NATURAL CONVECTION FLOW IN A RECTANGULAR ENCLOSURE WITH AN INTERNAL CYLINDER

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Abstract. *A numerical study of a two-dimensional natural convection flow in a rectangular enclosure with an internal cylinder is accomplished. The finite volume method is used to discretize the flow governing equations and to determine the fluid heat transfer. The bottom surface of the cavity is adiabatic. The top surface exchanges heat with the environment by convection, while the others are kept at a cooling temperature T_C. As for the cylinder surface, it is kept at a heating temperature T_H. The Nusselt number around the cylinder is evaluated for several cases and compared with experimental results. The influence of Rayleigh number and cavity geometry in the flows is analyzed as well .*

Keywords*. Natural Convection, Finite Volume*

1. Introduction

Natural convection problems in rectangular enclosures with an internal cylinder have some applications in engineering in general. We can mention the heat exchangers in solar heating systems, set of tubes in closed cavities for cooling. Next, some relevant works found in literature are presented.

Farouk et al (1981) studied the laminar natural convection in an isothermal horizontal cylinder by using the finite difference method. The Nusselt number is determined for some Rayleigh numbers. The results are compared with both experimental and numerical results available in literature.

Farouk et al (1981) studied the turbulent natural convection in an isothermal horizontal cylinder using the finite difference method too. The k-ε model was used as the turbulence model. The results presented for Rayleigh numbers ranging from $5x10^7$ to 10^{10} and also for the average Nusselt number are compared with some correlations and with some works available in literature.

Cesini et al (1999) performed an experimental and numerical analysis on heat transfer by natural convection in a cylinder within a closed enclosure. The temperature distribution and the heat transfer coefficients are experimentally found and compared to numerical results obtained using a finite element method calculation. The formulation used was the Vorticity-Streamfunction one. The aspect ratio and Rayleigh numbers influence on the heat transfer is investigated.

Tasnim et al (2002) studied the thermal and hydrodynamic flow behavior in a square enclosure with an isothermal cylinder inside. Some simulations are carried out for the following aspect ratios: 2.0, 4.0, and 5.0. The effects of the cylinder eccentricity are taken into consideration. The numerical method adopted is the finite volume method and the equations are discretized using cylindrical coordinates. The SIMPLE method is used. The local Nusselt number is calculated on the cylinder perimeter. The natural convection results and the results for the mixed convection study are shown. The average Nusselt number is calculated for various Rayleigh numbers. Moreover, the temperature and streamfunction distributions are obtained around the cylinder.

Padilla(2000) made a numerical study of natural and mixed convection around a heated rotating cylinder. Two turbulence models are employed: the Smagorinsk sub-grid and the dynamic sub-grid models for the complex flow modeling. They presented the distributions of temperature, velocities, turbulent viscosity, and the local heat transfer coefficient.

The present work has as its goal the heat transfer study with natural convection in an internal cylinder within a closed enclosure. The Nusselt numbers are found around the cylinder and compared with both experimental and theoretical ones from the work by Cesini et al (1999), which considered the laminar flow and also compared with the theoretical ones found in Padilla (2000), which studied the turbulent flow. It is used the k-ω turbulence model.

2. Problem description

Figure (1) shows the flow geometry and the typical mesh used in the present work. A rectangular enclosure with height H and width L is considered. The internal cylinder has diameter D. The surfaces S_2 and S_4 are isothermal with temperature T_c. In addition, the surface S_5 is also isothermal with temperature T_H, while the surface S_3 is isolated and the surface S_1 exchanges heat with the environment by convection.

