TRANSIENT INVERSE DESIGN OF RADIATIVE ENCLOSURES

Francis H. R. França
Department of Mechanical Engineering
Federal University of Rio Grande do Sul
Porto Alegre, RS, Brazil
frfranca@mecanica.ufrgs.br

Abstract. *This work investigates time-dependent inverse boundary design of radiative enclosures. The problem consists of finding the heat flux distribution on the heater located on the top of a two-dimensional enclosure that satisfies both the temperature and the heat flux prescribed on the design surface of the enclosure, which are time-dependent. The problem examines a two-dimensional rectangular enclosure formed by diffuse, gray surfaces. All the thermal properties are constant in time. The inverse analysis is described by a system of linear equations that is expected to be ill-conditioned since it involves the solution of a Fredholm integral equation of the first kind. To tackle the ill-conditioned system of equation, the TSVD (Truncated Singular Value Decomposition) regularization method. In addition to presenting a methodology to solve a transient inverse design, this work investigates the effect of the thermal capacity of the insulated wall in the enclosure. While the thermal capacity of the insulated surface is not relevant in steady state inverse design, it must be taken into account in transient conditions, since the heating of the insulated wall itself will affect the amount of heat input required in the heater to satisfy the two conditions in the design surface.*

Keywords. *Inverse analysis, thermal design, radiative enclosure, thermal processing., transient process.*

1. Introduction

Many industrial processes require controlled heating of materials, in which two thermal conditions are imposed on the surfaces of the materials. The thermal designer attempts to find the conditions on the unconstrained elements of the system such that the two specifications are satisfied. For instance, in the metallurgy field, a major issue in the design of furnaces and ovens is the guarantee of uniform heating during thermal processing of metals. In the semiconductor industry, the rapid thermal processing of silicon wafers requires the temperature of the wafer to be uniform during the heating. In addition, the heating curve, that is, the time-dependent temperature distribution, is also specified, as has been described in a number of recent papers (Choi and Do, 2001; Huang et al., 2000; Balakrishnan and Edgar, 2000; Fan and Qiu, 1998). Such conditions can be achieved only by means of carefully controlled heat flux on the surface of the processed material, so both the time-dependent temperature and heat flux are prescribed in the thermal process.

In the conventional design of such systems, namely *forward design*, where the mathematical formulation relies on the knowledge of one and just one condition on each element of the system, the designer needs to guess one thermal condition in the unconstrained elements and, for the imposed temperature on the process material, solve the corresponding heat flux. If it is not uniform, a new guess is made, and the calculations are rerun. This trial-and-error procedure can be cumbersome to deal with, and a great number of iterations may be necessary to achieve a satisfactory configuration, which can be especially undesirable in transient processes, the inverse problem needs to be solved for every instant of time.

Inverse design aims at finding the conditions in the unconstrained elements directly from the two specifications on the design surface, so it avoids the trial-and-error procedure of forward design. Inverse mathematical models allow some surfaces to have two prescribed boundary conditions, as well as some elements to have no prescribed condition at all. For problems that involve thermal radiation heat transfer, this type of formulation is described by a Fredholm integral equation of the first kind, known to result in an ill-posed problem, and can be solved only by means of regularization methods (Hansen, 1998). The matrix of coefficients presents singular values that decay continuously and steeply, and conventional matrix solvers will probably fail to provide a useful solution. The system must be regularized.

The research undertaken on inverse design can be divided into two classes. The first includes the inverse boundary problem, in which the conditions on some of the boundaries are sought to satisfy the specifications on the design surfaces; the second includes the inverse heat source problem, where the heat source distribution in the unconstrained medium is to be found. A comprehensive review of steady state transient design is presented in França et all (2002). The inverse design approach is applied to a number of different problems and conditions, including cases involving combined radiation, convection and conduction heat transfer. Recently, Ertürk et al. (2002) considered the transient inverse design employing the Monte Carlo method to solve the radiative heat exchanges in the enclosure, while the conjugate gradient regularization method was employed to solve the system of equations.

This paper considers a transient inverse design of a two-dimensional rectangular enclosure. The objective is to find the time-dependent heat input heater located on the upper wall that is capable of satisfying the specified heating history of the design surface, which is located on the bottom of the enclosure. All physical properties are assumed constant, and all the surfaces that form the enclosure are gray emitters and absorbers. A zonal type formulation is applied for the discretization of the radiative terms of the energy equation. The resulting system of equations is expected to be ill-conditioned due to the inverse approach that is involved. The set of equations is solved by first relating the known temperature and heat flux of the elements on the design surface directly to the unknown radiosity of the elements on the heater. This requires an iterative solution, since the effect of the insulated walls needs to be assumed and corrected The ill-conditioned nature of the system is treated by means of the truncated singular value decomposition (TSVD).

