# SEMI-EMPIRICAL MODELING OF RADIAL TURBINE USED IN TURBOCHARGER

# Martins, G.

School of Mechanical Engineering, Federal University of Uberlândia Campus Santa Mônica – 38400-902. Uberlândia – MG – Brazil – gmartins@mecanica.ufu.br

### Hernandez, O.S.M.

School of Mechanical Engineering, Federal University of Uberlândia Campus Santa Mônica – 38400-902. Uberlândia – MG – Brazil – oscarhm@ mecanica.ufu.br

#### Julio, D. A. J.

School of Mechanical Engineering, Federal University of Uberlândia Campus Santa Mônica – 38400-902. Uberlândia – MG – Brazil – dorisjulio@ mecanica.ufu.br

#### Melo, C.M. T.

School of Mechanical Engineering, Federal University of Uberlândia Campus Santa Mônica – 38400-902. Uberlândia – MG – Brazil – dorisjulio@ mecanica.ufu.Br

#### Tumialán, J.A. B.

School of Mechanical Engineering, Federal University of Uberlândia Campus Santa Mônica – 38400-902. Uberlândia – MG – Brazil – jtumialan@ mecanica.ufu.br

**Abstract.** Industrial use of small size radial turbochargers like those used by utility vehicles, demands a real modeling of their behavior. This work, use the axial flow turbine model proposed by document ASHRAE TC 4.7, applying it to a Braziliam axial turbocharger. The model proposed by ASHRAE requires; identification of several parameters such as: incidence flow angle, impeller diameter, exit throat area. The ASHRAE model was tested using real curves behavior, given by Brazilian manufacturers. Our work shows that the identified parameters can not be constants when the ASHRAE model is applied to small radial turbines, and also our work show that the throat area can not be identified using just the chock flow condition, it needs a Mach number correction to represent with veracity the real behavior. It is proposed a new methodology to identify parameters that improves and gives good results for the HVAC component. Results obtained and physical explanations are offered for the new methodology of parameter identification presented in this paper.

Keywords: centrifugal turbine, Turbocharger, modelization, HVAC component.

# 1. Introduction

The use of radial turbochargers by Diesel engines it is a well known practice, due mainly to the engine power increase in the order of 20%. These turbo machines can also be used in small cogeneration plants, using the reverse Joule/Brayton cycle, as shown in Fig. (1),as well as for refrigeration pour poses ,due mainly to its high volumetric flow and low cost. For the reasons above, the use of these turbo machines demands studies related with modeling of these components.



Figure 1 Brayton/Joule reverse cycle schematic representation

The operation of turbo machines is based on a continuos changing on angular momentum, between the rotating element and the flow; this continuos change is responsible for the high volumetric capacity when compared with reciprocating machines. The volumetric flows obtained go from 200 m<sup>3</sup>/hr to 3.000 m<sup>3</sup>/hr, with isentropic efficiencies up to 85% and pressure ratio per stage limited to 3, with 100.000 rpm wheel rotation, Gigiel et all (2001). Centrifugal radial compressor convert dynamic pressure into static pressure by a diffusion process, because of continuos changes in the impeller diameter.

The validation of turbo compressors models is done based on operational curves given by manufacturers. These curves give a polynomial or graphical relationship between pressure ratio, isentropic efficiency and volumetric or mass flow rate. Black box models can be easily developed using data given by manufacturers, but its use is limited to their ranges and there is not confidence on its behavior when it is desired to predict operation outside or between these curves. More complex models Kelly et all (2003), Baccar et all (2001) and Ranade (1997), can be found in literature, based on Euler equation analysis applied to elementary mass flow elements, using numerical solutions of partial differential equations of fluid mechanics to describe the physical phenomenon.

This paper analyses the use of the semi-empirical model proposed by ASHRAE in document TC-4.7, which is recommended for axial turbines, the model technique used replace the axial turbine for a D'Laval wheel ,operating in chock flow conditions, this model needs identification of some physical parameters, which can be obtained from data given by manufacturers, problems with the parameter identification procedure are clearly explained below.

## 2. Semi-Empirical Model

The ASHRAE toolkit model proposed, says that centrifugal axial turbines operates in similar way to a chocked D'Laval wheel, as shown in Fig. 2.