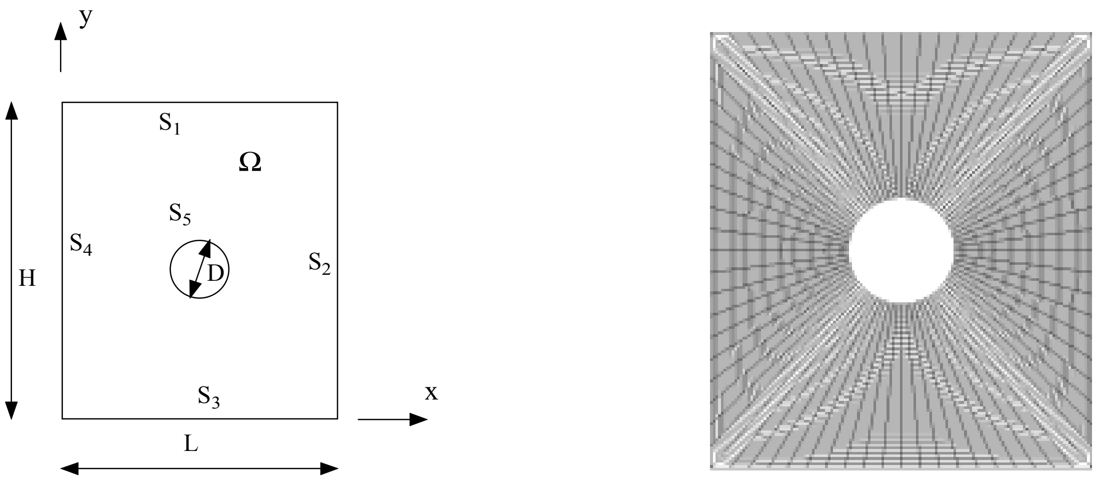


Figure 1. Geometry and typical grid of the present work

2.1. Problem hypotheses

The following hypotheses are considered in all cases studied here:

- unsteady regime;
- turbulent regime;
- bi-dimensional flow;
- incompressible flow;
- the fluid physical properties are constant, except the density in the buoyancy terms;

2.2. Conservation equations

Under the adopted hypotheses, the conservation equations are given by:

Continuity equation:

$$\frac{\partial u_i}{\partial x_i} = 0\,; \tag{1}$$

Momentum equations:

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_i} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + g_i \beta (T - T_{REF}) + \frac{\partial}{\partial x_i}\left((\nu + \nu_t) \frac{\partial u_j}{\partial x_i} \right); \tag{2}$$

Energy equation:

$$\frac{\partial T}{\partial t} + \frac{\partial (u_i T)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\left(\frac{\nu}{Pr} + \frac{\nu_t}{Pr_t} \right) \frac{\partial T}{\partial x_i} \right); \tag{3}$$

where u_i are the average velocities on x and y directions respectively, p is the pressure, t is the time, g_i are the gravity acceleration components, β is the volumetric expansion coefficient, ν is the fluid kinematic flow, ν_t is the fluid turbulent viscosity, T is the fluid temperature, T_{REF} is the reference temperature $T_{REF} = (T_H + T_C)/2$, ρ is the fluid density, and Pr_t is the turbulent Prandtl number.

2.3. k-ω turbulence model

In the present work, the k-ω turbulence model which was developed by Wilcox (1994) is used. It can be highlighted here the works by Peng (1999) and Bredberg (2000) which used this same model. These model equations are presented as following:

Turbulent kinematic energy equation:

$$\frac{\partial k}{\partial t}+\frac{\partial(u_i k)}{\partial x_i}=\frac{\tau_{ij}}{\rho}\frac{\partial u_i}{\partial x_j}-\beta^* k\omega+\frac{\partial}{\partial x_i}\left[\left(\nu+\sigma^*\nu_t\right)\frac{\partial k}{\partial x_i}\right]. \tag{4}$$

Specific dissipation rate:

$$\frac{\partial \omega}{\partial t}+\frac{\partial(u_i \omega)}{\partial x_i}=\alpha\frac{\omega}{k}\frac{\tau_{ij}}{\rho}\frac{\partial u_i}{\partial x_j}-\beta\omega^2+\frac{\partial}{\partial x_i}\left[\left(\nu+\sigma^*\nu_t\right)\frac{\partial \omega}{\partial x_i}\right]. \tag{5}$$

Turbulent viscosity:

$$\nu_t=\frac{k}{\omega}. \tag{6}$$

In the above equations, k is the turbulent kinematic energy, ω is the dissipation rate; and the empirical constants are:

$$\alpha=5/9;\qquad \beta=3/40;\qquad \beta^*=9/100;\qquad \sigma=1/2;\qquad \sigma^*=1/2; \tag{7}$$

The specific dissipation is given by:

$$\varepsilon=\beta^*\omega k. \tag{8}$$

The classical wall laws associated with the k-ω turbulence model to simulate the flow in the region within the turbulent boundary layer were used. The k-ω model and the wall laws are employed in the turbulent region and in the regions near the wall, respectively.

The Prandtl number, the turbulent Prandtl number are defined, respectively, by:

$$\mathrm{Pr}=\frac{\nu}{\overline{\alpha}}, \tag{9}$$

$$\mathrm{Pr}_t=\frac{\nu_t}{\overline{\alpha}_t}, \tag{10}$$

2.4. Initial and boundary conditions

The initial and boundary conditions imposed, according to fig. (1), are:

$$\text{In } \Omega: \ u(x,y,0)=0,\ v(x,y,0)=0,\ T(x,y,0)=0; \tag{11}$$

$$\text{on } S_1: \ \frac{\partial T}{\partial y}=h\left(T-T_A\right); \tag{12}$$