2. Physical and mathematical modeling

Figure 1 shows a schematic view of a two-dimensional rectangular enclosure having length L and height H. The material to be thermally processed is placed on the bottom of the enclosure, and is treated as a section of the bottom surface, being indicated as *design surface*. The heater, which must provide the thermal energy to the design surface, is located on the top of the enclosure. The lengths of the design and of the heater surfaces are L_D and L_H, respectively. The remaining surfaces of the enclosure are assumed to be insulated to the outside environment. All surfaces are diffuse, gray emitters and absorbers; the total hemispherical emissivities of the design, heater and insulated surfaces are indicated by ε_D, ε_H and ε_S, rerspectively. The present formulation concerns enclosures that are either evacuated or filled with a non-participating medium, so that the thermal energy is transported solely by surface thermal radiation. All the physical properties are assumed to be constant. The present case considers controlled heating of the desing surface, in which both the temperature and the heat flux are simultaneously imposed. The inverse design consists of finding the time-dependent heat input in the heater so that both the temperature and the heat flux on the design surface are attained at any instant of the heating process. When the thermal capacitance of the insulated walls are not negligible, the evaluation of the heat input also requires accounting for the energy that the insulated walls absorb during the heating process.

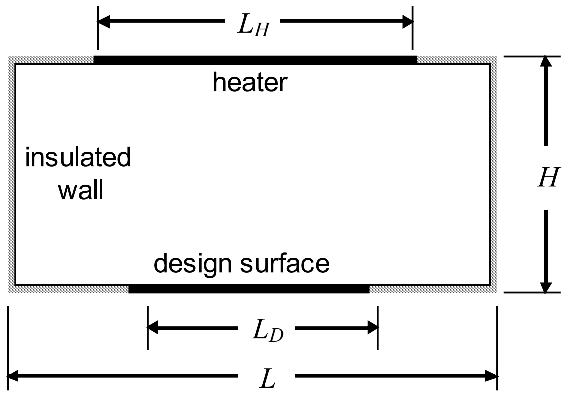

Fig. 1 Two-dimensional radiative enclosure

Consider that besides being uniform, the temperature on the design surface is required to obey a given time history. This condition can be achieved only by means of a carefully controlled heat flux on each element that forms the desing surface. Consider an infinitesimal element *j* at the design surface. The net radiative heat flux on this element, a result of the heat transfer with the heater and the insulated surfaces, must balance both the increase in the internal energy of the element and the heat dissipation to the outside of the enclosure. That is,

$$-q_{r,j} = C_D \frac{dT_j}{d\mathrm{T}} + \frac{(T_j - T_\infty)}{R} \tag{1}$$

where C_D represents the thermal capacity of the design surface per unit of area, T_j is the temperature of the design surface element, T_∞ is the outside temperature, T is time, and R is the equivalent thermal resistance of the material. In the above relation, it was assumed the lumped capacitance model for the design surface. While this may represent a strong simplification for most cases, what matters here is to establish a relation between the heat imposed and the temperature of the design surface. A more detailed analysis will be necessary to account for internal temperature distribution in the design surface, but that will not cause a change in the inverse design methodology, which is the purpose of this paper. The negative signal for $q_{r,j}$ in Eq. (1) is to be consistent with the convention that net heat flux is directed outward the surface element. Therefore, if the element receives heat, which is the case considered here, $q_{r,j}$ should be negative, and the first member becomes positive.

In dimensionless form, Eq. (1) becomes:

$$-Q_r = \frac{dt}{d\tau} + \frac{(t - t_\infty)}{r} \tag{2}$$

where the dimensionless terms are defined as: $Q_r = q_r / \sigma T_{ref}^4$, $t = T / T_{ref}$, $\tau = \mathrm{T}/(C_D \sigma T_{ref}^3)$, and $r = R\sigma T_{ref}^3$.

Consider the following temperature profile on the design surface:

$$t = 1.0 - 0.5 \times \exp(-4.6\tau) \tag{3}$$

According to the above equation, the temperature is required to increase exponentially from an initial value of $t = 0.5$ to $t = 1.0$ in a total dimensionless time of $\tau = 1.0$, then remaining constant. For this condition to be achieved, the necessary heat flux on the design surface can be found by simply solving Eq. (2) based on the time-dependent temperature history of Eq. (3). This results in:

$$-Q_r = 2.3 \times \exp(-4.6\tau) + 32 \times (t - t_\infty) \tag{4}$$

Once the heating on the design surface is characterized, the thermal designer must specify the heat input in the heater so that both conditions are satisfied on the design surface for every instant of time. Although the temperature and the heat flux are uniform on the design surface, one should not expect an uniform heat load in the heater to satisfy those conditions. When the heater and the design surface are located in the center of the top and bottom surfaces, as in the example shown in Fig. 1, the region in the center of the design surface is more affected by the heater than the elements on the ends, where the effect of the side walls become more important. To compensate the effect of the side walls on the ends of the design surface, it is likely that a higher heat input in the heater will be necessary in the positions facing the ends of the design surface.