Figure 2 D'Laval wheel schematic representation

The work performed in a Laval stage, according to Euler theorem is given by:

$$W_{sh} = C_{U1} \cdot U_1 - C_{U2} \cdot U_2 = U \cdot (C_{41} \cos(\alpha) - C_{42} \cos(\alpha))$$
(1)

Where

U - tangential expander velocity  $C_{41}$ -Inlet flow velocity  $C_{42}$ -outlet flow velocity  $\alpha_1$ - Incident angle with

 $U = \pi \cdot D \cdot N_2$ 

Where

*D* -expander Diameter *N* -Wheel rotating speed From velocity triangle

 $C_{41} \cdot \cos(\alpha_1) = U + W \cdot \cos(\beta_1)$ 

where

W - relative velocity  $\beta_1$  - inlet blade angle  $\beta_2 = \pi - \beta_1$  - outlet blade angle from where:

 $W_{Sh} = 2 \cdot U \cdot W \cdot \cos(\beta_1) = 2 \cdot U \cdot (C_{41} \cdot \cos(\alpha_1) - U)$ (3)

Velocity  $C_{41}$  shown in fig (1), can be calculated from thermodynamic relationship for isentropic expansion

$$C_{41} = \sqrt{2 \cdot Cp \cdot Tsu \cdot \left[1 - \left(\frac{Pex}{Psu}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$
(4)

Where

 $c_p$  - specific heat  $T_{su}$  - Supply temperature

 $p_{ex}$ ,  $p_{su}$  - pressure ratio

 $\gamma$  - specific heat ratio

The isentropic effectiveness ( $\eta$ ) can be calculated as follows:

$$\eta = \frac{W_{ad}}{w_{sh}} = \frac{2 \cdot U \cdot (C_{41} \cdot \cos(\alpha_1) - U)}{\frac{1}{2} \cdot C_{41}^2}$$
(5)

The mass flow rate is defined as function of nozzle throat area  $(A_{thr})$ , using compressible flow analysis and nozzle flow rate area  $(A_{Thr})$ 

$$\dot{M}Ar \cdot \frac{\sqrt{Tsu}}{Psu} = \frac{A_{Thr}}{\sqrt{r}} \cdot \sqrt{\frac{2 \cdot \gamma}{\gamma - 1}} \cdot (\pi_{E2})^{\frac{1}{\gamma}} \cdot \sqrt{1 - (\pi_{E2})^{\frac{\gamma - 1}{\gamma}}}$$
(6)

Where

 $P_{su}$  - Supply pressure r - Particular gas constant

$$\pi_{E2}$$
 - pressure ratio given by;  $\pi_{E2} = Max \left( \frac{Pex}{Psu}; \pi_{critical} \right)$ 

where the critical pressure ratio is given by ;  $\pi_{critical} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ 

From equation (5) we can observe that maximum efficiency is obtained when:

$$\eta_{\rm max} = \cos^2(\alpha_1) \tag{7}$$

The use of above model requires the ID of following parameters : Laval wheel diameter (D), nozzle throat ( $A_{Thr}$ ) area, and incident angle  $\alpha_1$ ; this can be accomplish through the appropriate use of curves given by manufacturers, as shown below

# 2.1 ID of parameters

- The incident angle  $\alpha_1$  is obtained with the maximum efficiency curve, which is considered the nominal or design condition using Eq. (7)
- The throat area is obtained, accepting that the nozzle works in chock condition (Mach=1.0), in this case the Eq. (6) is valid
- The Laval wheel diameter is found using the N speed and Eq.(1) and Eq. (2)

The model above described is recommended for axial turbines, although in this work, this model with some small changes can be easily used for small radial turbines.

# 2.2 Radial turbine model

It was observed, that the model proposed for axial turbines when applied to radial turbines showed a lack of precision, parameters were not constant, and near the chocked region the behavior was unstable, when the nozzle throat area was considered constant.

The new parameter proposed is the use of a corrected nozzle mass flow rate as shown below:

$$\dot{M}_{AR} = Cd \cdot A_f \cdot \sqrt{\rho \cdot Psu \cdot \left(1 - \frac{1}{RT}\right)}$$
(8)

Where

 $\dot{M}_{AR}$  -Mass flow rate

Cd -Flow discharge factor

 $A_f$  -Corrected nozzle throat area

 $\rho$  - Density

RT - Pressure ratio

The same equation can be applied to a chocked nozzle as before, using the critical pressure ratio.

$$\dot{M}_{AR,crit} = Cd \cdot A_{Thr} \cdot \sqrt{\rho_{crit} \cdot Psu \cdot (1 - \pi_{critical})}$$
(9)

Where:

 $\dot{M}_{AR\ crit} = \dot{M}_{AR}$  $A_{Thr}$  - fictitious nozzle throat area  $\rho_{crit}$  - Critical density  $\pi_{critical}$  - Critical pressure ratio

Combination of Eq. (8) and (9), followed by a previous determination of a average critical throat area  $A_{Thr}$ , allows the determination of a fictitious area ( $A_f$ ), for the nozzle to be used in radial turbine model.

$$A_{f} = \overline{A}_{Thr} \cdot \sqrt{\frac{1 - \pi_{critical}}{Cd \cdot \left(1 - \frac{1}{Rt}\right)}}$$
(10)  
where:

 $Cd = \left(\frac{4}{\gamma+1} + 2 \cdot \frac{\gamma-1}{\gamma+1} \cdot Ma\right)^{\frac{1}{1-\gamma}}$  $M_a$  =Mach number

The parameter identification procedure required the development of two computational codes in FORTRAN language; One code to identify parameters, using curves given by manufacturers and the other to simulate the turbine model. Fig. (3) show the flowchart of the two codes developed by authors.