$$\text{on } S_2: T=T_C; \tag{13}$$

$$\text{on } S_3: \ \frac{\partial T}{\partial y}=0; \tag{14}$$

$$\text{on } S_4: \ T=T_C; \tag{15}$$

$$\text{on } S_5: T=T_H. \tag{16}$$

3. Solution method

The conservation equations are discretized using the finite volume method. The mesh arrangement employed was the colocalized one where the variable values are placed at the control volume centers. In this arrangement, the fluxes on the surfaces are determined by the velocity interpolation at the centers of the volumes. The hybrid scheme is utilized in the convective fluxes and the central difference scheme in the diffusive fluxes. The non-orthogonal Cartesian mesh is used. The details of the surface flux calculations using appropriate interpolation, can be seen in the works by Ferziger et al (1997) and Tasnim et al (2002). The SIMPLE and TDMA methods are employed when coupling pressure-velocity and solving the system of equations, respectively.

The local Nusselt number is calculated based on the temperature gradients on the surface normal direction and is defined as:

$$Nu = \frac{D}{T_H - T_C}\sqrt{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2} \tag{17}$$

The average Nusselt number é given by:

$$\overline{Nu} = \frac{1}{\pi D}\int_0^{\pi D} Nu\, dS\,. \tag{18}$$

4. Validation

A study of natural convection in a rectangular enclosure with an internal cylinder is carried out to validate the computational code that was developed in FORTRAN using the finite volume method. First, a study of the mesh is done by taking into consideration the computational cost. Therefore, a mesh with 4800 volumes is chosen. The initial and boundary conditions, presented in topic 2.4, are the same as the ones used in the work of Cesini et al (1999).

The cavity dimensions are: the cavity height H=57[mm], and the cavity width L=30; 40 and 50 [mm]. The internal cylinder has a diameter D = 14 [mm]. The cylinder surface is kept at temperature T_H = 50 [ºC] while surfaces S_2 and S_4 are at temperature T_C = 10 [ºC]. The surface S_3 is adiabatic. The surface S_1 is subjected to a prescribed heat flux which is exchanged with the environment whose temperature is T_A=20 [ºC] and heat transfer coefficient given as h = 10 [Wm^2K^{-1}]. These parameters are taken to be the same for all cases studied throughout this work.

In the following, comparisons with the results from the work of Cesini et al (1999) are carried out by considering the regime to be laminar. The aspect ratio is given by W=L/D and it assumes the values W=2,1; 2,9, and 3,6. Furthermore, the Rayleigh numbers are ranged from Ra=$1,3x10^3$ to $7,5\ x10^4$.

Table (1) shows the average Nusselt number for the aspect ratios and Rayleigh numbers mentioned previously. It is verified a good agreement with the results found in Cesini et al (1999). A maximum deviation of 8,14% from the theoretical results is achieved.

Table 1 – Average Nusselt number ($\overline{Nu}$) on the cylinder surface.

	W = 2,1			W = 2,9			W = 3,6		
	Teor.*	Exp.**	Calc. †	Teor.*	Exp.**	Calc†	Teor*	Exp**	Calc. †
Ra=$1,3x10^3$	2,36	2,46	2,34	2,25	2,54	2,31	2,35	2,35	2,57
Ra=$2,4x10^3$	2,61	2,80	2,53	2,65	3,00	2,68	2,75	2,79	2,87
Ra=$3,4x10^3$	2,77	3,07	2,67	2,90	3,15	2,91	2,98	3,06	3,06
Ra=$5,0x10^3$	2,99	-	2,89	3,22	-	3,19	3,25	-	3,27
Ra=$1,0x10^4$	3,52	-	3,47	3,80	-	3,70	3,74	-	3,64
Ra=$2,0x10^4$	4,27	-	4,27	4,42	-	4,22	4,29	-	4,10
Ra=$3,0x10^4$	4,78	-	4,78	4,80	-	4,53	4,67	-	4,41
Ra=$4,0x10^4$	5,17	-	5,14	5,09	-	4,78	4,99	-	4,67
Ra=$5,0x10^4$	5,47	-	5,43	5,33	-	4,98	5,25	-	4,88
Ra=$7,5x10^4$	6,05	-	5,93	5,82	-	5,38	5,77	-	5,30

* Theoretical results from Cesini et al(1999).
** Experimental results from Cesini et al(1999).
† Results from present work.

For validation of the turbulence model, a natural convection study in a rectangular enclosure with an internal cylinder is carried out. The conditions from Cesini et al (1999), which are mentioned previously, are used again. The average Nusselt number on the cylinder surface is calculated and compared to the theoretical results found in Padilla (2000). Figure (2) shows the average Nusselt number versus the Rayleigh number for the aspect ratio W=3,6. The Rayleigh numbers considered are: Ra=$1,0x10^3$ a $1,0x10^8$. It is observed a good agreement with the values found in the work of Padilla (2000).