To solve for the needed heat input in the heater, an inverse analysis of the radiative heat exchanges in the enclosure is applied. The mathematical model relies on the radiative energy conservation (Siegel and Howell, 2002). The net radiative heat flux on an infinitesimal element of the design surface, which must be equal to the imposed heat flux on the design surface element j, is given by a balance between the income and the outcome radiative heat fluxes, given by:

$$Q_{r,j} = Q_{o,j} - Q_{i,j} \tag{5}$$

The indexes r, o and i stand for net, outgoing and income radiative heat fluxes. The income radiative heat flux accounts for the incident radiation on the element of the design surface, considering both emission and reflection form the other surface elements of the enclosure. The outgoing radiative heat flux takes into account both the emission and the reflection from the element in the design surface.

To establish relations for the income and outgoing radiative heat fluxes, a numerical approach is applied. The design, the heater and the insulated surfaces are divided into a finite number of elements, designated by indexes j, k and l respectively, where all thermal properties and quantities can be assumed constant. The outgoing radiative heat flux of a design surface element j, Q_{oj}, can be written as a function of the temperature and net heat flux according to:

$$Q_{o,j} = t_j - \frac{1-\varepsilon_D}{\varepsilon_D} Q_{r,j} \tag{6}$$

Since in the design surface both the temperature and heat flux are known, the outgoing radiative heat flux can be readily obtained from Eq. (5). Next step is relating the income radiative heat flux on element j to the outgoing radiative heat fluxes from the other elements in the enclosure, leading to:

$$Q_{i,j} = \sum_k F_{jk} Q_{o,k} + \sum_l F_{jl} Q_{o,l} \tag{7}$$

where F_{jk} and F_{jl} are the view factor between the elements j and k, and between elements j and l, respectively.

Combining Eqs. (5) and (7) results in:

$$Q_{o,j} - Q_{r,j} = \sum_k F_{jk} Q_{o,k} + \sum_l F_{jl} Q_{o,l} \tag{8}$$

The left-hand side of Eq. (8) is known, since both the outgoing and the net radiative heat fluxes on the design surface element j are already known. The unknowns are the outgoing radiative heat fluxes on the remaining elements of the enclosure, located in the heater and in the insulated walls.

In a next step, the energy balance can be applied to the insulated surfaces. The imposed condition on these surfaces is that there is no heat loss to the outside of the enclosure. Concerning the thermal response of the insulated surfaces, two cases are considered in this work. The first one is the ideal case in which the thermal capacity of the insulated surface is negligible in comparison to that of the design surface. In this way, the net radiative heat flux will be always zero, so that the outgoing and the income radiative heat fluxes are the same in any element l in the insulated wall:

$$Q_{o,l} = Q_{i,l} \tag{9}$$

In the second case, it is considered that the thermal capacity of the insulated wall is not negligible. Therefore, it absorbs part of the energy generated in the heaters, and so the net radiative heat flux is not zero. To account this effect, the energy balance can be first applied to an insulated wall element l to relate its outgoing radiative heat flux to its temperature and the income radiative heat flux:

$$Q_{o,l} = \varepsilon_S t_j^4 + (1-\varepsilon_S) Q_{i,l} \tag{10}$$

Applying Eq. (10) is convenient because the temperature distribution in the insulated surface can be followed during the heating, and so it is known in every moment of the process. In fact, the temperature can be found from the balance of energy:

$$-Q_{r,l} = Q_{i,l} - Q_{o,l} = C_S \frac{dt_l}{d\tau} \tag{11}$$

where C_S is the ratio between the thermal capacity of the insulated wall and that of the design surface. Again, the negative for Q_r arises from the convention that the direction of net radiative heat flux is outward with respect to the surface. Completing the needed relations for the insulated wall, the income radiative heat flux on the insulated surface element is given by:

$$Q_{i,l} = \sum_j F_{lj} Q_{o,j} + \sum_k F_{lk} Q_{o,k} + \sum_{l'} F_{ll'} Q_{o,l'} \tag{12}$$

The last term of the above equation accounts for the irradiation on the insulated surface element that originates in other insulated surface elements (indexed by l'), since the view factors between the elements on the insulated surface may not be null (Fig. 1).

Finally, the net radiative heat flux on a heater element k is given by the balance between the outcome and the income radiative heat fluxes:

$$Q_{r,k} = Q_{o,k} - \sum_j F_{kj} Q_{o,j} - \sum_l F_{kl} Q_{o,l} \tag{13}$$

The procedure to determine the net radiative heat flux on the heater, the goal of the inverse analysis, is discussed next.

3. Solution procedure

Based on the procedure proposed in França et al. (2001 and 2002), the following approach is adopted. The outgoing radiative heat flux on the elements located in the insulated walls are initially neglected in Eq. (8). Therefore, the unknowns in Eq. (8) are only the outgoing radiative heat fluxes on the heater, $Q_{o,k}$. Once Eq. (8) is written for each of the N_D elements that form the design surface, a system with N_D equations will be formed. The unknowns are the outgoing radiative heat fluxes on the N_H elements that form the heater. Therefore, one aspect of the inverse design arises: the number of equations and the number of unknowns are not necessarily the same, unless $N_D = N_H$. In addition, since the problem corresponds to a discrete form of a integral equation of the first-kind, one should expect the system of equations to be ill-conditioned. In most cases, such systems cannot be solved by standard matricial solvers, but rather should be treated by the so called regularization methods, like the Tikhonov method, the conjugate gradient, the truncated singular value decomposition (TSVD), the modified TSVD, among others. The chosen method of solution, TSVD, is discussed in the next section, which is a well explored method to deal with ill-conditoned non-square system of equations.