Figure 3 Flowchart of simulation codes

#### 2.2 Results and Discussions

Initially the model proposed by ASHRAE was evaluated; results are shown in Fig (4) (a),(b)



CHARACTERISTIC CURVE

It is observed in Figure (4) (a) that at 31312 rpm, wheel rotation, for a given mass flow rate, the difference in pressure ratio is less than 3%, when the rotation is increased to 53200 rpm, this difference goes up to 25%, it was observed that around 53200 rpm, the pressure ratio was close to the chock pressure ratio.

In order to get better results, it was introduced the new nozzle throat area  $(A_f)$  as new parameter instead of  $(A_{Thr})$ 

constant area used before, the obtained results are shown in Fig. (5),(a),(b)

Figure 4. ASHRAE model results

Caracteristic Curve



4 31312-Rea 43642-Real 43642-Mode 53200-Rea 61089-Rea 61089-Model 53200-Mode 67543-Rea -67543-Mode 73425-Real 79578-Real - 85188-Real 85188-Mode × △ 31312-erro × 43642-error 53200-error - 61089-error + 67543-error 73425-error × 79578-erro 85188-error ۰ (b)

•₊q;

1.80E+00

2.00E+00

Pressure ratio

2.20E+00

2.40E+00

2.60E+00

0.00%

3.00E+00

2.80E+00

Figure 5. Model behavior with corrections

1.20E+00

1.40E+00

1.60E+00

As seen in figures (5a,5b) the differences in pressure ratio estimation ,observed with the corrected model are always below 4% ,and when efficiency is estimated the differences observed are less than 2.5% in the whole range of rotation velocities.

# 3. Conclusions

0.00E+00

1,00E+00

The ASHRAE model, seems to be adequate for axial turbines, as shown in ASHRAE document TC-4.7, using pressure ratios bigger than 3.0. The throat area correction recommended in this work gives good results, for small Diesel engines radial turbines working with pressure ratios less than 3.0

The radial turbine model proposed, is very simple to implement, the parameter ID procedure is not difficult to follow and we strongly recommend the use of this model to explore applications of small radial turbines in non conventional air cooling systems.

### 5. References

Balje, O.E., 1981, "Turbo machines, A Guide to Design Selection, and Theory" Ed. Wiley Interscience

Cohen, H., Rogers, G.F.C., Saravanamuto, H.I.H, 1996, "Gas Turbine Theory", Ed. Longmam

"HVAC Systems and Equipment", ASHRAE HANBOOK 2000, SI Edition

Lebrun, J. Bourdouxhe, J. P., Grodent, M., 1994 "A Toolkit for Primary HVAC System Energy Calculation", TC 4.7 Energy Calculation, ASHRAE HANBOOK

Gigiel, A.J., Granryd, E., Novak, P., Nowacki, A.E., Stadtländer, C., Verschoor, M.J.E., 2001, "Guide for the Application and Design of Air Cycle Systems for Heating, Ventilating, and Air Conditioning in Buildings", Handbook TNO Novembro 2001

Kelly,W., Gigas, B., 2003, "Using CFD to Predict the Behavior of Power Law Fluids near Axial-flow Impellers Operating in the Transitional Flow Regime", Chemical Engineering Science, Pergamon, 58 (2003) 2141-2152

Baccar, M., Mseddi, M., Abid S.M., 2001"Contribution Numérique à L'étude Hydrodynamique et Thermique des Écoulements Turbulents Induits par une Turbine Radiale en Cuve Agitée", Int J. Therm. Sci (2001) 40,753-772

Ranade, V. V., 1997, "An efficient computational model for simulating flow in stirred vessels: a case of Rushton turbine", Chemical Engineering Science, Pergamon, Vol. 32, No 24. pp. 44773 4484

Mseddi, M., Baccar, M., Kchaou, H., Abid, M.S., 2000, "Modelisation des Turbines Radiales de Suralimentation", Mécanique & Industries

Struchtrup, H., Rosen, M.A.,2001, "How Much Work is Lost in a Irreversible Turbine?", Exergy Dixoir, S., 1967, "Fluid Mechanics, Thermodynamics of Turbo machinery", Oxford, Pergamon Press