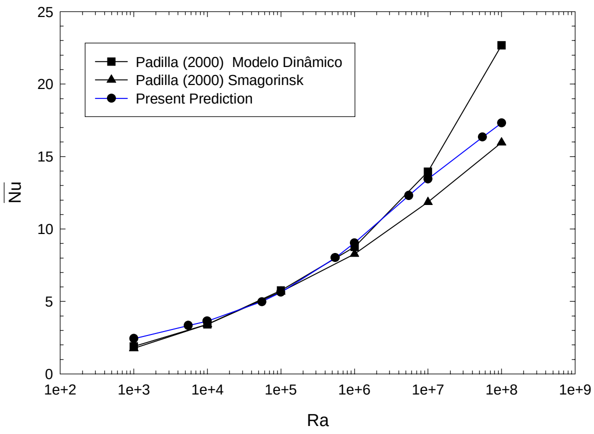


Figure 2. Average Nusselt number versus Rayleigh number.

5. Results

The results of a laminar natural convection study in an enclosure with an internal cylinder are presented. Fig. (3) shows the behavior of the local Nusselt number on the cylinder perimeter. Three aspect ratios are considered: W=2,1; 2,9 e 3,6. The Rayleigh number is Ra = $3,4\text{x}10^3$.

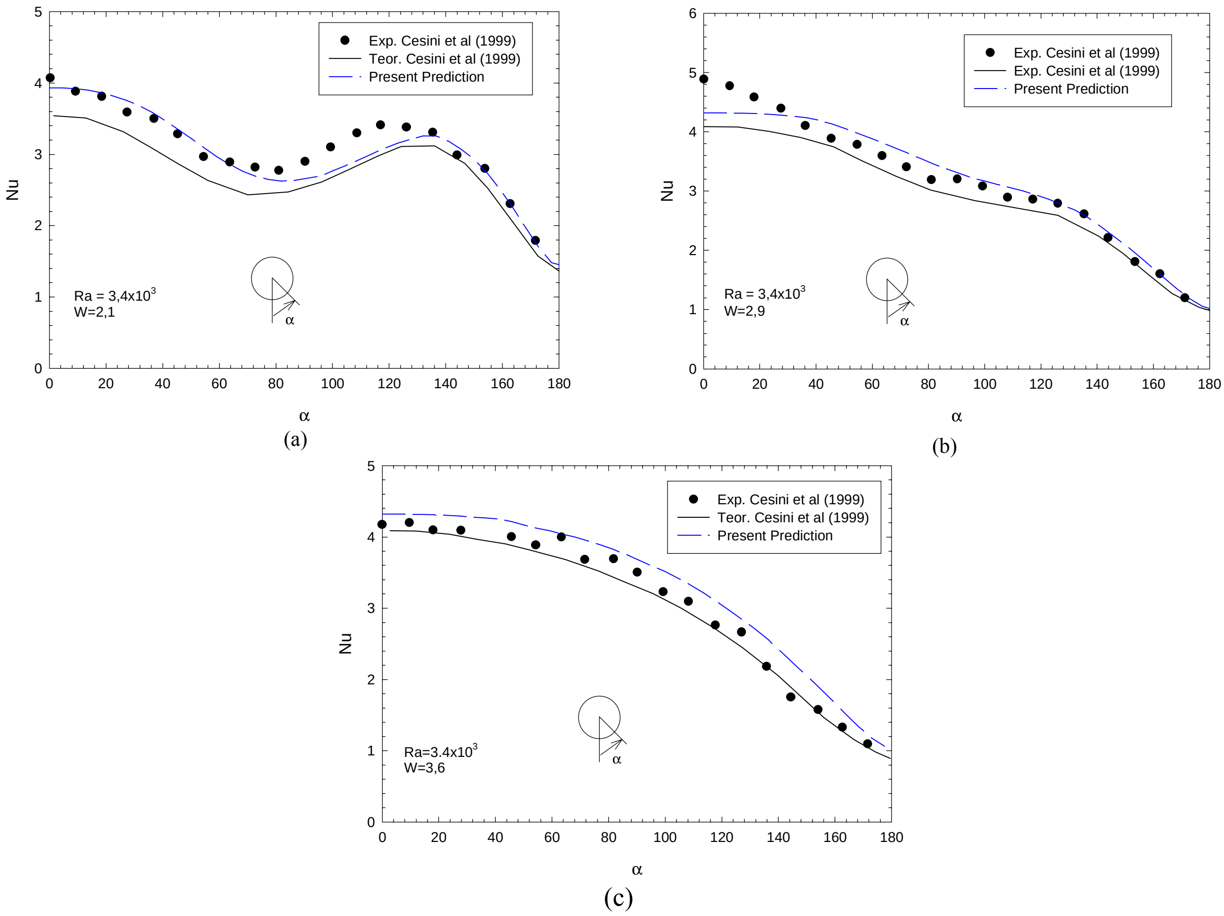
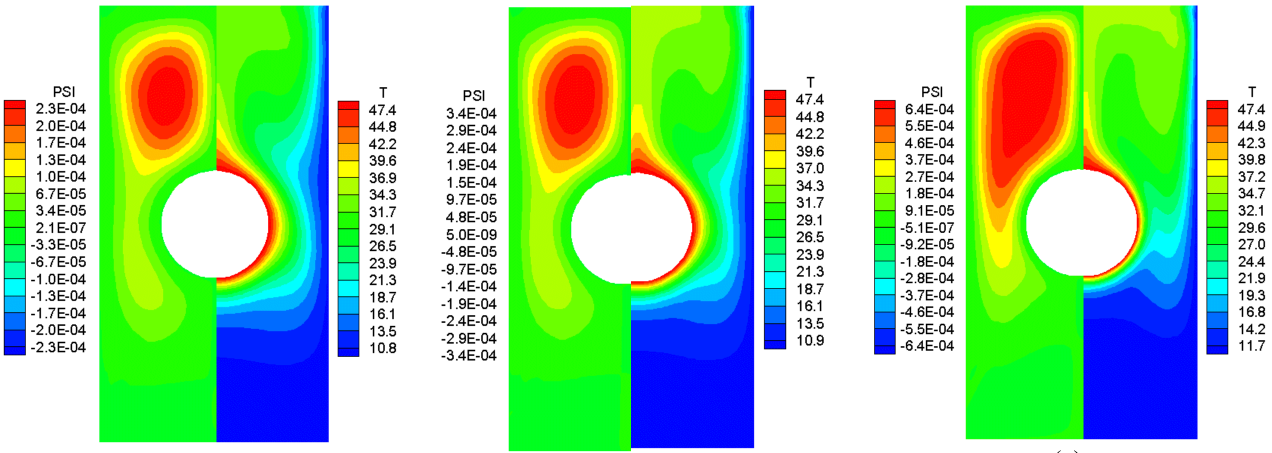