Although a system of equations can be assembled in such a way to include the equations for the elements on the insulated wall, an aspect of interest in the aforementioned procedure is that the elements in the design surface, where two conditions are imposed, are directly linked to the elements in the heater, which are left unconstrained. In other words, the ill-posed part of the problem is separated from the remaining of the problem, allowing an isolated application of the appropriate methods of solution of ill-posed system of equations.

The solution is achieve by means of the following procedure. For a given instant of time, the temperature, the net and the outgoing radiative heat flux on the design surface element j are computed by means of the specified heating histories, Eqs. (3) and (4), and by Eq. (6), respectively. As discussed above, the outgoing radiative heat flux on the insulted wall elements, $Q_{o,l}$, are initially set equal to zero. Then, the system of equations formed by Eqs. (8) is solved for the outgoing radiative heat fluxes on the heaters, $Q_{o,k}$. After so, Eq. (12) is applied to estimate the income radiative heat flux on the insulated walls, $Q_{i,l}$. For the case where the thermal capacity of the insulated wall is neglected (that is, when $C_S = 0$), Eq. (9) is invoked to estimate $Q_{o,l}$. For the case when the thermal capacity of the insulated wall cannot be neglected, then $Q_{o,l}$ is estimated by Eq. (10). The newly calculated $Q_{o,l}$ are inserted into Eqs. (8), and the system is solved again for the outgoing radiative heat flux on the heater, $Q_{o,k}$. The procedure is repeated until convergence is achieved.

Finally, Eq. (13) is used to find the net radiative heat flux on the heater, $Q_{r,k}$, which is the required energy input to provide the two conditions on the design surface at a given instant of time. For the next time step, new values of the design surface temperature and net radiative heat flux are computed again by means of Eqs. (3) and (4), and the above procedure is repeated to find the required heat input on the heater. Concerning the two cases for the behavior of the insulated wall, an important difference arises. When its thermal capacity cannot not be neglected, the temperature of a given element on the insulated wall is computed from a discrete form of Eq. (11):

$$t_l^{m+1} = t_l^m + \frac{\Delta\tau}{C_s}(Q_{i,l}^m - Q_{o,l}^m) \tag{14}$$

where $\Delta\tau$ is the time step, and the superscript m indicate the time step. Because the knowledge of the insulated wall temperature distribution is required to solve for the cases in which $C_S > 0.0$, it is necessary marching on time. For the other case, in which $C_S = 0.0$, the condition that $Q_{r,l} = 0.0$ is valid in every instant of time. For that reason, to solve for the necessary heat input on the heater, it is only necessary to know what are the required temperature and heat flux on the design surface, as given by Eqs. (3) and (4); marching on time is not necessary for this case.

3.1 Regularization of the system of equations

The procedure discussed above involves the solution of a system of linear equations on the radiosities of the heater elements, which can be represented by:

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}(\mathbf{x}) \tag{15}$$

where the coefficient matrix **A** is formed by the view factors between the elements on the heater and the design surface, vector **x** represents the unknown radiosities of the heater elements, and vector **b** contains the terms guessed in the previous iteration, being dependent on **x**.

The system of Eq. (15) is the discretized form of a set of equations that includes Fredholm integral equations of the first kind, and so it is expected to present the characteristics of ill-posed problems. In general, the components of the exact solution vector **x** present steep oscillations with very large positive and negative numbers, and small perturbations cause a much amplified change in the solution. The solution of ill-posed problems does not aim at an exact solution, but rather to impose additional constraints to reduce the size (norm) of **x,** and achieve a smooth solution. However, the greater the smoothness imposed on the solution, the greater will be the residual. This is the basic idea behind the regularization methods for the solution of ill-posed problems.

Among the regularization procedures, the truncated singular values decomposition (TSVD) is employed here. First, the matrix **A** is decomposed into three matrices:

$$\mathbf{A} = \mathbf{U} \cdot \mathbf{W} \cdot \mathbf{V}^T \tag{16}$$

U and **V** are orthogonal matrices, and **W** is a diagonal matrix formed by the singular values w_j. The solution vector **x** can be computed by:

$$\mathbf{x} = \sum_{j=1}^{N} \left(\frac{b_k \cdot u_{kj}}{w_j} \right) \mathbf{v}_j \tag{17}$$

where N stands for the number of unknowns.

