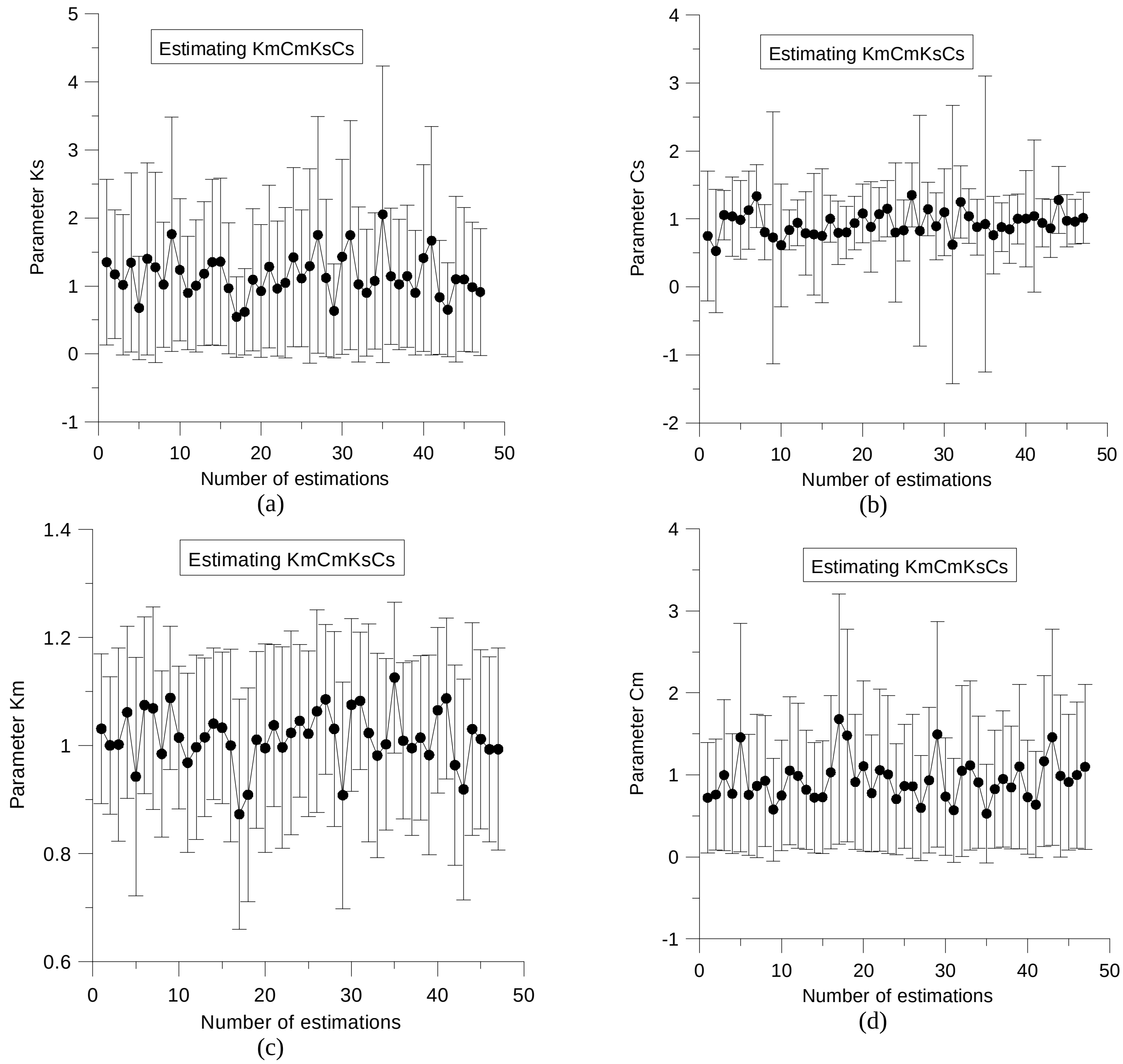
(a) (b)

Figures 5. Results for inverse problem type (ii)

The results obtained for inverse problem type (iii) are presented in figures 6.a-d, for 50 different runs of the inverse problem solution procedure, with different simulated measurements of standard deviation of 0.5 ^{o}C. The average estimated parameters and 99% confidence intervals, obtained with the 50 runs, are presented in table 1 for this case.

Differently from the results obtained for inverse problems (i) and (ii), the parameters estimated for this case had large confidence intervals. Such a fact was expected from the analysis of the determinant of the information matrix. On the other hand, the average values estimated for each of the parameters is in reasonable agreement with the exact ones. Therefore, inverse problem type (iii) can be used to generate low accuracy initial estimates for the properties of the probe and of the material. We note that the exact parameters were recovered with initial guesses ranging from 0.1 to 2, when errorless measurements were used in the inverse analysis.

The results presented in figures 4 to 6 were obtained with a measurement rate of 1 reading every 0.1 seconds.



Figures 6. Results for inverse problem type (iii)

Table 1. Results obtained for inverse problem type (iii) with optimal heating and final times of $t_h = 2.4$ and $t_f = 7.2$

Parameter	Estimated Value	99% Confidence Interval
K_s	1.2	2.0
C_s	0.9	3.9
K_m	1.0	0.2
C_m	0.9	1.3

7. Conclusions

In this paper we examined the solution of three different inverse parameter estimation problems, involved with the development of a line heat source probe. The parameter estimation problems were solved with the Levenberg-Marquardt method of minimization of the least-squares norm. The heating and final times were optimally identified through the analysis of the determinant of the information matrix, by using the D-optimum approach.

Results obtained with simulated data indicate the accuracy and stability of the present approach and, specially, the beneficial effects of appropriately designing the experiments. The effects of uncertainties on *a priori* assumed known parameters is currently under investigation, via the minimization of the *maximum a posteriori objective function* with the *sequential parameter estimation technique* (Beck and Arnold, 1977) Also, the results obtained with the use of actual measured data from the line heat source probe under development in the Laboratory of Heat Transmission and Technology of COPPE are being examined.

8. Acknowledgements

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9. Dedication

This work is dedicated to the memory of Prof. Roberto de Souza, who was a great enthusiast of this technique and started the development of the line heat source probe in the Laboratory of Heat Transmission and Technology of COPPE.

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