Figure 3. Local Nusselt number (Nu) versus angle α.

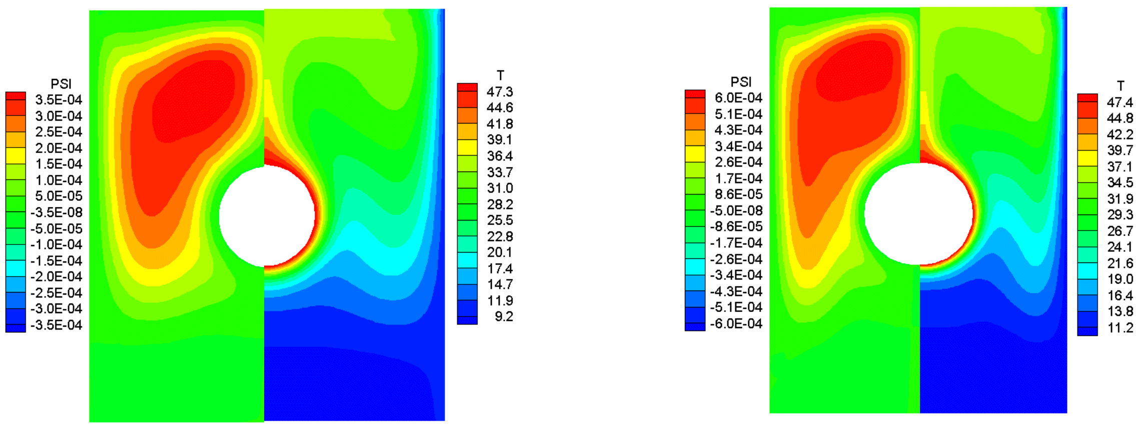
Figure (4) presents the streamlines PSI and also the temperature distributions for Rayleigh numbers Ra=$1,3\text{x}10^3$; Ra = $5,0\text{x}10^3$ e Ra = $7,5\text{x}10^4$ considering the aspect ratio W=2,1. It is verified that as the Rayleigh number increases, the gradients on the cylinder surface also do it too, hence bigger heat exchanges are present. A recirculation cell appears in the upper part of the enclosure.



(a) (b) (c)

Figure 4. Streamfunction PSI and Temperature T[ºC] for W=2,1. (a) Ra = $1,3x10^3$ (b) Ra = $5,0x10^3$ (c) Ra = $7,5x10^4$.