In ill-posed problems, the singular values w decay continuously to very small values. As they are in the denominator of Eq. (14), the components of **x** can present very large absolute numbers. However, the smaller the singular value w_j is, the closer the corresponding vector $\mathbf{v}_j$ is to the null-space of **A**. In other words, the terms related to the smaller singular values can be eliminated from Eq. (14) without introducing a large error to the solution. This is the principle idea of the TSVD: only the terms related to the p-th largest singular values are kept on Eq. (14), instead of all N terms. The solution is the vector **x** with the smallest norm subjected to minimum deviation $|\mathbf{A} \cdot \mathbf{x} - \mathbf{b} \cdot (\mathbf{x})|$.

Another aspect of the inverse solution concerns the number of unknowns and the number of equations. Equation (8) is set for all the elements on the design surface, while each element on the heater contains one unknown radiosity. Therefore, the number of equations equals the number of unknowns only when the numbers of elements on the design surface and heater are the same. This is not always the case. An important feature of the TSVD method is that it can also be applied to the case where the numbers of unknowns and equations are not the same, as will be shown in the results section.

3.2 Verification of the solution

Due to the need for regularization of the system of equations, an exact solution is not expected. The following procedure is used for the verification of the solution. Once the heat flux on the heater is obtained, a forward problem is solved where the heat flux on heater is known, and only the temperature on the design surface is imposed. The heat flux on each element k of the design surface is then calculated, and compared to the specified heat flux by:

$$\gamma_j = \left| \frac{Q_{r,j,specified} - Q_{r,j,inverse}}{Q_{r,j,specified}} \right| \tag{18}$$

where $Q_{r,j,specified}$ is the specified heat flux, and $Q_{r,j,inverse}$ is the heat flux resulting from the conditions on the heater that were obtained in the inverse solution. Once γ_j is calculated for each element j in the design surface, the maximum and arithmetic average errors γ_{max} and γ_{avg} can be readily found.

4. Results and discussion

To illustrate the proposed transient inverse methodology, the following example is considered. The aspect ratio of the two-dimensional enclosure shown in Fig. 1 is $H/L = 0.2$. The design surface and the heater are located in the center of the bottom and top surfaces, respectively. The dimensionless length of the design surface is $L_D/L = 0.6$, while the length of the heater is allowed to vary. The emissivities of the design, heater and insulated wall surfaces are all equal to $\varepsilon_D = \varepsilon_H = \varepsilon_S = 0.8$. The specified temperature profile on the design surface is given by Eq. (3), in which the dimensionless temperature of the surface is required to increase exponentially from 0.5 to 1.0 in a dimensionless period of 1.0, then remaining constant. Assuming that the outside environment is at a dimensionless temperature of $t_\infty = 0.5$, the required heat flux on the design surface is $Q_r = -16.0$ when the steady state is reached ($\tau > 1.0$). The negative value indicates that the heat flux direction is towards the design surface, to balance the heat loss to the environment.

To better understand some aspects of the inverse design, it is first considered the steady state condition. The conditions in any element j of the design surface are $t_j = 1.0$ and $Q_{r,j} = -16.0$, while for the insulated wall element $Q_{r,l} = 0.0$, no matter if its thermal capacity is neglected or not. The heater is unspecified, and the inverse design aims at finding its heat flux, $Q_{r,k}$. Consider first that the length of the heater is the same of the design surface, $L_H/L = L_D/L = 0.6$. Considering that all the enclosure surfaces are divided into zones of uniform size of $\Delta x/L = 0.1$, the number of elements in the design surface and in the heater are the same, $N_D = N_H = 30$, leading to a system of linear equations [formed by Eqs.(8)] with the same number of unknowns and equations. However, employing standard matrix solvers will lead to unrealistic solutions; that is, the outgoing radiative heat fluxes on the heater $Q_{o,k}$ present steep oscillations between very large absolute values with alternating signals. This can be explained with the aid of Fig. 2, which presents the singular values of matrix **A**. For this first case, labeled in the figure with LH = 0.6, the singular values decayed to very small numbers. Inserting those values into the denominator of Eq. (17) led to very large terms, which explains why the solution of ill-posed problems were plagued by undesirable, steep oscillations between large numbers. Following the procedure described in Section 3.1, the TSVD regularization was applied to the series of Eq. (17), keeping only the first p terms related to the p largest singular values (instead of all the N terms).

Figure 3 presents the solutions for regularization parameters of p = 2, 4 and 6. (For p larger than 6, the solution became unrealistic with the presence of some negative values of $Q_{o,k}$.) As seen in the figure, choosing different values of the regularization parameter led to different answers for the heat input on the heater. Since any regularized solution is an approximation (the smaller the p, the larger the expected error of the solution), the inverse solutions were verified according to the proposition in Section 3.2. For the three solutions, the error γ_j was computed for all the elements j of the design surface, and then the maximum and the average errors were found. For p = 2, 4 and 6, respectively, it was found that: γ_{max} = 21.7 % and γ_{avg} = 9.21 %,γ_{max} = 6.25 % and γ_{avg} = 2.08 %, and γ_{max} = 1.82 % and γ_{avg} = 0.624 %. Therefore, if the maximum acceptable error is1.0 %, then none of the three solutions in Fig. 3 satisfies the two specifications in the design surface. This illustrates the fact that an inverse design may not present any solution that satisfies the problem within a specified error.