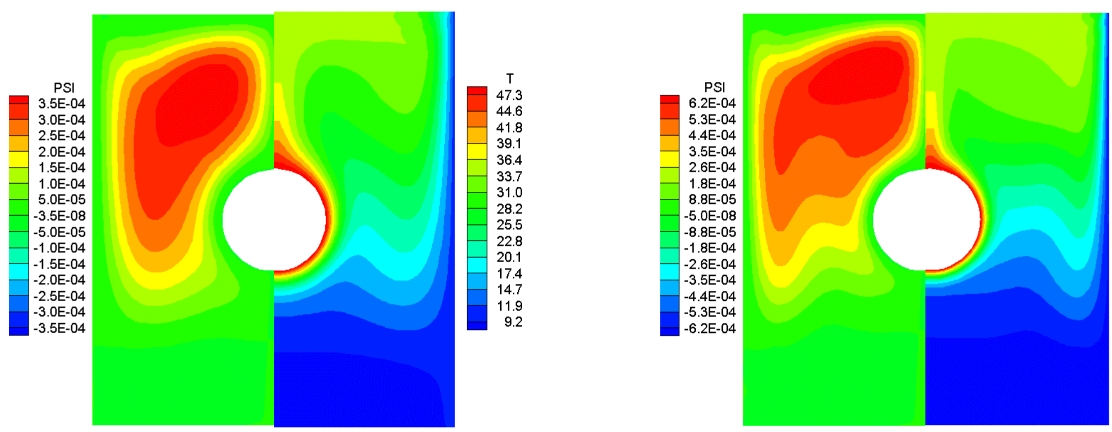
Figure (5) shows the streamfunction PSI together with the temperature distributions for Rayleigh numbers Ra=$1,0x10^4$; Ra = $7,5x10^4$ and aspect ratio W = 2,9. It is also noted that when the Rayleigh number increases, the gradients on the cylinder surface have the same behavior, that is, they increase too. Again, a larger recirculation cell is formed in the upper surface of the enclosure.



(a) (b)

Figure 5. Streamfunction PSI and Temperature T[ºC] for W=2,9. (a) Ra = $1,0x10^4$ (b) Ra = $7,5x10^4$

Figure (6) depicts the streamline PSI and the temperature distributions for Rayleigh numbers Ra = $1,0x10^4$; Ra = $7,5x10^4$ for an aspect ratio W = 3,6. More intense temperature diffusion is present in the upper part of the enclosure. Again, a bigger recirculation cell is formed in the superior wall occupying almost the entire part.



(a) (b)

Figure 6. Streamfunction PSI and Temperature T[ºC] para W=3,6 (a) Ra = $1,0x10^4$ (b) Ra = $7,5x10^4$

Next, the results for a natural convection study in an enclosure with an internal cylinder for both turbulent and natural regimes are given.

Table (2) has the results for the average Nusselt number on the cylinder surface. The Rayleigh numbers considered here are: Ra $1{,}0\text{x}10^3$ a $1{,}0\text{x}10^8$ and aspect ratio W=3,6.

Table 2 – Average Nusselt number ($\overline{\text{Nu}}$) on the cylinder surface.

	Present Prediction
Ra=$1{,}0\text{x}10^3$	2,42
Ra=$5{,}5\text{x}10^3$	3,32
Ra=$1{,}0\text{x}10^4$	3,65
Ra=$5{,}5\text{x}10^4$	4,98
Ra=$1{,}0\text{x}10^5$	5,64
Ra=$5{,}5\text{x}10^5$	8,02
Ra=$1{,}0\text{x}10^6$	9,04
Ra=$5{,}5\text{x}10^6$	12,30
Ra=$1{,}0\text{x}10^7$	13,44
Ra=$7{,}5\text{x}10^7$	16,34
Ra=$1{,}0\text{x}10^8$	17,31

Figure (7) presents the behavior of the local Nusselt number (Nu) around the cylinder considering the aspect ratio W= 3,6. The Rayleigh numbers considered are: Ra = $1{,}0\text{x}10^3$; $1{,}0\text{x}10^5$; $1{,}0\text{x}10^6$; $1{,}0\text{x}10^7$, and $1{,}0\text{x}10^8$. It is observed that when the Rayleigh number is increased, the local Nusselt number becomes higher in terms of numerical values and unstable on the cylinder surface. Moreover, in the cylinder position around 90º, that is, on the upper part of it, there are lower temperature gradients and, hence, lower local Nusselt numbers. It can be observed that for Rayleigh numbers Ra = $1{,}0\text{x}10^8$, which means a moderate turbulent flow, the local Nusselt number shows more intense instabilities describing a typical behavior of turbulent regimes.