The inverse design can be further explored by increasing the length of the heater, since an inspection of the solutions in the Fig. 3 revealed that the maximum error occurred in the ends of the design surface, most likely due to the effect of the insulated walls. Consider that L_H/L = 0.8. Keeping the same grid mesh, the number of unknown $Q_{o,k}$´s became N_H = 40, while the number of equations remained equal to N_D = 30. The problem is underspecified, but the TVSD method can still be applied without any additional modification. The singular values of matrix **A** are presented in Fig. 3 under the label of LH = 0.8. As seen, the singular values presented the same behavior of the cases in which LH = 0.6. The only difference is that, for the present case, ten singular values were equal to zero (corresponding to the difference between N_H and N_D), and were not included in Fig. 3. Applying the TSVD method with p = 6, 8 and 10 led to the heat input on the heaters shown in Fig. 4. In all the three cases, the maximum error γ_{max} were less than 1.0 %, and therefore they all satisfy the steady state inverse design in which t_j = 1.0 and $Q_{r,j}$ = -16.0. This fact illustrates the other side of the inverse design: while in some cases no solution can be found, in others a number of different solutions can satisfy the problem within a specified error.

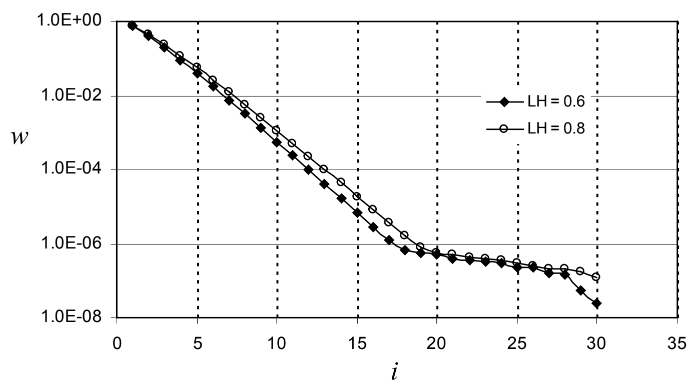

Figure 2. Singular values of matrix **A**.

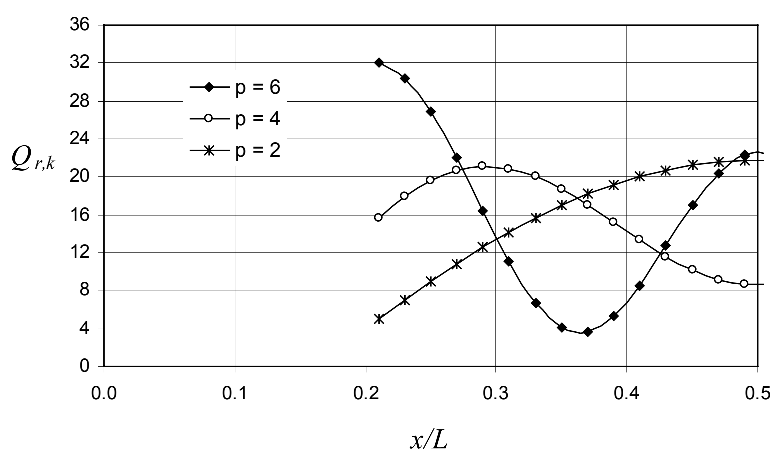

Figure 3. Heat flux distributions on the heater for different values of the regularization parameter p (LH = 0.6).

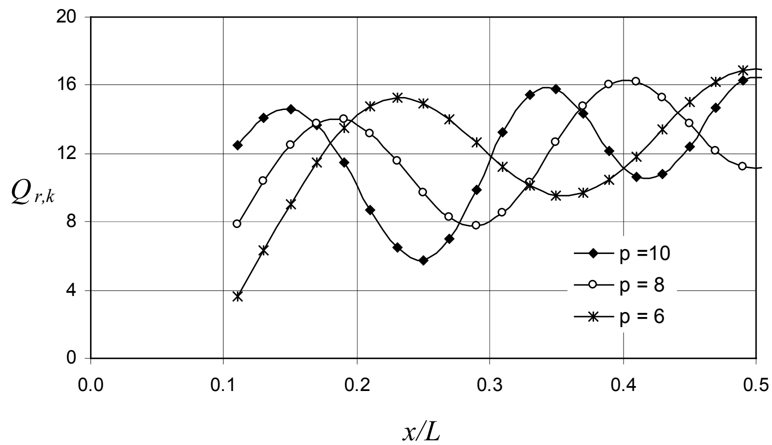

Figure 4. Heat flux distributions on the heater for different values of the regularization parameter p (LH = 0.8).