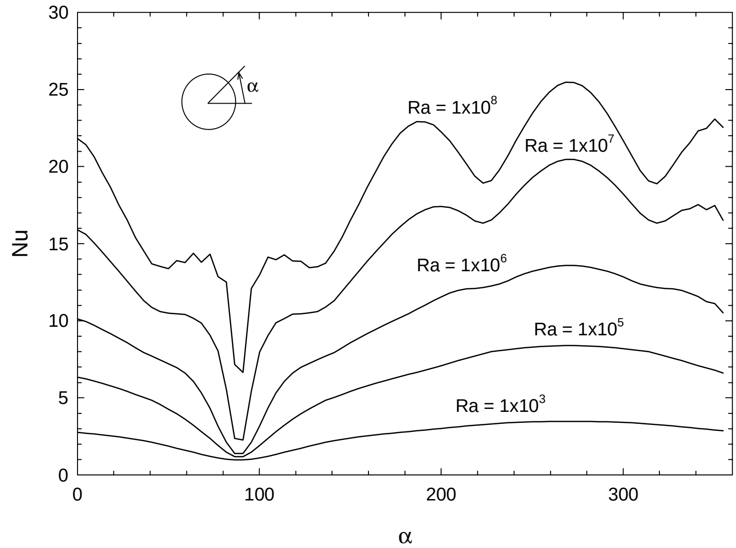
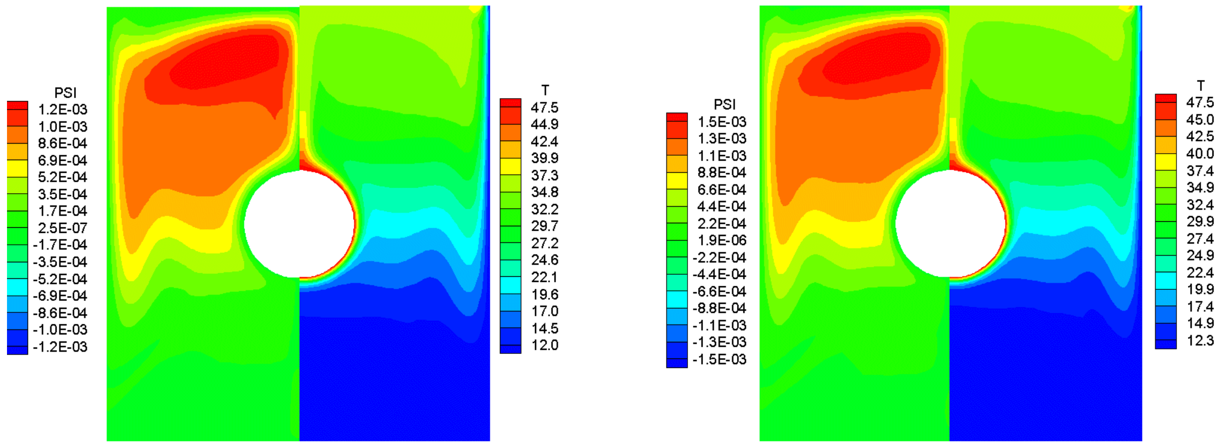


Figure 7. Local Nusselt number Nu versus angle α for Ra = $1{,}0\text{x}10^3$ to $1{,}0\text{x}10^8$ and aspect ratio W=3,6

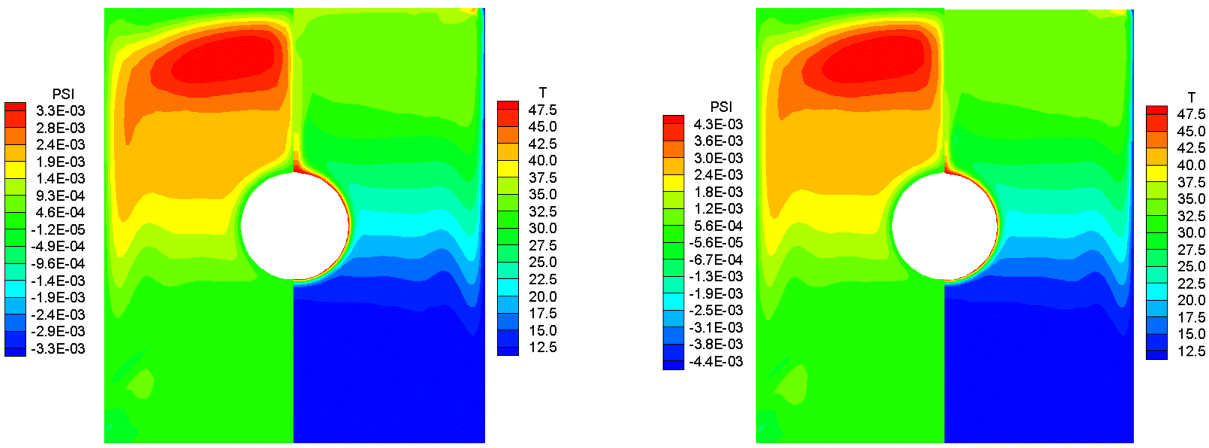
Figure (8) depicts the streamline PSI and the temperature behavior for Rayleigh numbers Ra = $5{,}5\text{x}10^5$ and Ra= $1{,}0\text{x}10^6$ and aspect ratio W = 3,6. The temperature distribution becomes more stratified on the horizontal direction of the enclosure. It is seen the formation of a great recirculation cell on the upper surface of the enclosure.