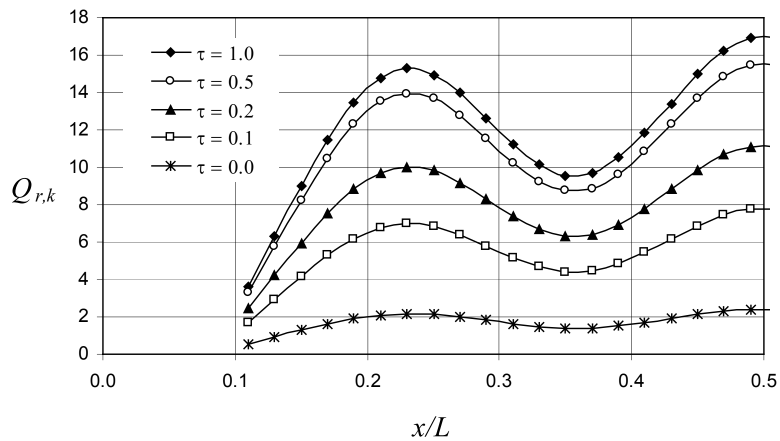

Figure 5. Heat flux distributions on the heater for different instants of time τ (LH = 0.8, p = 6, C_S = 0.0).

The next step is tackling the transient inverse design. Consider first the case in which the thermal capacity of the insulated wall is negligible (C_S = 0.0). From the previous discussion, the heat input in the heater at any instant of time can be found by simply determining, from Eqs. (3) and (4), the prescribed temperature and heat flux on the design surface for the given instant of time, and then applying the inverse solution as if the problem were in steady state regime. Since the heater with a dimensionless length of L_H/L = 0.8 proved capable of satisfying the two conditions on the design surface, this length was selected for the transient problem. In addition, since the steady state solution with a regularization parameter of p = 6 was sufficiently accurate (γ_{max} less than 1.0 %), this parameter was selected for the transient problem. Figure 5 presents the heat input in the heater for dimensionless times of τ = 0.0, 0.1, 0.2, 0.5 and 1.0. In all cases, the maximum error of the inverse solution was verified to be less than 1.0 %, so they are all considered to be satisfactory. As seen in the figure, the heat input distribution in the heater keeps a similar profile for every instant of time.

A more interesting and practical situation is that in which the thermal capacity of the insulated wall cannot be neglected. In this case, the heat input in the heater needs to compensate the heating of the insulated wall itself. Only when the steady regime is reached, when the temperatures anywhere in the enclosure remain constant, the heat input will equal the specified heat rate on the design surface. The procedure to solve this case was discussed in Section 3, and requires marching on time, since the temperature distribution on the insulated wall needs to be followed. It is considered that the initial temperature of the insulated wall is the same as that of the design surface, $t_I(\tau = 0) = t_J(\tau = 0) = 0.5$. Figure 6 presents the time dependent heat input distribution in the heater for the case in which thermal capacity of the insulated wall is 20 % of the design surface (that is, C_S = 0.2) for dimensionless times of τ = 0.0, 0.1, 0.2, 0.5 and 1.0. Comparing the heat input distributions with those where C_S = 0.0 (shown in Fig. 5), one can observe that for less advanced times, τ = 0.0, 0.1 and 0.2, the heat input distributions for C_S = 0.2 were considerably larger than that for C_S = 0.0, since the insulated wall was initially at a lower temperature and needs to be heated. For more advance times, τ = 0.5 and 1.0, the heat input distributions were approximately the same in the two cases, indicating that the insulated wall had already absorbed most of the needed energy to achieve the steady state condition.

A better insight of the effect of the thermal capacity of the insulated wall can be achieved by comparing the time-dependent total heat rate (THR) in the heater, given by integrating the heat flux distribution along its length, for different values of the thermal capacities of the insulated wall, as presented in Fig. 7. For the case in which $C_S = 0.0$, the heat rate is always equal to the heat rate in the design surface, since the insulated wall absorbs no energy, and so it increases in accordance with the heat rate in the design surface, given by integrating the prescribed heat flux of Eq. (4) along the design surface length. When the thermal capacity of the insulated wall is not negligible, the heat rate in the heater must compensate the heating of the insulated wall. As seen in the figure, for all values of C_S the initial heat load is always the same, since it is governed solely by the initial thermal condition in the enclosure, where both the insulated wall and the design surface have a dimensionless temperature of $t_j = t_l = 0.5$. For long times ($\tau \rightarrow \infty$) , the enclosure is expected to reach steady state, and so the total heat rate in the heat should equal the heat required in the design surface no matter the value of C_S. Figure 7 shows that the lowest the value of C_S the sooner the total heat rate in the heater becomes closer to that for $C_S = 0.0$. For large values of the thermal capacity of the insulated wall, $C_S \rightarrow \infty$, the heat load in the heater does not reach steady state with the design surface, since the insulated wall requires a much longer time to achieve steady state.

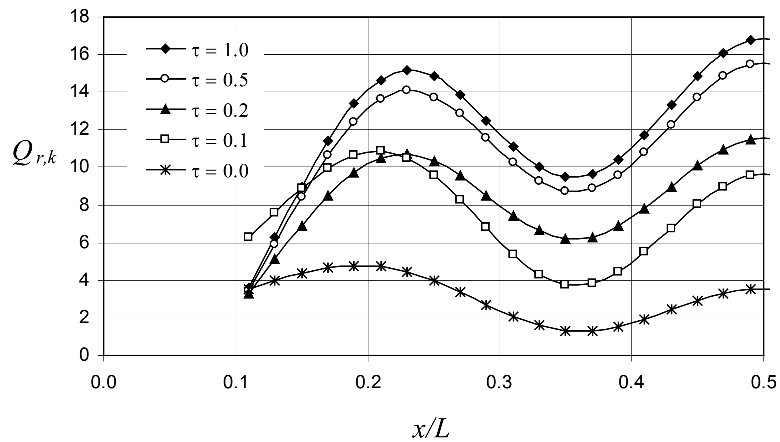

Figure 6. Heat flux distributions on the heater for different instants of time τ (LH = 0.8, $p = 6$, $C_S = 0.2$).

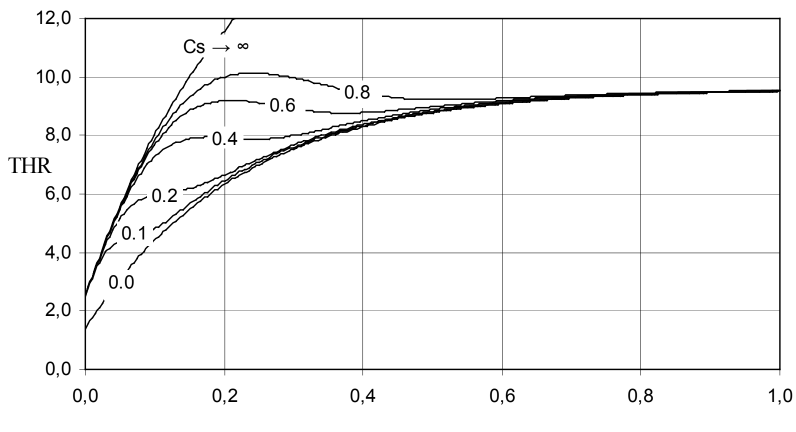

Figure 7. Total heat rate (THR) in the heater during the heating process for different values of C_S. (LH = 0.8, $p = 6$)

As a final remark, the present transient solution involved two types of discretization. The solution of the radiative energy balance involved the discretization of the spatial domain into surface zones with dimension of $\Delta x/L = 0.1$. The solution of the heating of the insulated wall required the discretization of the time in Eq. (14), taking $\Delta\tau = 0.001$. It was verified that, reducing $\Delta x/L$ and $\Delta\tau$ from the above values, no significant changes were observed on the solutions.

5. Conclusions

This work considered a transient inverse boundary design where the time-dependent heat input on the heater was determined to satisfy the specified temperature and heat flux time histories on the design surface. The numerical discretization of the problem led to an ill-conditioned system of equations as it is usual of an inverse design approach. Such systems cannot be solved by conventional matrix solvers, requiring regularization methods, such as the truncated singular valued decomposition (TSVD). The TSVD eliminates the linear combinations related to the smaller singular values of the system, keeping only the *p* largest ones. The regularization of the system inevitably introduces some error, but makes it possible to obtain useful and accurate solutions.

The proposed procedure set a system of equations relating the design surface elements directly to the heater elements, calculating the remaining terms, that is, the outgoing radiative heat flux on the insulated wall, from the conditions of the previous iterative step. Because the ill-conditioned part of the problem was isolated, it was necessary to apply regularization only to a reduced system of equations.

The examples treated the case where both the prescribed temperature and heat flux on the design surface were uniform, and should follow prescribed time distribution. The necessary heat input on the heater was found for different lengths of the heater and of the regularization parameter *p*. As a general rule, the smaller the regularization parameter *p*, the smoother is the inverse solution but the largest is the error. Therefore, every regularized must be verified to find whether the error is acceptable. In this work, the error was computed by comparing, for the design surface, the specified heat flux distribution and the heat flux resulting from the obtained heat input in the heater. As typical of inverse design, there were situations where no solution and where more than one solution existed for the problem.

Concerning the thermal behavior of the radiative enclosure, two cases were considered. The first one involved the ideal situation where the insulated wall thermal capacity was negligible, and therefore the heat input in the heat should always equal the total energy required on the design surface. In this case, the transient solution can be obtained by simply finding the steady state inverse design for the temperature and heat flux specified for every instant of time. There was no need to execute a marching solution on time. The other case took into account the more realistic situation in which the thermal capacity of the insulated wall not negligible, and the heat input in the heater should account for the heat absorbed by the insulated wall itself. Such a problem required that the temperature distribution on the insulated wall be followed for every instant of time, therefore involving a marching solution. In both cases, the presented inverse design methodology was successful to provide accurate, realistic solutions.

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