(a) (b)

Figure 8. Streamfunction PSI and Temperature T[ºC] for W=3,6 (a) Ra = $5{,}5\times10^{5}$ (b) Ra = $1{,}0\times10^{6}$

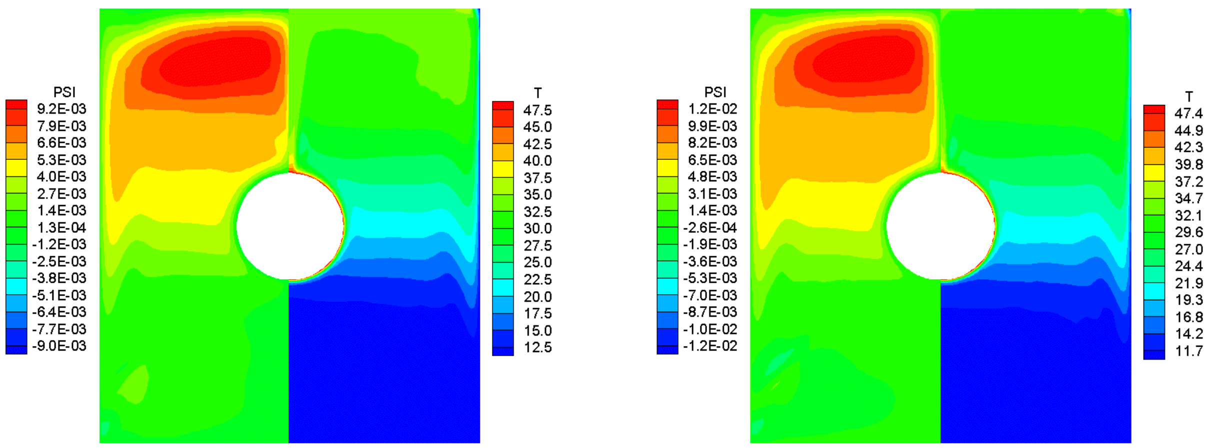
Figure (9) shows the streamline PSI and the temperature distributions for Rayleigh numbers Ra = $5{,}5\times10^{6}$ e Ra=$1{,}0\times10^{7}$ with aspect ratio W = 3,6. It is also verified the development of a great recirculation cell on the upper wall and smaller cells on the bottom wall.



(a) (b)

Figure 9. Streamfunction PSI and Temperature T[ºC] for W=3,6 (a) Ra = $5{,}5\times10^{6}$ (b) Ra = $1{,}0\times10^{7}$

Figure (10) gives the streamline PSI and the temperature distributions for Rayleigh numbers: Ra = $5{,}5\times10^{7}$ e Ra=$1{,}0\times10^{8}$ with aspect ratio W = 3,6. As expected, the formation of a great recirculation cell is observed on the superior surface and smaller ones on the inferior surface.



(a) (b)

Figure 10. Streamfunction PSI and temperature T[ºC] for W=3,6 (a) Ra = $5{,}5\times10^{7}$ (b) Ra = $1{,}0\times10^{8}$

6. Conclusions and comments

In this work, the study of a laminar and turbulent natural convection in a rectangular enclosure with an internal isothermal cylinder is carried out with the use of the k-ω turbulence model.

In the validation test, considering the laminar regime, it is determined the average Nusselt number on the cylinder surface which successfully agree with the results from Cesini et al (1999). Moreover, in the second validation test, considering now the laminar and turbulent regime, it is also determined the average Nusselt number on the cylinder surface, with a good agreement with the results presented by Padilla (2000).

It is noted that the Rayleigh number increases with the increase of the Nusselt number. One can refer to Fig. (2) and table (2) to check this behavior. This behavior is more intense as the turbulence becomes stronger.

For the k-ω turbulence model, the results seemed to be as good as the ones obtained with the Smagorinsk sub-grid model. One of the advantages of the k-ω turbulence model is that the processing time is smaller than the one achieved with sub-grid models.

The streamline and temperature distributions are presented for a range of Rayleigh numbers. It is certified that the wake formed in the flow becomes thinner as the Rayleigh number increases.

7. Acknowledgements

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8. References

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